Reducing the out of Band Radiation of OFDM Using an Extended Guard Interval

A. D. S. Jayalath and C. Tellambura
SCSSE, Faculty of Information Technology
Monash University, Clayton Victoria 3168, Australia
Email: chintha@dgs.monash.edu.au

Abstract—**In this paper we investigate the spectrum, out of band radiation (OBR) and the use of extended guard interval (EGI) to reduce the out of band radiation of an OFDM signal when passing through different nonlinear devices. Spectra of the OFDM signal with different EGI lengths after passing through nonlinear devices are obtained through computer simulations. Mathematical expressions for the power spectral density of a conventional OFDM signal and an OFDM signal with different EGI lengths are also presented. Theoretical and simulated results are compared. Although the EGI reduces the OBR, this reduction is not significant in some cases. The effect of EGI on the OBR of an OFDM signal differs for different nonlinear devices. The EGI is also not capable of reducing the OBR caused by excessive clipping of the OFDM signal. Finally, it is also found that the length of the EGI does not have a considerable effect on the amount of OBR reduced when passing through a nonlinear device.**
Index terms: OFDM, Out of band radiation, Non-linear devices.

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is an attractive technique for the transmission of high-bit-rate data in a radio environment [1, 2]. It has recently gained great popularity due to its robustness in mitigating impairments such as inter symbol interference (ISI) due to multipath fading and impulsive noise in such systems. If a guard interval (GI) greater than length of the channel impulse response is chosen, the ISI is eliminated entirely. Another advantage of OFDM is its simple realization by means of efficient fast Fourier transform (FFT) algorithms. OFDM is already being investigated for new wireless standards and current OFDM applications include digital audio broadcasting, asynchronous digital subscriber lines, digital video broad-casting and wireless LANs. A recent addition to this list of applications is the IEEE 802.11 standard. However, any multicarrier signal with a large number of subchannels is burdened with a large peak-to-average power ratio (PAR). When passed through a nonlinear device, such as a transmitter power amplifier, the signal suffers significant power spectral spreading and in-band distortion. One solutions to this problem is to use a highly linear amplifier with a sufficient input backoff (IBO), resulting in significant power efficiency penalty. Very high backoff demanding by OFDM signals are a major disadvantage where peak signal power levels are constrained by design factors such as battery power in portable equipment. Another alternative to overcome this problem is to reduce the PAR of the OFDM signal before it is passing through the nonlinear device (amplifier).

Several alternative solutions have been proposed to reduce the PAR of an OFDM signal. Since high peaks occur with very low probability, the high peaks may be clipped when they occur [3]. However, this deliberate clipping of the signal causes significant distortion and increases the OBR and the BER. A model to analyze the effect of clipping of discrete multitone signal is presented in [4]. Model provides information on the reduction of the signal level, the total noise power due to clipping and spectral properties of the noise. Filtering after clipping can reduce the spectral splatter but may cause some peak re-growth. Clipping and filtering on the performance of OFDM has been investigated [5]. There are techniques capable of achieving significant PAR reductions in the expense of increased system complexity. Even with reduced PAR, OFDM signal needs significantly high IBO at the transmitter power amplifier. Therefore it is important to look at the spectrum of the OFDM signal, distortion caused by different nonlinear devices and the suitable means to reduce the high OBR.

Conventional OFDM signal itself possesses a high OBR due to sharp variations at the OFDM signal boundaries. This level of OBR increases when the OFDM signal passes through a nonlinear device. Studies of the spectrum and the OBR of OFDM signals can be found in the literature, with several OBR reduction methods. Symbol envelope shaping to reduce the OBR is the most popular scheme, which is used in magic WAND project [6] under the European union's advanced communications technologies and services (ACTS). Oversampled inverse discrete Fourier transform (IDFT) output with a guard prefix is multiplied by a root-raised-cosine pulse-shaping window to achieve the expected performance. Although the reduction in spectrum splatter or OBR is achieved by this method, an increase in the bit error rate (BER) may occur due to shaping of the wanted signal.

An envelope-shaping and windowing schemes are proposed in [7] and [8]. In [7], signal envelope is multiplied with a weighting function composed of sum of Gaussian pulses. Different windowing functions are used for the same purpose in [8]. Both these techniques are used to remove the sharp transitions in the signal. The method of introducing EGI before windowing is analytically described in [7]. Instead of shaping the wanted signal, two extended portions are introduced at both ends of the OFDM signal and shaped using a raised cosine window. Effect of the use of EGI on the OBR of an OFDM signal is also analysed. Raised cosine windowing function is used. It is shown that the use of an EGI would not have any effect on the BER performance but reduces OBR significantly.

An another analytical result is presented in [9], where an expression is obtained for the spectrum of the OFDM signal after passing through a saturated high-power amplifier. Effect of hardware nonlinearities on OFDM transmissions is presented in many papers [10–12]. None of these papers considered common nonlinear devices into account and compared the spectrum and the OBR. No papers can be found which gives a proper account of the effect of EGI in the presence of common non-linearities.

This paper investigates the effect of EGI in the presence of different non-linearities. Next section derives the power spectral density of an OFDM signal with an EGI. Section 3 describes the different nonlinear devices used in the simulations. Section 4 presents the results. Finally, Section 5 concludes the paper.

II. System Description

In OFDM, a block of N symbols, $X_n, n = 0, 1, ..., N-1$, is formed with each symbol modulating one of a set of N subcarriers, $f_n, n = 0, 1, ..., N-1$. The N subcarriers are chosen to be orthogonal, that is $f_n = n\Delta f$, where and T is the original symbol period. The resulting signal can be expressed as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi(f_n)t}, 0 \leq t \leq T. \qquad (1)$$

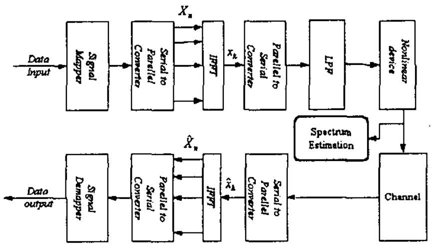

Fig. 1. System block diagram.

A cyclic prefix (called guard interval) is added to the resulting signal in order to avoid the intersymbol interference (ISI) which occur in multipath channels. At the receiver the guard interval is removed and only the time interval $[0, T]$ is evaluated. The guard interval usually a periodic extension of the symbol over the interval $[-T_{CP}, 0]$, resulting in a symbol of length $[-T_{CP}, T]$.

These time-domain samples in the equivalent complex valued low-pass domain are approximately Gaussian distributed, because of the statistical independence of carriers. Resulting high PAR is given by

$$\text{PAR} = \frac{\max|x(t)|^2}{E[|x(t)|^2]}. \tag{2}$$

where $E\{.\}$ denotes expectation. This does not depend on the signal set X_n used to modulate the signal. Theoretical maximum PAR occurs when the data is mapped to same signal point. The theoretical maximum of the PAR for N number of sub-carriers is $10\log(N)$dB. For the system investigated in this paper, the signal is oversampled by a factor of 8 and an extended guard interval (EGI) is introduced. Now, the time interval T_S consists of a following time segments.

$$T_S = T + T_{CP} + T_g \tag{3}$$

where T, T_{CP} and T_g are the data symbol duration, cyclic prefix and extended guard interval respectively.

This signal is then multiplied by a raised cosine pulse shaping window, having a unity gain during the interval $[-T_{CP}, T]$ and roll-off at both extended guard intervals. Finally the OFDM signal is overlapped in an interval of $T_g/2$. Power spectral density (PSD) of signal is then estimated after passing through a non linear device, using Welch's averaged periodogram method with Hanning window. The power spectral density of each OFDM block is measured and then averaged over 10^4 blocks. System block diagram is shown in Fig. 1. A theoretical expression for the PSD of an OFDM signal with an EGI is derived in the next section.

III. Power Spectral Density of an OFDM Signal

Consider a block of data defined as $\{X_n, n = 0, 1, \dots, N-1\}$ at the time interval k. The OFDM signal transmitted is then expressed as

$$x(t) = \sum_{k=-\infty}^{\infty} w(t-kT) \sum_{n=0}^{N-1} X_{n,k} e^{j2\pi n\Delta f(t-kT)} \tag{4}$$

where $w(t)$ is the windowing function. For an ordinary OFDM signal windowing function is rectangular. Now, consider the autocorrelation function $R(\lambda)$ of $x(t)$, given by

$$\begin{aligned} R(\lambda) &= E\{x(t+\lambda)x(t)^*\} \\ &= \left\{ \sum_{k=-\infty}^{\infty} w(t+\lambda-kT) w^*(t-kT) \right\} \sum_{n=0}^{N-1} e^{j2\pi n\Delta f\lambda} \end{aligned} \tag{5}$$

The Fourier transform of the auto correlation function is equal to the power spectral density. This can be expressed as

$$S(f) = \int \left\{ \bar{R}(\lambda) \sum_{n=0}^{N-1} e^{j2\pi n\Delta f\lambda} \right\} e^{j2\pi f\lambda} d\lambda \tag{6}$$

where

$$\bar{R}(\lambda) = \frac{1}{T} \int_0^T w(t+\lambda) w^*(t) dt. \tag{7}$$

We have

$$S(f) = \frac{1}{T} \sum_{n=0}^{N-1} W^2 [(f - n\Delta f)T] \tag{8}$$

where W is the Fourier transform of the windowing function. The spectrum of an ordinary OFDM signal with a rectangular window can be expressed as

$$S(f) = \frac{1}{T} \sum_{n=0}^{N-1} \{\text{sinc}\,[(f - n\Delta f)T]\}^2 \tag{9}$$

The raised cosine window function used to shape the OFDM signal can be expressed as follows

$$w(t) = \begin{cases} 0.5(1 + \cos\left\{\pi(1 + \frac{t}{\beta T_S})\right\}) & 0 \le t \le \beta T_S \\ 1 & \beta T_S \le t \le T_S \\ 0.5(1 + \cos\left\{\pi\frac{(t-T_S)}{\beta T_S}\right\}) & T_S \le t \le (1+\beta)T_S \end{cases} \tag{10}$$

where $\beta = T_g/(2T_S)$ is the roll off factor. The spectrum of an OFDM signal with both GI and EGI multiplied by a raised cosine windowing function is given by

$$S(f) = \frac{1}{T_S} \sum_{n=0}^{N-1} \left\{ \text{sinc}\,[(f - n\Delta f)T_S] \frac{\cos[\pi\beta(f - n\Delta f)T_S]}{1 - 4\beta^2[(f - n\Delta f)T_S]^2} \right\}^2 \tag{11}$$

In this case the OBR decreases more rapidly with the role off factor. However, increase in the EGI will reduce the efficiency [7] of the OFDM system which is defined as

$$\eta = \frac{T}{T_S} \tag{12}$$

where T is the useful symbol time and T_S is the total symbol time. Therefore it is a compromise between expected OBR reduction and efficiency. Simulations are performed for EGI lenghts of 3%, 6% 12% and 24%, which corresponds to efficiencies of 91%, 89%, 84% and 76%.

For PSD results, it is convenient to define the normalized bandwidth B_n

$$B_n = \frac{f}{N(\Delta f)} = fT \tag{13}$$

IV. Description of Memoryless Nonlinearity

Examples of non-linearities are signal clipping in the analog to digital converter (A/D) converter, signal clipping in the IFFT and FFT processors with a limited word length, AM/AM and AM/PM distortion in the radio frequency (RF) amplifiers. Here, AM and PM stand for amplitude modulation and phase modulation respectively. The effect of hardware non-linearities has been studied in [13, 14]. Out of band power of OFDM signals increase, when amplified with nonlinear power amplifiers operating at lower back-offs. High PAR of OFDM require for high back-offs at the amplifiers. The non-linearity present in the amplifier can be expressed by its AM/AM component $F[\rho]$ and AM/PM component $\Phi[\rho]$ [15], where ρ is the magnitude of the signal. Common non-linear devices found in practice are as follows.

A. Soft limiter (SL)

The AM/AM and AM/PM nonlinear characteristics of a soft limiter (SL) can be written as

$$F[\rho] = \begin{cases} \rho & \rho \leq A, \\ A & \rho > A. \end{cases} \tag{14}$$

Since the AM/PM component is zero the nonlinear characteristics of a SL can be written as

$$g(x) = \begin{cases} x & |\rho| \leq A, \\ Ae^{j\varphi} & |\rho| > A. \end{cases} \tag{15}$$

Although most physical components will not exhibit this piecewise linear behavior, the SL can be a good model if the nonlinear element is linearized by a suitable predistortor.

B. Solid-state power amplifiers (SSPA)

The input out relationship of many solid-state power amplifiers (SSPA) can be modelled as

$$\begin{aligned} F[\rho] &= \frac{\rho}{[1 + (\frac{\rho}{A})^{2P}]^{\frac{1}{2P}}} \\ \Phi[\rho] &= 0. \end{aligned} \tag{16}$$

where the parameter P controls the smoothness of the transition from the linear region to the limiting or saturation region. When, $P \to \infty$ the SSPA model approximates the SL characteristics.

C. Travelling-wave tube (TWT)

The AM/AM and AM/PM characteristics for a TWT, according to [16] are,

$$\begin{aligned} F[\rho] &= \frac{\rho}{1 + (\frac{\rho}{2A})^2} \\ \Phi[\rho] &= \frac{\pi}{3}\frac{\rho}{\rho^2 + 4A^2}. \end{aligned} \tag{17}$$

All these models give a maximum output signal of A. Input back off (IBO) used in these nonlinear devices can be defined in terms of A

$$\text{IBO} = 10\log_{10}\left\{\frac{A^2}{E|x(t)|2}\right\}. \tag{18}$$

V. Results

A 512 carrier OFDM system with quadrature phase shift keying (QPSK) and a GI of 32 is used in the simulations. The OFDM signal is passed through nonlinear devices described in Section 4. Both theoretical and simulated spectra of a conventional OFDM signal with 16, 64, 256 and 512 carriers are depicted in Fig. 2. Continues lines represent the simulated results while discontinues lines represent the theoretical results given by 9. No significant difference is observed between theoretical and simulated results. Both theoretical and simulated results show low OBR for larger N and the spectrum decreases more rapidly in the beginning, which is caused by the fact that the sidelobes are closer together. For 16, 64 256 and 512 subcarriers, the OBR at $4B_n$ are, -34dB, -40dB, -45dB and -50dB respectively. However, relatively large OBR is observed even for 512 subcarriers. The amount of OBR tends to increase further, if the OFDM signal passes through a nonlinear device. Next, the effect of EGI on the OBR of an OFDM signal is observed.

Theoretical and simulated spectra of 512 carrier OFDM signal with EGI of 3%, 6%, 12% and 24% of the symbol duration, multiplied with a raised cosine window are depicted in Fig. 3. Despite

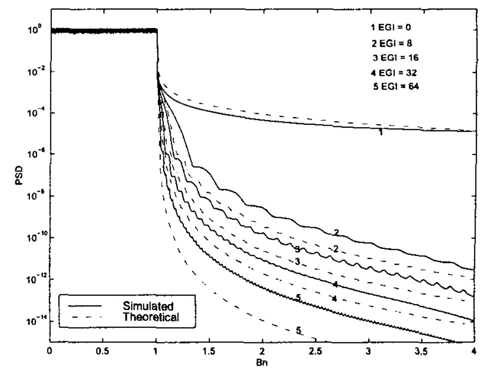

Fig. 2. Power spectral density of an OFDM signal with different number of carrier

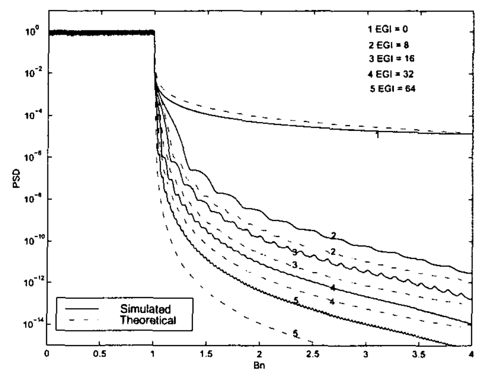

Fig. 3. Power spectral density of an OFDM signal with an extended guard interval.

marginal differences, both between theoretical and simulated results agree for different lengths of EGI. The OBR of an ordinary OFDM signal reduces with the EGI. For EGI of 3%, 6%, 12% and 24% the OBR reductions at $4B_n$ are 43dB, 56dB, 68dB and 80dB respectively.

Next, the effect of EGI on the OBR of an OFDM signal in the presence of nonlinear devices is observed through simulations. Fig. 4 depicts the PSD for a SL with different IBOs. Curve 2 represents the OBR without an EGI. All other curves (1,3,4,5) represent the PSD for with an EGI of 6%. Simulations results were obtained for IBO values of 5dB, 9dB and 11dB. OBR is not reduced even after the introduction of EGI, when the IBO is 5dB. Therefore, the OBR caused by an SL due to insufficient IBO can not be reduced using an EGI. When the IBO is increased up to 9dB and 11dB, OBR reductions of 20dB and 70dB at $2B_n$ are observed compared to the OBR of the conventional OFDM without EGI.

Fig. 5 and Fig. 6 depict the PSD when the signal is passing through SSPA and TWT respectively. Unlike in SL, a reduction in OBR is observed even for an IBO of 5dB. But these reductions are observed at 2.5Bn in the case of SSPA and after 1.75Bn in the case of TWT. Here again greater reductions are observed for larger IBOs. One important observation made at this point was the high OBR caused by TWT close to the signal band. As it can be seen in Fig. 6 high OBR exists within B_n and $2B_n$. This OBR can not be reduced significantly by using EGI. However, high OBR reduction is observed in the region beyond 2Bn in the case of TWT compared to other two non-linearities. Next we study the effect of the length of EGI on the OBR when the signal is passing through a nonlinear device.

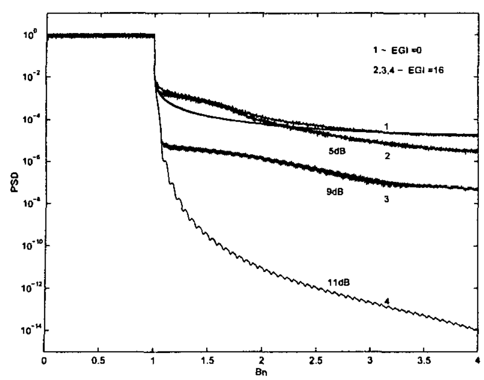

Fig. 4. Power spectral density of an OFDM signal passing through a SL with different IBOs (EGI=0,16).

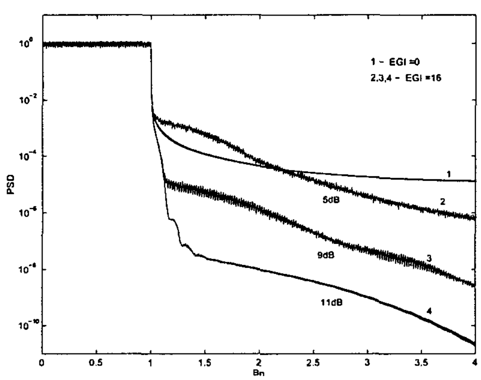

Fig. 5. Power spectral density of an OFDM signal passing through a SSPA with different IBOs (EGI=0,16).

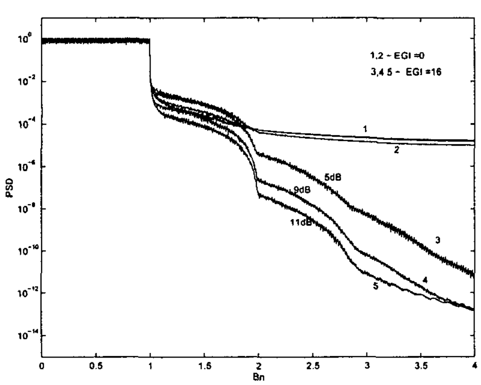

Fig. 6. Power spectral density of an OFDM signal passing through a TWT with different IBOs (EGI=0,16).

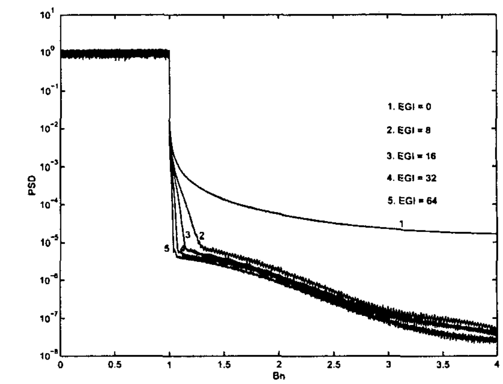

Fig. 7. Power spectral density of an OFDM signal with different EGIs passing through a SL (IBO=9dB).

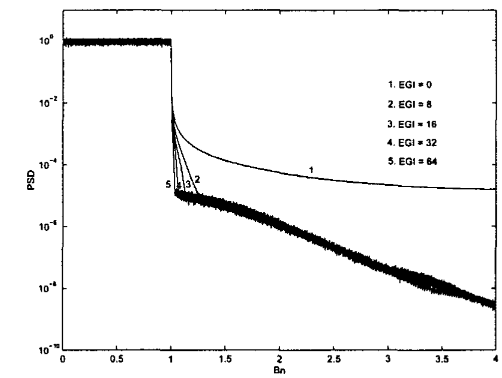

Fig. 8. Power spectral density of an OFDM signal with different EGIs passing through a SSPA (IBO=9dB).

Fig. 7 Fig. 8 and Fig. 9 depict the effect of the size of the EGI on OBR. Curves 1 to 5 present the spectra corresponds to EGI values 0, 8, 16, 32 and 64. The IBO of the amplifiers are 9dB. All three figures show the results, that the OBR is not reduced further with increase of the length of the EGI. These results clearly show that there is no significant effect of the size of EGI on OBR, when non-linearities are present. An EGI as small as 3% would be enough to get the desired results thereby maintaining the efficiency.

Different nonlinearities have different performances with respect to OBR. Fig. 10 depicts a comparison of PSD for different nonlinear devices. The EGI is fixed to 6% for this simulation result. When the IBO is just 5dB, the OBR levels from all three nonlinearities are same until the normalized frequency of 2. Beyond this point TWT non-linearity has less OBR compared to SSPA and SL. This difference is about 50dB at normalized frequency of 4. When SSPA and SL is compared, it is observed that both has almost similar performances close to the useful frequency band, while SSPA has about 10dB less radiation at $4B_n$. When the IBO is increased to 9dB, the OBR caused by SSPA and SL decrease by about 20dB at $2B_n$. There is only about 5dB reduction in OBR due to TWT in this region. Results indicate that the use of EGI effect differently on different non-linearities. It also suggests that we have to maintain a sufficient IBO in order to reduce OBR caused by non-linearities. Fig. 11 shows the reduction in OBR with different windowing functions. Four types of windowing functions, Hamming, Hanning, raised cosine and Blackman are simulated. Here, the Blackman window is defined as

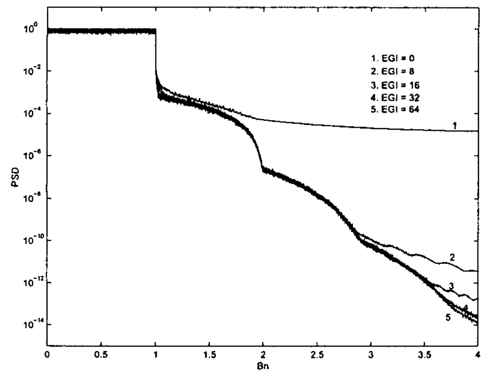

Fig. 9. Power spectral density of an OFDM signal with different EGIs passing through a TWT (IBO=9dB).

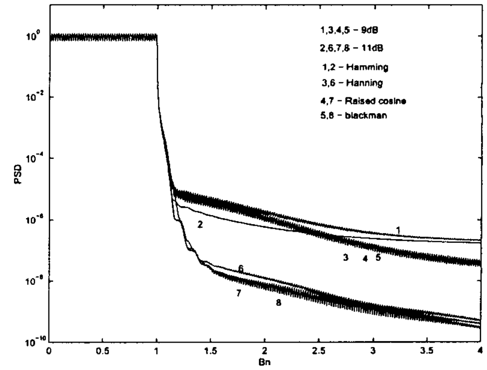

Fig. 11. A comparison of OBR with different windowing functions(EGI=16).

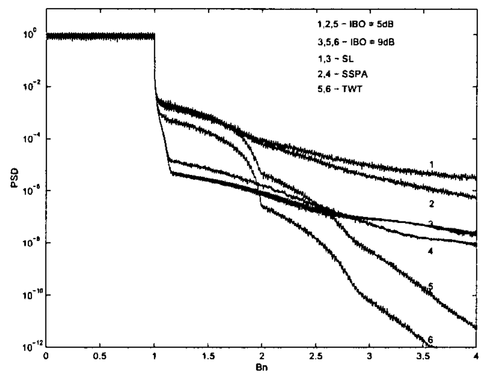

Fig. 10. A comparison of OBR of an OFDM signal with different nonlinearities (EGI=16).

$$w[n] = \begin{cases} 0.42 - 0.5\cos(\frac{2\pi n}{M}) + 0.08\cos(\frac{4\pi n}{M}) & 0 \le n \le M, \\ 0 & \text{else} \end{cases} \tag{19}$$

where M is the length of the window. Simulations are carried out with a SL having IBO of 9dB and 11dB. The EGI is fixed at 16. The results show that except Hamming window, all the other types of window has similar OBR reduction capability. At 11dB back off other windowing methods are capable of achieving about 20dB reduction in OBR at $4B_n$ compared to Hamming windowing. Although Hamming window has significantly lower side lobes compared to that of rectangular window, side lobes are higher compared to other windows considered here. The higher side lobes in Hamming window contributes to the higher OBR level of the OFDM signal. Raised cosine, Hanning and Balckman windows in conjunction with EGI is good in reducing OBR.

VI. Conclusion

This paper investigates the spectrum and the out of band radiation of an OFDM signal and the use of EGI in the presence of different nonlinear devices. Simulation and theoretical spectra of an OFDM signal are presented. Use of EGI together with a suitable windowing function yields reduced OBR with increased system complexity and reduced efficiency of an OFDM system. Reduction in OBR does not depend on the length of the EGI when a non-linearity exists. EGI is not capable of reducing the OBR caused by non-linearities having insufficient (5dB) IBO. These observations are true for all the three nonlinear devices considered. We conclude that by using a small extended guard interval, the out band radiation of an OFDM signal can be reduced significantly, if sufficient input back off is set at the nonlinear device. By introducing the EGI required backoff at the nonlinear device can be reduced significantly. Raised cosine, Hanning and Blackman windows show similar performances in reducing OBR.

References

[1] W. Y. Zou and Y. Wu, "COFDM: An overview," *IEEE Trans. Broadcast.*, vol. 41, pp. 1-8, March 1995.

[2] H. Rohling, T. May, K. Bruninghaus, and R. Grunheid, "Broad-band OFDM radio transmission for mutimedia applications," *Proc. IEEE*, vol. 87, pp. 1778-1788, Oct. 1999.

[3] R. O'Neill and L. N. Lopes, "Envelope variation and spectral splatter in clipped multicarrier signals," in *IEEE PIMRC*, pp. 71-75, Sept. 1995.

[4] R. Gross and D. Veeneman, "SNR and spectral properties for a clipped DMT ADSL signal," in *IEEE ICC*, (New York, NY, USA), pp. 843-847, 1994.

[5] X. Li and L. J. Cimini, "Effects of clipping and filtering on the performance of OFDM," *IEEE Commun. Lett.*, vol. 2, pp. 131-133, May 1998.

[6] G. A. Awater and R. van Nee, "Implementation of the magic WAND wireless ATM modem," in *IEEE GLOBECOM*, pp. 1-6, 1999.

[7] M. Pauli and H.-P. Kuchenbebecker, "On the reduction of the out-of-band radiation of OFDM signals," in *IEEE ICC*, (New York, NY, USA), pp. 1304-1038, 1998.

[8] R. van Nee and A. D. Wild, "Reducing the peak-to-average power ratio of OFDM," in *IEEE VTC*, (New York, NY, USA), pp. 2072-2076, 1998.

[9] E. Costa, M. Midrio, and S. Pupolin, "Impact of amplifier nonlinearities on OFDM transmission system performance," *IEEE Commun. Lett.*, vol. 3, pp. 37-39, Feb 1999.

[10] A. Chini, M. S. El-Tanny, and S. A. Mahmoud, "Hardware nonlinearities in digital TV broadcasting using OFDM modulation," *IEEE Trans. Broadcast.*, vol. 44, pp. 12-21, March 1998.

[11] D. Dardari, V. Tralli, and A. Vaccari, "A theoretical approach to error probability evaluation for OFDM systems with memoryless non-linearities," in *IEEE GLOBECOM*, (Sydney, Australia), pp. 207-212, 1998.

[12] S. Merchan, A. G. Armada, and J. L. Garcia, "OFDM performance in amplifier nonlinearity," *IEEE Trans. Broadcast.*, vol. 44, pp. 106-114, Mar. 1998.

[13] R. Gross and D. Veeneman, "Clipping distortion in DMT systems," *IEE Elect. Lett.*, vol. 29, no. 24, pp. 2080-2081, 1993.

[14] M. Friese, "On the degradation of OFDM-signals due to peak-clipping in optimally predistorted power amplifiers," in *IEEE GLOBECOM*, (Sydney, Australia), pp. 936-944, 1998.

[15] H. E. Rowe, "Memoryless nonlinearities with gaussian inputs: Elementary results," *BELL Systems Technical Journal*, vol. 61, pp. 1519-1525, Sept. 1982.

[16] A. A. Saleh, "Frequency independant and frequency dependent nonlinear models of TWT amplifiers," *IEEE Trans. Commun.*, vol. COM-29, pp. 1715-1720, Nov. 1981.