PEAK-TO-AVERAGE POWER RATIO REDUCTION OF AN OFDM SIGNAL USING DATA PERMUTATION WITH EMBEDDED SIDE INFORMATION

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ABSTRACT

The use of a set of fixed permutations has recently been proposed for peak-to-average power ratio (PAR) reduction of an OFDM signal [1]. Sending the identity of the used permutation to the receiver reliably is critical in this method. This paper presents the use of data permutation with embedded side information (SI) to reduce the PAR of an OFDM signal. SI is coded using a simple forward-error-correction code and inserted into the information sequence. The PARs of the permuted sequences and the original information sequence are then computed using IDFTs. The sequence with the lowest PAR is chosen for the transmission. The PAR statistics do not degrade due to the inclusion of SI. This paper also presents theoretical expressions for the complementary cumulative density function (CCDF) of the PAR of an interleaved OFDM (IOFDM) signal and for the average number of interleavers in the adaptive interleaving approach.

Key Words: OFDM, peak-to-average power ratio, data permutation, side information.

1. INTRODUCTION

While on the whole, the global wireless market has been expanding rapidly, high speed wireless data services have not been very successful thus far. Current cellular systems mainly carry voice traffic. Even high-speed wireless access to internet is not widely available. The slow growth of wireless data services is due to low data rates, costly services and lack of "killer" applications. These problems are being addressed by newly emerging wireless standards which also include support for multimedia traffic requirements, such as quality of service parameters.

OFDM is a promising solution for such wireless systems operating over frequency selective fading radio channels [2]. OFDM is already being investigated for new wireless standards and current OFDM applications include digital audio broadcasting, asynchronous digital subscriber lines, digital video broadcasting and wireless LANs. OFDM is commonly implemented using Discrete Fourier Transform (DFT) techniques. One substantial disadvantage of OFDM is the potentially high PAR values. In practice, peak signal levels are constrained by design factors such as battery power (on portable equipment) or regulatory limits that prevent adjacent channel interference. The use of non-linear amplifiers and digital hard limiting causes inefficiency, interference and performance degradation.

The PAR related drawbacks have motivated the search for PAR reduction techniques and a large number of solutions have been proposed to date. The partial transmit sequences (PTS) and the selected mapping (SLM) approaches, proposed by Muller et al.[3], are based on the phase shifting of clusters of data symbols and the multiplication of the data frame by random vectors respectively. Unfortunately, finding the best phase factors for PTS is a and non-linear optimization problem. For this reason, some attempts have been made to reduce the complexity of the optimization: [4] has presented a suboptimal iterative flipping algorithm, while [5] has derived an alternative optimization criterion. Yet even with full optimization, the 0.1% PAR (i.e. the PAR exceeds this value for less than 0.1% of the OFDM blocks) is reduced by about 4.2dB for a 256 QPSK-carrier system with 16 subblocks. The real complexity of PTS is clear when one considers the fact that if the $M-1$ phase factors are limited to 0 and π ,2^{M-1} PAR computations are still required in order to find the optimum phase factors.

A technique involving interleaving of data in a frame to reduce the PAR is presented in [1, 6]. Highly correlated data frames of an OFDM system have large PARs, which could thus be reduced, if the long correlation patterns were broken down. $K-1$ interleavers are employed to permute or reorder each input data frame. The size of each of these interleavers is N. The frame with the lowest PAR of all the K frames (the original plus $K-1$ permutations) is selected for transmission with a pointer (integer between 0 and $K-1$) to the corresponding interleaver used. For an N subcarrier QPSK system, the fraction of overhead created by additional information (pointer) is $\log_2 K/2N$, which is negligibly small for large N. While both symbol interleaving and bit interleaving can be used, bit interleaving proves to be better. Performance of the interleaving technique with embedded SI has not been presented before.

Furthermore, an adaptive technique, which can be employed to reduce the complexity of the above scheme, is also presented in [6]. OFDM frames with high PAR are generally rare. As such, the overall complexity would be greatly reduced if only the OFDM frames with high PAR values were processed by the PAR reduction scheme. Hence, the key idea in adaptive interleaving (AIL) is to establish an early terminating threshold. That is, the search is terminated as soon as the PAR falls below the threshold, rather than searching all the interleaved sequences ($K-1$). Of course, if the threshold is set to a small value, AIL will be forced to search most of the permutations. Likewise, if the threshold is set to a large value, AIL will search only a fraction of the $K-1$ permutations. So the adaptive method trades PAR reduction for complexity.

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In this paper the performance of the interleaving technique with embedded SI in reducing the PAR is presented. Some theoretical results for both interleaving and adaptive interleaving techniques are also presented. The paper is organized as follows. Section 2 presents the definition of an OFDM system and the PAR. Section 3 describes the proposed system model and Section 4 derives the theoretical curves of complementary cumulative distribution function (CCDF) of the PAR of an OFDM signal. Results are presented in Section 5, and the paper concludes in Section 6.

2. AN OFDM SYSTEM AND PAR

In OFDM, a block of N symbols, $X_n, n = 0, 1, ..., N-1$, is formed with each symbol modulating one of a set of N subcarriers, $f_n, n = 0, 1, ..., N-1$. The N subcarriers are chosen to be orthogonal, that is $f_n = n\Delta f$, where and T is the original symbol period. The resulting signal can be expressed as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi(f_n)t}, 0 \le t \le NT. \qquad (1)$$

The PAR of the transmitted signal in (1) can be defined as

$$PAR = \frac{\max |x(t)|^2}{E[|x(t)|^2]}. \qquad (2)$$

The PAR of the continuous-time OFDM signal cannot be computed precisely [5] by the use of the Nyquist sampling rate, which amounts to N samples per symbol. In this case, signal peaks are missed and PAR reduction estimates are unduly optimistic. Oversampling by a factor of 4 is sufficiently accurate and is achieved by the simple computation of the $4N$-point zero-padded IDFT of the data frame. Herein we shall assume oversampling by a factor of 4 for all further calculations.

3. INTERLEAVER APPROACH

In this approach $K-1$ interleavers are used at the transmitter. These produce $K-1$ permuted frames of the input data frame and the identity of the interleaver is added to the data sequence before mapping into QPSK symbols. The four times oversampled IDFT of each frame (including the original frame) is used to compute its PAR. The minimum PAR frame of all the K frames is selected for transmission. Figure 1 depicts the system block diagram.

As all the interleavers process the data in parallel and SI pre encoded, the delay in processing the data is small. Fixed carriers in OFDM symbol are assigned for SI such that it can be recovered without de-interleaving at the receiver.

3.1. adaptive interleaving

In the adaptive approach, up to $K-1$ interleavers are considered. As a first step, the PAR of the signal is computed without interleaving. If it is less than a set threshold PAR_T, then PAR minimization stops immediately. If not, the data sequence is interleaved and the resulting PAR is recomputed. If it is less than PAR_T, the minimization process is stopped. The algorithm continues in this fashion until the PAR is less than PAR_T or all $K-1$ of the interleavers have been searched. If no permuted sequence is found to have a PAR less than PAR_T, the one with the smallest PAR is selected for transmission.

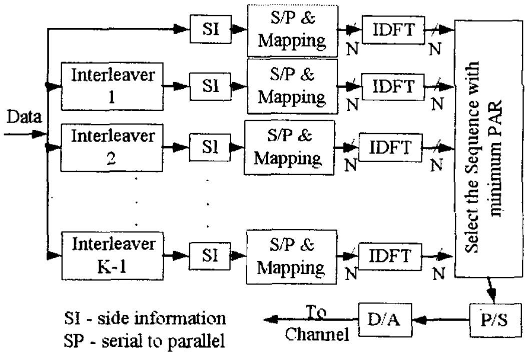


Figure 1: system model.

4. THEORETICAL CCDF OF PAR OF AN OFDM SIGNAL

Theoretical results for the CCDF of PAR can be derived using results reported in [7]. For an OFDM symbol with N carriers, the samples of the complex baseband signal is given by (1). From the central limit theorem it follows that for large values of N, the real and imaginary values of power normalized $x(t)$ become Gaussian distributed. The amplitude of the OFDM signal therefore has a Rayleigh distribution. The CDF of the peak power per OFDM symbol can be derived by assuming the samples are mutually uncorrelated. This is true for the Nyquist sampling rate. The probability that the PAR ratio is below the threshold level PAR_0 can be written as

$$\Pr(\mathrm{PAR} \le \mathrm{PAR}_0) = (1 - \exp(-\mathrm{PAR}_0))^{N}. \qquad (3)$$

As (3) does not hold for the oversampling case, an approximation is presented in [7]. Adding a certain number of extra independent samples approximates the effect of oversampling. The distribution of the PAR is then given by

$$\Pr(\mathrm{PAR} \le \mathrm{PAR}_0) = (1 - \exp(-\mathrm{PAR}_0))^{\alpha N}. \qquad (4)$$

The CCDF of the PAR of and OFDM signal can then be expressed as

$$\Pr(\mathrm{PAR} > \mathrm{PAR}_0) = 1 - (1 - \exp(-\mathrm{PAR}_0))^{\alpha N}. \qquad (5)$$

Reference [7] shows that, $\alpha = 2.8$ is a good approximation for the oversampled OFDM signal in general. But we have found that $\alpha = 2$ gives more accurate results for $N = 256$. As the simulated results agree with the theoretical results, (5) will be used to analyze the performance of the interleaving method.

4.1. Theoretical CCDF of PAR of an IOFDM signal

If we assume fully random interleaving so that the K interleaved outputs are uncorrelated, the CCDF of the PAR of the interleaved OFDM signal can then be expressed as

$$\Pr(\mathrm{PAR} > \mathrm{PAR}_0) = (1 - (1 - \exp(-\mathrm{PAR}_0))^{\alpha N})^{K}. \qquad (6)$$

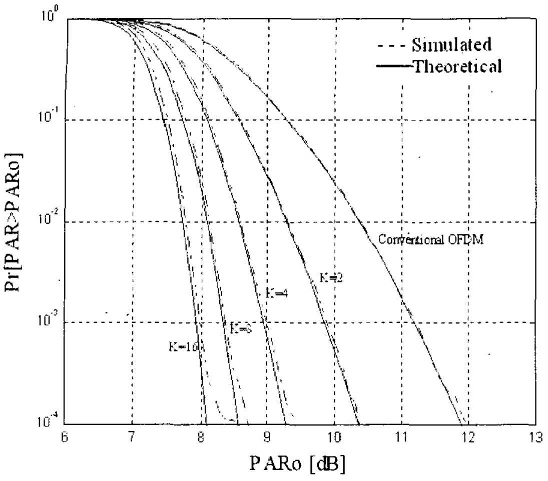


Figure 2: Theoretical and simulated CCDF of the PAR of an interleaved OFDM signal ($N = 256$).

In the adaptive approach permutations are stopped as soon as the PAR reaches below the given threshold. Let $P(k, \mathrm{PAR}_T) = Pr(\mathrm{PAR} > \mathrm{PAR}_T)$ be the probability that the PAR greater than the threshold with k permutations. Then the average number of permutations needed to achieve the PAR less than threshold can be written as

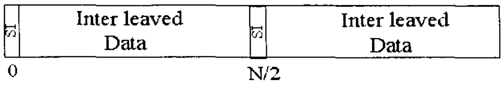
$$\mathrm{AIL} = 1 + \sum_{k=2}^{K} k\{[\prod_{i=1}^{k-1} \Pr(i, \mathrm{PAR}_T)](1 - \Pr(k, \mathrm{PAR}_T))\}. \quad (7)$$

5. RESULTS

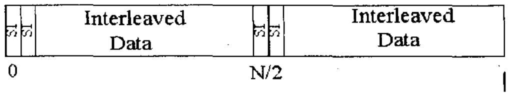
A 256 subcarrier OFDM system with QPSK modulation is simulated to obtain the following results. Figure 2 depicts the simulated performance of random interleaves compared with the theoretical performance predicted by (6) with $\alpha = 2$. The simulated and theoretical results agree each other very well.

Sending SI to the receiver is a main requirement of our PAR reduction scheme. If SI is not reliable, the bit error rate can increase drastically. Figure 3 shows the proposed scheme to embed SI in to the data sequence. SI is added at the beginning and the middle of the data frame. The PAR is optimized with the embedded SI. This is an advantage of using the interleaving technique.

Next, we propose the use of a forward error correction (FEC) code to encode SI before inserting in to the data sequence. In our simulation we used a (7,4) Hamming code with single error-correcting capability. This is proposed to increase the reliability of the SI. In the uncoded case two carrier (0 and $N/2-1$ are reserved fixed for SI and in the coded SI case four carriers ($0, 1, N/2-1, N/2$) are reserved. Coded SI increases the additional fractional overhead to $\log_2 K/N$. Figure 4 and Figure 5 depict the CCDF curves for the OFDM signal with embedded SI. Figure 4 shows the performance with uncoded SI and Figure 5 shows the performance with coded SI. Only an insignificant degradation in the CCDF is observed in both figures. This is because SI is inserted before the optimization process.



a). Uncoded SI



b). Coded SI

Figure 3: Data frame format with embedded side information.

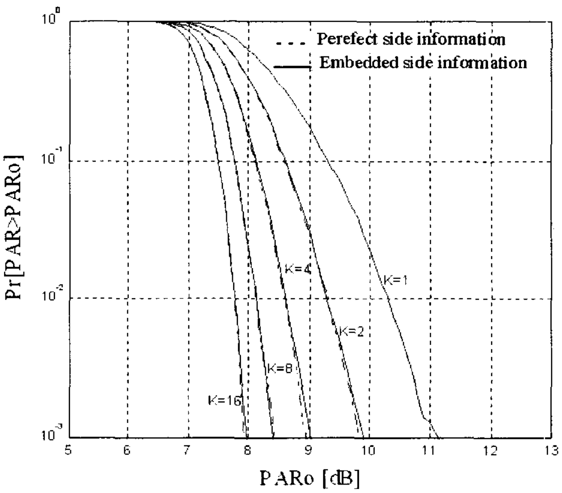


Figure 4: CCDF of PAR of an OFDM signal with embedded uncoded side information.

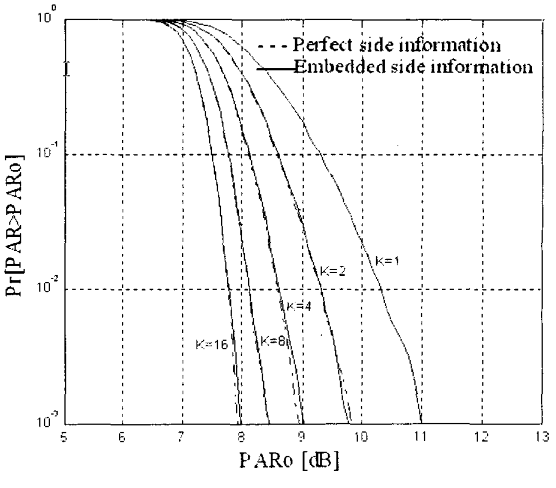


Figure 5: CCDF of PAR of an OFDM signal with embedded coded side information.

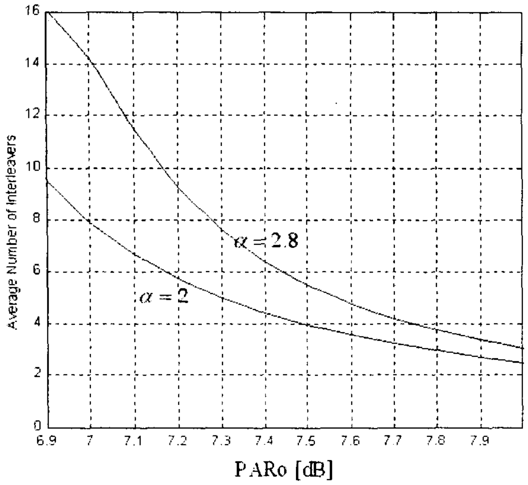


Figure 6: Theoretical curve for AIL.

Figure 6 depicts the theoretical average number of interleavers predicted by (Figure 7). Two curves represent the values correspond to $\alpha = 2$ and $\alpha = 2.8$ in (6). It is observed that $\alpha = 2.8$, which is proposed in [7], predicts the gives average number of interleavers close to the simulation results as shown in TABLE I.

Table 1: Average number of interleavers in adaptive interleaving approach.

		AIL			
Threshold		7.9dB	7.7dB	7.5dB	7.3dB
Theoretical	$\alpha = 2$	2.7	3.2	3.9	4.9
	$\alpha = 2.8$	3.3	4.2	5.4	7.6
Simulated		3.8	5.8	6.5	15.6

TABLE I shows the average number of interleavers needed for adaptive interleaving with maximum number of interleavers fixed at 32 ($K = 32$). Use of 32 interleavers improves the PAR of OFDM by 4dB at 0.01% CCDF. Results for AIL with values 7.9dB, 7.7dB, 7.5dB and 7.3dB correspond to average number of interleavers 3.8, 5.8, 6.5 and 15.6, which reduce complexity up to 88%, 82%, 70% and 51% respectively. The complexity can be reduced greatly if the number of iterations is limited by selecting a suitable threshold value. Lower threshold values yield better performances but have higher complexity. Threshold value selection has to be done using the CCDF curve for $K = 32$. The theoretical average permutations predicted by 7 are smaller than the simulated results. This is due to the correlation between interleaved sequences in practice.

6. CONCLUSION

This paper presents the performance of an interleaving technique with embedded side information to reduce the PAR of an OFDM signal. The use of side information does not degrade PAR reduction capability of this technique. A block code with a single error correcting capability is proposed to increase the reliability of side information. The statistics of the PAR do not degrade even with coded side information. This is due to the insertion of side information before the optimization process. Theoretical expressions are obtained for both interleaving and adaptive interleaving cases.

7. REFERENCES

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