Forward Error Correction Codes to Reduce Intercarrier Interference in OFDM

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ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) is sensitive to the carrier frequency offset (CFO), which destroys orthogonality and causes intercarrier interference (ICI). Recently, a simple rate 1/2 repeat coding scheme has been shown to be effective in suppressing ICI. That such a simple coding scheme is so effective raises an interesting question. Can more powerful error correcting codes with less redundancy be used just as effectively for the same purpose? In this paper, we propose the use of rate-compatible punctured convolutional (RCPC) codes.

Keywords: Orthogonal Frequency Division Multiplexing (OFDM), Carrier Frequency Offset (CFO), Intercarrier Interference (ICI)

1. INTRODUCTION

The explosive growth in mobile users and demand for new services, such as wireless multimedia services and wireless Internet access, require high data rate wireless communication system. OFDM has favorable properties such as robustness to channel fading and intersymbol interference and immunity to impulse noise [1]. OFDM has therefore been accepted for several wireless LAN standards, as well as mobile multimedia applications [2]. However, OFDM is sensitive to CFO, which is caused by misalignment in carrier frequencies or Doppler shift, and phase noise. CFO violates the orthogonality of subcarriers and results in ICI [3]. The BER performance degrades as a result.

In the literature, three kinds of approaches have been developed to reduce the effect of CFO on OFDM. First, the frequency offset can be estimated at the receiver and corrected [4,5]. This approach requires pilot symbols. Second, the transmitted signal is multiplied by a suitable window function to avoid ICI [3]. The other approach is a repetition coding technique called self-ICI cancellation coding [3,6] or polynomial cancellation coding [3,7]. Each data symbol X_k transmitted on two adjacent subcarriers as X_k and $-X_k$ in order to cancel ICI and hence the data throughput is half of that of ordinary OFDM. In this paper, we will refer to this method as rate ½ repetition coding.

From an error correcting point of view, this repetition code has rather limited capability (the minimum Hamming distance is 2). However, it enables an ICI cancellation receiver structure that computes the difference between the received samples for pairs of adjacent subcarriers. Since ICI coefficients (Eq. 2) vary slowly between adjacent subcarries, this receiver is highly effective in canceling ICI, albeit at a loss of 50% data throughput. This raises an interesting question. Can a more powerful error correction code with less redundancy be used for ICI cancellation just as effectively? Note that such a code would not have a direct ICI cancellation structure. Instead, higher values of $d_{\min}$ enables it to suppress more errors.

A rich collection of error correction codes is at our disposal for this application. For example, convolutional codes, Bose Chaudhuri and Hocquenghem (BCH) codes, Reed Soloman (RS) codes, trellis coded modulation (TCM) and turbo codes are used in many applications. The application of BCH codes to reduce ICI effects is reported in [8]. However, hard decision decoding of BCH codes does not offer much coding gain over repetition coding although data throughput is better [8]. We study the use of convolutional codes to reduce ICI effects in this paper because of the availability of soft decision decoding algorithms.

Convolutional codes are widely used in many practical applications such as space and satellite communication and GSM. For these, convolutional codes achieve the required performance for some desired information rate with low complexity decoder [9]. Rate-compatible punctured convolutional codes (RCPC codes) offers simple Viterbi decoding for high rate convolutional codes and code rate can be adjusted according to channel variation without changing the decoder [10]. Therefore, RCPC codes offer flexibility to change code rate for varying frequency offset. For example, the code rate can be increased when there is no Doppler effect in the system.

The organization of this paper is as follows: In Section 2, the repetition coding technique and the theoretical analysis of error correction codes to reduce ICI effect is explained. Simulation results are reported in Section 3 and concluding remarks are presented in Section 4.

2. REDUCING ICI BY USING ERROR CORRECTION CODES

In an OFDM system in AWGN channel, the received signal for the k-th subchannel after the receiver fast Fourier transform processing can be written as [3,6]

$$y_k = X_k S_0 + \sum_{l=0, l \neq k}^{N-1} S_{l-k} X_l + n_k \; ; \; k = 0,..,N-1 \quad (1)$$

where X_k denotes the transmitted symbol for the k-th subcarrier, n_k is a complex Gaussian noise sample (with its real and imaginary components being independent and identically distributed with variance σ^2) and N is the number of subcarriers. The second term in (1) is the ICI term caused by the CFO. The

0-7803-6685-9/01/$10.00©2001 IEEE

sequence $\{S_k\}$ (the ICI coefficients) depends on the CFO and is given by [3,6]

$$S_k = \frac{\sin \pi(k+\varepsilon)}{N \sin \frac{\pi}{N}(k+\varepsilon)} \exp\left[j\pi(1-\frac{1}{N})(k+\varepsilon) \right] \quad (2)$$

where ε is the normalized frequency offset. For zero frequency offset, S_k reduces to the unit impulse sequence.

We assume the data symbols X_k are independent and identically distributed random variables (RVs). For M-ary signaling, X_k is equally likely to assume one out of M levels.

If the ICI is assumed to be a Gaussian distributed random variable with a zero mean, then the effective signal to noise ratio for the k-th subcarrier can be expressed as

$$\gamma_{eff} = \frac{S_k^2 \sigma_k^2}{\sigma_{ICI}^2 + \sigma^2} \quad (3)$$

where σ_k^2 and σ^2 are the signal power of the k-th subcarrier and the noise power, respectively. σ_{ICI}^2 is the variance of the interference signal on the k-th subcarrier and can be expressed as

$$\sigma_{ICI}^2 = \sum_{l=0, l\neq k}^{N-1} \left| S_{l-k} \right|^2 \sigma_s^2 \quad (4)$$

where σ_s^2 is the variance of the signal constellation. This effective SNR ban be used directly to calculate the probability of symbol error.

In the repetition coding technique, the modulating symbols are repeated such that $X_1 = -X_0, X_3 = -X_2$, $X_5 = -X_4$ and so on. The decision variable at the receiver is the combined received values at subcarrier k and $(k+1)$ and can be expressed as [3,7]

$$r_k = y_{2k} - y_{2k+1} \quad ; k = 0,1,\ldots,N/2-1 \quad (5)$$

and this can be written as

$$r_k = (2S_0 - S_{-1} - S_1)X_0 + \sum_{\substack{l=0 \\ l\neq k}}^{N/2-1} (2S_{2l} - S_{2l-1} - S_{2l+1})X_l + (n_{2k} - n_{2k+1}) \quad (6)$$

The first term is the desired component and the second is the total reduced ICI. This is similar to (1), except for modified weighting coefficients of $X_k, k = 0,\ldots,N/2-1$.

Approximate calculation of BER or SER using (3) is acceptable for BPSK modulation as precise calculation of BER or SER yields almost same results. However, this approximation gives optimistic results for higher order modulation scheme [8].

The Viterbi upper bound for the bit error probability of RCPC codes in AWGN channel is expressed as [9]

$$P_b \le \frac{1}{P} \sum_{d=d_{free}}^{\infty} c_d P_d \quad (7)$$

where P is the puncturing period, d_{free} is the free distance of the code and $\{c_d\}$ is the distance spectra. P_d is the probability that wrong path at distance d is selected and is given by [9]

$$P_d = \frac{1}{2} erfc(\sqrt{dRE_b/N_o}) \quad (8)$$

where R is the code rate. This upperbound is only valid for Gaussian noise. If the data symbols are modulated by BPSK, then the ICI can be approximated as Gaussian. Therefore, the upper bound for bit error probability of RCPC codes with frequency offset can be expressed as

$$P_b \le \frac{1}{2P} \sum_{d=d_{free}}^{\infty} c_d erfc(\sqrt{dR\gamma_{eff}}) \quad (9)$$

This can be used to study the capability of RCPC codes to reduce ICI if the values of c_d are known.

Note that we are interested in comparing the effectiveness of several coding schemes to suppress ICI. Therefore, we do not consider other channel impairments such as fading and shadowing etc. As a result, performance in Gaussian noise channels with CFO is evaluated.

We study the performance of 1/2, 2/3 and 4/5 rate convolutional codes as a function of the CFO and AWGN. The NASA standard (2,1,6) convolutional code with constraint length seven is considered and 2/3 and 4/5 rate codes were obtained according to rate-compatible puncturing [10]. QPSK modulation is considered in the simulation and we use soft decision Viterbi decoding to improve the coding gain. The Viterbi algorithm is a sequential trellis search for performing Maximum Likelihood sequence detection. The decoder selects the sequence $X^{(m)}$ that minimizes the Euclidean distance metric [9]

$$D(y, X^{(m)}) = \sum_k \left| (y_k - X_k^{(m)}) \right|^2 \quad (10)$$

The complexity of RCPC codes is higher than that of the repetition code. However, as hardware costs decreases rapidly, this may become less of an issue.

3. SIMULATION RESULTS

Figure 1 and Figure 2 show the BER performance of RCPC codes [Table II(c), 10]. Repetition coding and normal OFDM are shown for comparison. Note that the performance of the repetition coding and normal OFDM are precise whereas that

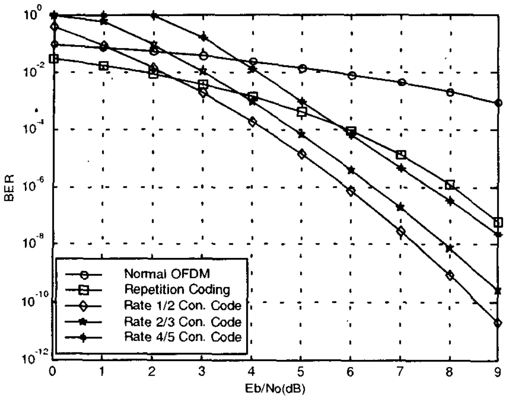

Figure 1. Probability of Bit Error with Normalized Frequency Offset of 0.1 (Theory).

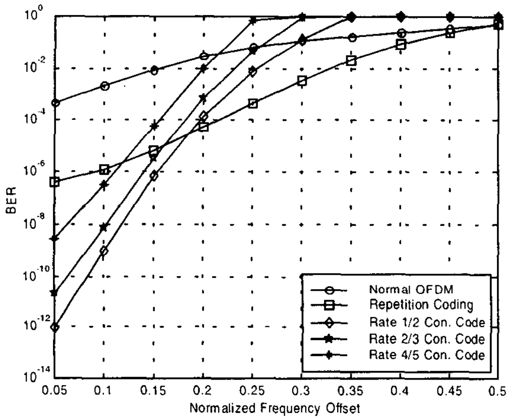

Figure 2. Variation of BER with Normalized Frequency Offset, Eb/No=8dB (Theory).

of RCPC is upper bounded. RCPC codes removes the error floor caused by ICI in normal OFDM and they performs better than repetition coding for small values of CFOs with gain in data throughput. In fact, these upper bounds confirm that RCPC codes are capable of correcting ICI errors effectively over repetition coding for small values of CFOs and at high SNRs. Often, CFOs are small in practice. Further, actual BER performance of RCPC codes will offer more coding gain, as the upper bounds are not tight.

Figure 3 shows the BER performance of a convolutional coded OFDM system with N=128 for QPSK modulation scheme in the presence of normalized frequency offset of 0.1. Repetition coding approach is also shown in Figure 3 for comparison. Here, we used RCPC codes with code rates of 1/2, 2/3 and 4/5.

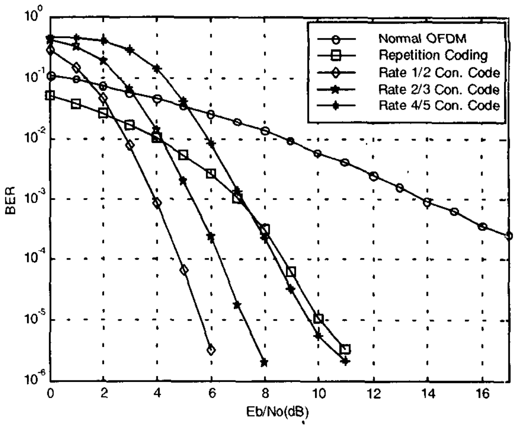

Figure 3. Probability of Bit Error with Normalized Frequency Offset of 0.1 (Simulation).

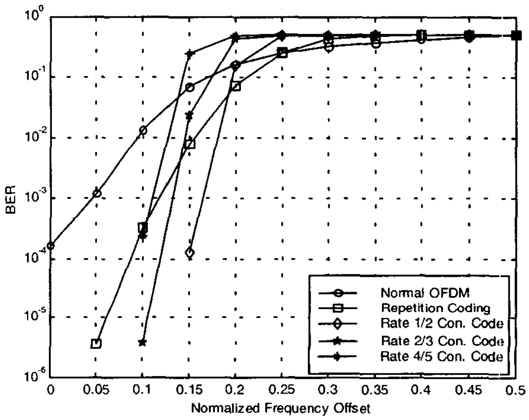

Figure 4. Variation of BER with Normalized Frequency Offset, Eb/No=8dB (Theory).

Convolutional codes perform worse than normal OFDM for small values of SNR where as repetition coding always performs better than normal OFDM. This is due to noise dominating the ICI at low *SNR*. However, the coding scheme rapidly improves the BER performance at high SNR and convolutional codes perform better than that of repetition coding and normal OFDM. This is due to increase in the minimum distance of convolutional codes and the repetition coding does not have this property. SNR gain of repetition coding and convolutional codes with code rate 1/2, 2/3 and 4/5, at 10^{-3} BER over normal OFDM with the normalized frequency offset of 0.1 is 6.8dB, 10dB, 8.5dB and 6.5dB respectively. The ½ rate convolutional code offers SNR gain of more than 3dB at BER less than 10^{-3} over the repetition coding but both have same data throughput. In fact, these three convolutional codes perform better than repetition coding for

BER less than 10^{-4}. These phenomena can be expected for small value of frequency offset as well.

Figure 4 shows the variation of BER with normalized frequency offset for $SNR = 8dB$ respectively. Convolutional codes performs better than repetition coding and normal OFDM for normalized frequency offset less than 0.125. In practice, the frequency offset caused by Doppler shift and inaccuracies in fine-tuning of oscillators are very small.

4. CONCLUSION

In this paper, we have applied RCPC codes to suppress the ICI caused by the CFO. The 4/5 rate convolutional code offers 0.5dB SNR gain at the 10^{-4} BER and 30% gain in data throughput over the repetition code. Using RCPC codes, we may adaptivley vary the data throughput based on ICI effects. This can be realized in OFDM based ARQ systems such as wireless LAN. The price for these benefits is decoding complexity. However, this may be acceptable as Viterbi decoders are widely used in many communication systems.

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