Simple and Accurate Methods for Outage Analysis in Cellular Mobile Radio Systems—A Unified Approach

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***Abstract*—Two unified expressions for computing the refined outage criterion (which considers the receiver noise) in cellular mobile radio systems are derived using the Laplace and Fourier inversion formulas. Since these expressions do not impose any restrictions on the signal statistics while being easy to program, they provide a powerful tool for outage analysis over generalized fading channels. We also assess compatibility and applicability of previously published approaches that treat noise as cochannel interference (noise-limited model) or consider a minimum detectable receiver signal threshold and receiver noise. The outage probability in an interference-limited case can be evaluated directly by setting the minimum power threshold to zero. The analysis of correlated interferers is presented. Results are also developed for a random number of interferers. Several new closed-form expressions for the outage probability are also derived. Some previous studies have suggested approximating Rician desired signal statistics by a Nakagami-m model (with positive integer fading severity index) to circumvent the difficulty in evaluating the outage in Rician fading. The suitability of this approximation is examined by comparing the outage performance under these two fading conditions. Surprisingly, some basic results for Nakagami-m channel have been overlooked, which has led to misleadingly optimistic results with the Nakagami-m approximation model. However, similar approximation for the interferer signals is valid.**

***Index Terms*—Cochannel interference, fading channels, land mobile cellular radio systems.**

I. Introduction

IN CELLULAR radio engineering, the system designers are frequently required to evaluate the probability of outage, which is a useful statistical measure of the radio link performance in the presence of cochannel interference (CCI) [1]–[12]. Development of mathematical tools for outage analysis has received a lot of attention during the past two decades—since the deployment of the cellular architecture in the early 1980s for commercial mobile radio systems. Several commonly used definitions of the outage exist. In its simplest version, the outage is the probability that the ratio of the desired signal power to the total interference power falls below a given threshold that depends on the modulation type and other system design requirements. More refined definitions take into consideration additive noise levels and minimum signal requirements.

Despite many years of research, closed-form expressions for the probability of outage (even for the simplest definition) do not exist for some fading channel models because of the lack of a simple expression for the probability density function (pdf) of a sum of L interfering signals. For example, the pdf of a sum of L unequal Rician signal powers is not available in closed form. Similarly, the pdf of a sum of unequal Nakagami signal powers with noninteger fading severity indices is not available in simple, closed form. As a result, many previous studies make restrictive assumptions and approximations (e.g., Nakagami fading index limited to integer values, statistically equal interferers, a Rician variable approximated by a Nakagami variable and so on). Although the assumption that all the received signals (both desired and undesired) have the same statistical characteristics is quite reasonable for medium and large cell systems, its validity for pico- and microcellular systems is questionable. This is because an undesired signal from a distant cochannel cell may well be modeled by Rayleigh statistics but Rayleigh fading assumption may not hold for the desired signal since a line-of-sight path is likely to exist in a microcell. Therefore, it is evident that different statistics are needed to characterize the desired user signal and the interfering signals in a micro- or picocellular radio systems.

A simple yet general numerical technique for computing the outage seem useful and attractive to address the problem on hand. *However, a general unified expression for computing the outage (simple and refined criterion) where the system designer can simply plug in the required fading channel models with arbitrary parameters has not been developed previously.* This is the main contribution of this paper. Our numerical technique is very easy to compute and handles arbitrary fading models, as well as both simple and refined definitions of the outage are considered. We also derive exact closed-form formulas for some special cases. These formulas are reported because they are new to the best of the authors' knowledge.

The literature on outage analysis is extensive (see references in [1]–[3]). Closed-form expressions for outage in Nakagami fading is derived in [2] for integer fading parameters. Results

Paper approved by K. B. Letaief, the Editor for Wireless Systems of the IEEE Communications Society. Manuscript received February 2, 1999; revised April 28, 2000. This work was supported in part by a Strategic Project Grant from the Natural Sciences and Engineering Research Council (NSERC) of Canada and by Telus Mobility. This paper was presented in part at the IEEE International Conference on Communications, Vancouver, BC, Canada, June 1999.

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Publisher Item Identifier S 0090-6778(01)01299-5.

0090–6778/01$10.00 © 2001 IEEE

for a Nakagami microcell interference model are presented in [3]. An approach based on complex variable theory for the Nakagami outage problem is developed in [5]. A multiple infinite series for outage in Nakagami fading is derived in [6] for real fading parameters. A Rician/Rician fading model and log-normal shadowing is considered in [7]. In [8], the Rician-fading desired signal is approximated by a Nakagami distribution in order to develop outage expressions. In [10]–[12], numerical approaches, rather than closed-form solutions, for the outage problem are developed.

The main contributions of this paper are the following. First, we derive exact finite-range integral expression for the outage, for both the noise-limited and the minimum signal power models, in generalized fading environments. The moment generating functions (MGFs) of the desired signal, noise, and the interfering signal powers constitute the integrand for the noise-limited case. A marginal MGF of the desired signal power is required for the minimum signal power case and this is derived for all the common fading channel models. The analysis of correlated interferers is also presented. Second, we derive exact closed-form expressions for the outage (noise-limited or minimum signal power constraint) where the signals are described by the Nakagami distribution with an integer fading index. This approach requires the derivatives of a product of MGFs. A procedure for evaluating the derivatives is outlined. Third, we extend the analysis to the case where there are a random number of interferers. Finally, we provide extensive numerical results to assess the suitability of Nakagami-m approximation for a Rician random variable (RV), compare the compatibility and applicability of the noise-limited case and minimum signal requirement case and to investigate the effect of correlated interferers on the outage performance.

The outline of this paper is as follows. In Section II, we develop a general numerical procedure for computing the probability of outage. Section III deals with the derivations of closed-form solutions for several restrictive cases. Section IV considers the random number of interferers. Selected numerical results are provided in Section V. Finally in Section VI, the main points are summarized and conclusions restated.

II. Outage Analysis

Scanning the literature, we find that most of the previous studies on the outage calculations has been primarily focused on interference-limited systems. In other words, satisfactory reception is assumed to be achieved as long as the short-term signal-to-interference ratio (SIR) exceeds the power protection ratio, thereby neglecting the receiver background noise.[1] Hence, the outage event is given by

$$P_{\rm out} = \Pr\{qI > p_0\} \tag{1}$$

where the instantaneous signal powers p_k $(k = 0, 1, 2, \ldots, L)$ are modeled as random variables with mean $\bar{p}_k$, L denotes the total number of interferers, $I = \sum_{k=1}^{L} p_k$, and q is the power protection ratio (which is fixed by the type of modulation and transmission technique employed and the quality of service desired). The subscript $k = 0$ corresponds to the desired user signal, and $k = 1, \ldots, L$ are for the interfering signals.

But in practice, thermal noise and receiver threshold exists which may be of concern particularly in large cells (macrocell). There are two approaches to deal with this scenario. In the first technique, noise is treated as CCI (e.g., [1]). Alternatively, a minimum signal power requirement is imposed as an additional criterion for satisfactory reception (e.g., [2], [6]). In the remaining part of this section, we will develop new unified expressions (i.e., handles arbitrary fading parameters for the desired user signal and for the interfering signals) to compute the outage probability for each of the two distinct approaches to treat the receiver background noise.

Since our unified approach for computing the outage performance requires the knowledge of only the MGF of the received signal power (i.e., when the receiver noise is treated as interference) or both MGF and cumulative distribution function (cdf) of the received signal power (i.e., for the dual-threshold model), the MGF and pdf of signal power for commonly used fading channel models are listed in Table I.

A. Treating Noise as Interference

Using a more refined criterion than (1), we can redefine the outage event as the likelihood that the desired signal strength drops below the total interference power I by a CCI power protection margin q, and the total noise power N by the noise power protection margin r, viz.,

$$P_{\rm out} = \Pr\{p_0 < qI + \Lambda\} = \Pr\left\{\frac{p_0}{q} - \sum_{k=1}^{L} p_k < \frac{\Lambda}{q}\right\} \tag{2}$$

where $\Lambda = rN$ is a constant. It is apparent from (2) that the evaluation of outage simply requires the cdf of $\gamma = (p_0/q) - \sum_{k=1}^{L} p_k$, which is a linear sum of random powers. Since the MGF or the characteristic function (CHF) of the sum can be determined very easily, the outage follows at once from the Laplace or Fourier inversion formulas. Hence, following the development of [12, eq. (8)], the exact outage probability is given by

$$\begin{aligned} P_{\rm out} &= \frac{1}{\pi}\int_0^{\infty} \\ &\quad\times \frac{\mathrm{Real}[(c - j\omega)\phi_\gamma(c + j\omega)\exp\{(c + j\omega)\Lambda/q\}]}{c^2 + \omega^2}\, d\omega \\ &= \frac{1}{2\pi}\int_0^{\pi} \mathrm{Real}[(1 - j\tan(\theta/2))\phi_\gamma(c + jc\tan(\theta/2)) \\ &\quad\times \exp\{[1 + j\tan(\theta/2)]\Lambda c/q\}]\, d\theta \end{aligned} \tag{3}$$

[1]However, such an assumption is valid in small cell because radio link performance in pico and microcellular radio systems are usually limited by interference rather than noise and, therefore, the probability of CCI is of primary concern.

TABLE I
PDF AND MGF OF SIGNAL POWER FOR COMMON FADING CHANNEL MODELS

Channel Model	PDF $f_k(.)$ and MGF $\phi_k(.)$ for fading signal power of the k-th interferer
Rayleigh	$f_k(x) = \frac{1}{\bar{p}_k}\exp\left(\frac{-x}{\bar{p}_k}\right), x \ge 0$ $\phi_k(s) = \frac{1}{1+s\bar{p}_k}$ where $\bar{p}_k = E[x]$ = average SNR per symbol
Rician	$f_k(x) = \frac{1+K_k}{\bar{p}_k}\exp\left[-K_k - \frac{(1+K_k)x}{\bar{p}_k}\right]I_0\left[2\sqrt{\frac{K_k(K_k+1)x}{\bar{p}_k}}\right], x \ge 0$ $\phi_k(s) = \frac{1+K_k}{1+K_k+s\bar{p}_k}\exp\left(\frac{-sK_k\bar{p}_k}{1+K_k+s\bar{p}_k}\right)$ where $K_k \ge 0$ is the Rice parameter
Nakagami-q	$f_k(x) = \frac{1}{\bar{p}_k\sqrt{1-b_k^2}}\exp\left[\frac{-x}{(1-b_k^2)\bar{p}_k}\right]I_0\left[\frac{b_k x}{(1-b_k^2)\bar{p}_k}\right], x \ge 0$ where $-1 \le b_k = \frac{1-q_k^2}{1+q_k^2} \le 1$ $\phi_k(s) = \frac{1}{\sqrt{[s\bar{p}_k(1+b_k)+1][s\bar{p}_k(1-b_k)+1]}}$ where $0 \le q_k \le \infty$ is the fading parameter
Nakagami-m	$f_k(x) = \frac{1}{\Gamma(m_k)}\left(\frac{m_k}{\bar{p}_k}\right)^{m_k} x^{m_k-1}\exp\left(\frac{-m_k x}{\bar{p}_k}\right), x \ge 0$ $\phi_k(s) = \left(\frac{m_k}{m_k+s\bar{p}_k}\right)^{m_k}$ where $m_k \ge 0.5$ is the fading figure
Lognormal	$f_k(x) = \frac{1}{\sqrt{2\pi}\sigma_k x}\exp\left[\frac{-\ln^2(x/\mu_k)}{2\sigma_k^2}\right], x \ge 0$ $\phi_k(s) = \frac{1}{\sqrt{\pi}}\sum_{i=1}^{N} w_i \exp[-s\mu_k\exp(\sqrt{2}\sigma_k x_i)] + R_H$ σ_k is the logarithmic standard deviation of shadowing μ_k is the local mean power
Suzuki	$f_k(x) = \int_0^\infty \frac{1}{\Omega}\exp\left(-\frac{x}{\Omega}\right) \times \frac{1}{\sqrt{2\pi}\sigma_k\Omega}\exp\left[\frac{-\ln^2(\Omega/\mu_k)}{2\sigma_k^2}\right]d\Omega, x \ge 0$ $\phi_k(s) = \frac{1}{\sqrt{\pi}}\sum_{i=1}^{N}\frac{w_i}{1+s\mu_k\exp(\sqrt{2}\sigma_k x_i)} + R_H$ x_i and w_i are abscissa and weight of the i-th root of an N-th order Hermite polynomial R_H is a remainder term.
Lognormal Rice	$f_k(x) = \int_0^\infty \frac{1+K_k}{\Omega}\exp\left[-K_k - \frac{(1+K_k)x}{\Omega}\right]I_0\left[2\sqrt{\frac{K_k(K_k+1)x}{\Omega}}\right] \times \frac{1}{\sqrt{2\pi}\sigma_k\Omega}\exp\left[\frac{-\ln^2(\Omega/\mu_k)}{2\sigma_k^2}\right]d\Omega$ $\phi_k(s) = \frac{1}{\sqrt{\pi}}\sum_{i=1}^{N}\frac{w_i(1+K_k)}{1+K_k+s\mu_k\exp(\sqrt{2}\sigma_k x_i)}\exp\left[\frac{-sK_k\mu_k\exp(\sqrt{2}\sigma_k x_i)}{1+K_k+s\mu_k\exp(\sqrt{2}\sigma_k x_i)}\right] + R_H$
Lognormal Nakagami-m	$f_k(x) = \int_0^\infty \left(\frac{m_k}{\Omega}\right)^{m_k}\frac{x^{m_k-1}}{\Gamma(m_k)}\exp\left(\frac{-m_k x}{\Omega}\right) \times \frac{1}{\sqrt{2\pi}\sigma_k\Omega}\exp\left[\frac{-\ln^2(\Omega/\mu_k)}{2\sigma_k^2}\right]d\Omega, x \ge 0$ $\phi_k(s) = \frac{1}{\sqrt{\pi}}\sum_{i=1}^{N}\frac{w_i}{[1+s\mu_k\exp(\sqrt{2}\sigma_k x_i)/m_k]^{m_k}} + R_H$

where $j = \sqrt{-1}, 0 < c < a_{\min} = \min\{a_i \,|\, 1 \le i \le L\}$, with a_i being the ith pole of $\phi_\gamma(s)$ in the left half plane (i.e., $a_i > 0$) and the MGF $\phi_\gamma(\cdot)$ is defined by [12, eq. (7)]

$$\phi_\gamma(s) = \phi_0(s/q)\phi_I(-s) = \phi_0(s/q)\prod_{k=1}^{L}\phi_k(-s) \qquad (4)$$

where $\phi_k(s)$ is the MGF of p_k. Note that (3) emanates from the relation that the Laplace transform of cdf is $\phi_\gamma(s)/s$, and therefore (3) is simply a Laplace inversion integral. When $\Lambda = 0$, (3) reduces to [12, eq. (9)] as one would expect. The above integral may also be replaced by a Gauss–Chebychev quadrature (GCQ) sum using the procedure detailed in [12]. Alternatively, by invoking the Gil-Pelaez inversion theorem [16], we have yet another exact expression for the refined outage criterion

$$P_{\text{out}} = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{\text{Imag}[\phi_\gamma(-jt)\exp(-jt\Lambda/q)]}{t}\,dt. \qquad (5)$$

B. Minimum Signal Power Constraint

The presence of thermal noise and the receiver threshold imply that the desired signal power must simultaneously exceed the total interference power by a protection ratio and a minimum power level. In other words, the effect of noise was included implicitly by setting a minimum reception threshold for the desired signal. Therefore, the probability of satisfactory reception can be expressed as the intersection of two probability events

$$\Pr\{S\} = \Pr\left\{\left[q\sum_{k=1}^{L} p_k < p_0\right] \cap [p_0 > \Lambda]\right\} = \int_{\Lambda}^{\infty} \Pr\left\{ q\sum_{k=1}^{L} p_k < p_0 \,\middle|\, p_0 \right\} f(p_0)\, dp_0 \quad (6)$$

where $\Lambda = rN$ is the minimum power requirement due to receiver noise floor. Then the probability of outage is given by

$$P_{\text{out}} = 1 - \int_{\Lambda}^{\infty} f_{p_0}(p_0) \int_{0}^{p_0/q} f_I(y)\, dy\, dp_0 = 1 - \int_{0}^{\infty} f_{p_0}(p_0+\Lambda) F_I\left(\frac{p_0+\Lambda}{q}\right) dp_0 \quad (7)$$

Note that specifying a minimum signal requirement will cause a floor on the outage probability with or without CCI because the deep fades will result in signal power level below the specified minimum.

Different from (7) (which generally will require evaluation of an L-fold convolution integral), we will solve the outage problem in the framework of hypothesis-testing and determine the outage probability directly from the MGF of a decision variable as in Section II-A. The conditional probability illustrated by (6) can be expressed using a Laplace integral

$$\Pr\left\{ q\sum_{k=1}^{L} p_k < p_0 \,\middle|\, p_0 \right\} = 1 - \Pr\left\{ -\sum_{k=1}^{L} p_k < \frac{-p_0}{q} \,\middle|\, p_0 \right\} = 1 - \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{1}{s}\exp(-sp_0/q)\left[\prod_{k=1}^{L}\phi_k(-s)\right] ds.$$

Then, we have

$$\Pr\{S\} = 1 - F_0(\Lambda) - \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{G_0(s)}{s} \times \left[\prod_{k=1}^{L}\phi_k(-s)\right] ds \quad (8)$$

where the incomplete MGF $G_0(s) = \int_{\Lambda}^{\infty} \exp(-sp_0/q) f(p_0)\, dp_0$ is convergent. The above development does not follow from [5].

Finally the probability of outage with minimum power requirement is given by

$$\begin{aligned} P_{\text{out}} &= F_0(\Lambda) + \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{G_0(c+j\omega)}{c+j\omega} \times \prod_{k=1}^{L}\phi_k(-c-j\omega)\, d\omega \\ &= F_0(\Lambda) + \frac{1}{2\pi}\int_{0}^{\pi} \Phi_\gamma(\theta)\, d\theta \\ &= F_0(\Lambda) + \frac{1}{2n}\sum_{i=1}^{n} \Phi_\gamma\left[\frac{(2i-1)\pi}{2n}\right] + R_n \end{aligned} \quad (9)$$

where $\Phi_\gamma(\theta) = \text{Real}[(1 - j\tan(\theta/2))G_0(c + jc\tan(\theta/2))\prod_{k=1}^{L}\phi_k(-c - jc\tan(\theta/2))]$. Note that when $\Lambda = 0$, (9) reverts back to [12, eq. (10)] as anticipated.

Notice that (9) involves the evaluation of a double integral for the most general case. However, for the practically important cases of Rician, Nakagami-m, Nakagami-q, or Rayleigh faded desired signal amplitude, we can express $G_0(\cdot)$ in terms of familiar mathematical functions

$$G_0(s) = \frac{1+K_0}{s\bar{p}_0/q + K_0 + 1}\exp\left[\frac{-K_0 s\bar{p}_0/q}{s\bar{p}_0/q + K_0 + 1}\right] \times Q\left(\sqrt{\frac{2K_0(K_0+1)}{s\bar{p}_0/q + K_0 + 1}}, \sqrt{\frac{2(s\bar{p}_0/q + K_0 + 1)\Lambda}{\bar{p}_0}}\right) \quad \text{(Rician)} \quad (10)$$

$$G_0(s) = \frac{1}{\Gamma(m_0)}\left(\frac{m_0}{m_0 + s\bar{p}_0/q}\right)^{m_0} \times \Gamma[m_0, \Lambda(s/q + m_0/\bar{p}_0)] \quad \text{(Nakagami-}m\text{)} \quad (11)$$

$$G_0(s) = \frac{q}{\sqrt{[s\bar{p}_0 + q]^2 - [s\bar{p}_0 b_0]^2}} - \frac{q}{s(1-b_0^2)\bar{p}_0 + q} \times I_e\left(\frac{b_0 q}{s[1-b_0^2]\bar{p}_0 + q}, \frac{\Lambda[1-b_0^2]\bar{p}_0 q}{s[1-b_0^2]\bar{p}_0 + q}\right) \quad \text{(Nakagami-}q\text{)} \quad (12)$$

where $Q(\sqrt{2a}, \sqrt{2b}) = \int_b^{\infty} \exp(-t-a) I_0(2\sqrt{at})\, dt$ denotes the first-order Marcum-Q function, $\Gamma(a,x) = \int_x^{\infty} \exp(-t)\, t^{a-1}\, dt$ is the complementary incomplete Gamma function, and Rice's I_e-function is related to the Marcum-Q function as $I_e(V/U, U) = (U/W)[Q(\sqrt{U+W}, \sqrt{U-W}) - Q(\sqrt{U-W}, \sqrt{U+W})]$, where $W = \sqrt{U^2 - V^2}$.

Substituting $K_0 = 0$ in (10) or $m_0 = 1$ in (11), we get a closed-form expression for $G_0(\cdot)$ (which corresponds to the case of Rayleigh-faded desired signal amplitude), i.e.,

$$G_0(s) = \frac{\exp[-\Lambda(s/q + 1/\bar{p}_0)]}{1 + s\bar{p}_0/q}. \quad (13)$$

Moreover if m_0 is a positive integer, then (11) reduces to

$$G_0(s) = \left(\frac{m_0}{m_0 + s\bar{p}_0/q}\right)^{m_0} \exp(-s\Lambda/q - m_0\Lambda/\bar{p}_0) \times \sum_{k=0}^{m_0-1} \frac{[\Lambda(s/q + m_0/\bar{p}_0)]^k}{k!} \tag{14}$$

since $\Gamma(n+1, x) = n! \exp(-x) \sum_{k=0}^{n} (x^k/k!)$ for integer n.

Alternatively, the probability of satisfactory reception illustrated by (6) may be restated as

$$\Pr\{S\} = \frac{1}{2} - \frac{F_0(\Lambda)}{2} - \frac{1}{\pi}\int_0^\infty \frac{1}{t}\text{Imag} \times \left[\int_\Lambda^\infty \exp(-jtp_0) f(p_0)\, dp_0 \prod_{k=1}^{L} \phi_k(-jqt)\right] dt \tag{15}$$

by invoking Gil-Pelaez inversion theorem. Hence

$$P_{\text{out}} = \frac{1}{2} + \frac{F_0(\Lambda)}{2} + \frac{1}{\pi}\int_0^\infty \frac{1}{t}\text{Imag} \times \left[G_0(jtq) \prod_{k=1}^{L} \phi_k(-jqt)\right] dt. \tag{16}$$

It should be emphasized that the outage expressions (9) and (16) hold for arbitrary desired user and interfering signal statistics. Besides, our derivation of the general results is short and simple. To the best of the our knowledge, no analytical expressions for computing the outage probability with minimum signal power constraint has been derived previously desired signal amplitude is Rician distributed. In fact, even for the simplest interference-limited case $(\Lambda = 0)$ Stuber [9] pointed out that a detailed analysis for the case of Rician faded desired signal with multiple Rician/Rayleigh is required because the existing analytical approaches do not lend themselves to analyze this case. He quotes, "…Unfortunately, this does not result in a simple multiplication of Laplace transform as before, and hence, alternative methods for finding the exact pdf must be employed. This is an open research problem." ([9], pp. 140).

C. Interference-Limited Scenario with Correlated Interferers

In [12], we present an efficient numerical approach for computing the probability of outage in an interference-limited scenario with independent interferers. The final result [12, eq. (10)] can be obtained as special case of the refined outage criterion derived in the preceding sections [i.e., by letting $\Lambda = 0$ in (3) or (9)]

$$P_{\text{out}} = \frac{1}{2n}\sum_{i=1}^{n} \tilde{\phi}_\gamma\left[\frac{(2i-1)\pi}{2n}\right] + R_n \tag{17}$$

where $\tilde{\phi}_\gamma(\theta) = \text{Real}\{[1 - j\tan(\theta/2)]\phi_0[(c + jc\tan(\theta/2))/q]$ $\phi_I[-c - jc\tan(\theta/2)]\}$ and the remainder term can be bounded using

$$R_n = \frac{\pi^2}{6n^2}\tilde{\phi}_\gamma^{(2)}(\zeta), \qquad \text{for some } 0 < \zeta < \pi. \tag{18}$$

Equation (18) is appealing for numerical evaluations since it only involves a second-order derivative of the MGF instead of $2n$th-order derivative of the MGF using the formula furnished in ([14], pp. 889). Furthermore, it is apparent that the remainder term decays at a rate $1/n^2$ as n increases because the derivative term is independent of the sample size.

Notice that (17) also holds when the interferers are generically correlated. However, in this case we need to determine the corresponding MGF $\phi_I(\cdot)$. Fortunately, the MGF of the sum of interfering signal powers for generically correlated Rayleigh, Rician or Nakagami-m interferers can be determined quite easily, and they are summarized below.

1) Correlated Nakagami-m Interferers: The MGF of $I = \sum_k p_k$ for arbitrarily correlated Nakagami-m interferers (assuming that the fading severity index is common to all the interferers) can be readily shown as

$$\phi_I(s) = \det(I + sR\xi)^{-m} = \prod_{k=1}^{L} \frac{1}{(1 + s\lambda_k)^m} \tag{19}$$

where I is the $L \times L$ identity matrix (L denotes the number of interferers), ξ is a positive definite matrix of dimension L (determined by the covariance matrix), and R is a diagonal matrices defined as $R = \text{diag}(\bar{p}_1/m, \ldots, \bar{p}_L/m)$, m is the fading parameter and λ_k is the kth eigenvalue of matrix $R\xi$. For the special case of constant correlation model with identical fading index and mean signal power for all the interferers, (19) may be rewritten as

$$\phi_I(s) = \frac{m^{mL}}{[\bar{p}s(1 - \rho + L\rho) + m]^m [\bar{p}s(1 - \rho) + m]^{m(L-1)}} \tag{20}$$

where ρ denotes the correletion coefficient and L is the number of cochannel interferers.

2) Correlated Rician Interferers: Let ν denote a complex normal random vector with mean $\mu = [\mu_1, \ldots, \mu_L]^T$ and covariance $C = E[(\nu - \mu)(\nu - \mu)^H]$ (Hermitian positive definite). If ν corresponds to the gain vector of the interferers, then the instantaneous total interference power is given by $I = \nu^H\nu$. Then the MGF of I in correlated Rician fading can be expressed in a Hermitian quadratic form [20, eq. (B-3-4)], namely

$$\phi_I(s) = \exp\left[\sum_{k=1}^{L} \frac{-s\lambda_k}{1 + s\lambda_k}|R_k|^2\right] \prod_{k=1}^{L} \frac{1}{1 + s\lambda_k} = \prod_{k=1}^{L}\left\{\frac{1}{1 + s\lambda_k}\exp\left[\frac{-s\lambda_k}{1 + s\lambda_k}|R_k|^2\right]\right\} \tag{21}$$

where λ_k is the kth eigenvalue of the covariance matrix C, R_k is the kth element of vector R defined as $R = U^H C^{-1/2}\mu$, $C^{-1/2}$ is the inverse of Hermitian square-root of C, and U^H can be obtained from the covariance matrix by singular value decomposition, viz., $C = U\xi U^H$.

3) Correlated Rayleigh Interferers: For the particular case of $m = 1$, (19) reduces to

$$\phi_I(s) = \prod_{k=1}^{L} \frac{1}{(1 + s\lambda_k)}. \tag{22}$$

The above expression can also be obtained from (21) by recognizing that R is a zero vector since ν_k $(k = 1, \ldots, L)$ are zero-mean Gaussian RVs $(\mu_k = 0)$ for the specific case of Rayleigh fading. Using these MGFs (19)–(22), we can now investigate the effects of correlated interferers on the probability of outage in mobile radio systems in conjunction with (17).

III. Derivation of Closed-Form Formulas for the Outage Probability

In this section, we derive exact closed-form expressions for the outage probability assuming that a) the desired signal amplitude is Rayleigh or Nakagami-m (integer m) distributed, or b) the signal amplitudes for the nonidentical interferers follow Nakagami-m distribution with positive integer fading severity index or the Rayleigh distribution. The underlying principle of the derivation with condition a) is that the inversion of MGF of I to obtain the pdf $f_I(\cdot)$ can be circumvented if the desired signal power is a sum of exponential RVs. On the other hand, the condition b) allows us to write $F_I(\cdot)$ in the interference-limited case in terms of a sum of exponential RVs and thus facilitates the evaluation of the final integral in a closed form.

A. Treating Noise as Interference

Equation (2) may also be rewritten as

$$P_{\text{out}} = 1 - \int_0^\infty f_I(y) \int_{qy+\Lambda}^\infty f_{p_0}(p_0)\, dp_0\, dy = \int_0^\infty f_I(y) F_0(qy+\Lambda)\, dy. \tag{23}$$

When $\Lambda = 0$, (23) reduces to the familiar expression for outage in an interference-limited scenario (i.e., [12, eq. (2)]).

If we assume the desired signal to be Nakagami-faded with integer m, then a closed-form expression for the refined outage probability can be directly obtained from Laplace transform of a derivative property

$$\begin{aligned} P_{\text{out}} &= \int_0^\infty \left[1 - \exp\left(\frac{-m_0(qy+\Lambda)}{\bar{p}_0}\right) \times \sum_{i=0}^{m_0-1} \frac{1}{i!}\left(\frac{m_0(qy+\Lambda)}{\bar{p}_0}\right)^i\right] f_I(y)\, dy \\ &= 1 - \exp\left(\frac{-m_0\Lambda}{\bar{p}_0}\right) \sum_{i=0}^{m_0-1} \frac{1}{i!} \sum_{l=0}^{i} {}^iC_l \left(\frac{m_0\Lambda}{\bar{p}_0}\right)^{i-l} \times \int_0^\infty \left(\frac{m_0 qy}{\bar{p}_0}\right)^l \exp\left(\frac{-m_0 qy}{\bar{p}_0}\right) f_I(y)\, dy \\ &= 1 - \exp\left(\frac{-m_0\Lambda}{\bar{p}_0}\right) \sum_{i=0}^{m_0-1} \frac{1}{i!} \sum_{l=0}^{i} {}^iC_l \left(\frac{m_0\Lambda}{\bar{p}_0}\right)^{i-l} \times (-s)^l \frac{d^l}{ds^l}\left[\prod_{k=1}^{L} \phi_k(s)\right]\Bigg|_{s=m_0 q/\bar{p}_0} \end{aligned} \tag{24}$$

where ${}^xC_y = (x!/y!(x-y)!)$ is the coefficient of binomial expansion. For the special case of Rayleigh-faded desired signal amplitude $(m_0 = 1)$, (24) reduces to

$$P_{\text{out}} = 1 - \exp(-\Lambda/\bar{p}_0) \prod_{k=1}^{L} \phi_k(q/\bar{p}_0). \tag{25}$$

It is worth pointing out that in arriving to (24) and (25) we do not impose any restrictions on the interferers signal statistics. Hence, these formulas are useful to gain some insights as to how the interferer statistics affect the outage performance and predict the irreducible outage probability due to the presence of the thermal noise. Also, the outage performance by considering generically correlated Rician, Nakagami-m, or Rayleigh interferers in the presence of receiver background noise may be evaluated using (24) by replacing $\prod_{k=1}^{L} \phi_k(\cdot)$ with $\phi_I(\cdot)$ given in Section II-C. Furthermore, substituting $\Lambda = 0$ in (24) and (25), we get new closed-form expressions for the outage probability in the interference-limited case, given that the desired signal amplitude follows Nakagami-m distribution with positive m and Rayleigh fading, respectively.

Although the computation of (24) appears to be difficult because we need to perform an lth-order derivative of the MGF $\phi_I(\cdot)$ with respect to s, but this task can be performed quite easily using common mathematical software packages such as Maple or Mathematica. Next, we will show that it is also possible to replace the lth-order derivative by finite sum of algebraic functions for all common fading channel models (Rayleigh, Rician, Nakagami-m, and Nakagami-q).

By applying Leibnitz's differentiation rule [15, eq. (0.42)], it can be shown that

$$\begin{aligned} \frac{d^l}{ds^l}\left[\prod_{k=1}^{L} \phi_k(s)\right] &= \sum_{n_1=0}^{l} \sum_{n_2=0}^{n_1} \cdots \sum_{n_{L-1}=0}^{n_{L-2}} \binom{l}{n_1}\binom{n_1}{n_2} \cdots \\ &\quad \binom{n_{L-2}}{n_{L-1}} \phi_1^{(l-n_1)}(s)\phi_2^{(n_1-n_2)}(s) \ldots \phi_L^{(n_{L-1})}(s) \end{aligned} \tag{26}$$

where notation $\phi_k^{(n)}(\cdot)$ denotes the nth-order derivative of $\phi_k(\cdot)$. For instance, when $L = 3$, (26) reduces to

$$\frac{d^l}{ds^l}\left[\prod_{k=1}^{3} \phi_k(s)\right] = \sum_{n_1=0}^{l} \sum_{n_2=0}^{n_1} \binom{l}{n_1}\binom{n_1}{n_2} \times \phi_1^{(l-n_1)}(s)\phi_2^{(n_1-n_2)}(s)\phi_3^{(n_2)}(s).$$

From the above, it is apparent that only the knowledge of $\phi_k^{(n)}(\cdot)$ $(n \le l)$ is further required to circumvent the need to perform $\phi_I^{(l)}(\cdot)$ in (24) numerically. Fortunately, closed-form expressions for $\phi_k^{(n)}(\cdot)$ in a variety of fading channel models can be determined quite easily and they are tabulated in Table II. The correlated interferers case may also be treated in a similar fashion because the MGF $\phi_I(\cdot)$ can be written in a product form similar to (26).

TABLE II
nTH-ORDER DERIVATIVE OF THE MGF OF SIGNAL POWER $\phi_k^{(n)}(\cdot)$

Channel Model	n-th order derivative of the MGF of the signal power
Rayleigh	$\phi_k^{(n)}(s) = \frac{(-\bar{p}_k)^n n!}{(1+s\bar{p}_k)^{1+n}}$
Rician	$\phi_k^{(n)}(s) = \frac{(-\bar{p}_k)^n (n!)^2 (1+K_k)}{(1+K_k+s\bar{p}_k)^{1+n}} \exp\left(\frac{-sK_k\bar{p}_k}{1+K_k+s\bar{p}_k}\right) \sum_{i=0}^{n} \frac{1}{(i!)^2(n-i)!}\left(\frac{K_k(1+K_k)}{1+K_k+s\bar{p}_k}\right)^i$
Nakagami-q	$\phi_k^{(n)}(s) = \frac{n!}{2^{2n}\sqrt{[1+s\bar{p}_k(1+b_k)][1+s\bar{p}_k(1-b_k)]}}\left[\frac{-\bar{p}_k(1+b_k)}{1+s\bar{p}_k(1+b_k)}\right]^n \times \sum_{w=0}^{n} \frac{(2w)![2(n-w)]!}{[w!(n-w)!]^2}\left[\frac{(1-b_k)[1+s\bar{p}_k(1+b_k)]}{(1+b_k)[1+s\bar{p}_k(1-b_k)]}\right]^w$
Nakagami-m	$\phi_k^{(n)}(s) = \frac{(-\bar{p}_k)^n m_k^{m_k}\Gamma(m_k+n)}{(m_k+s\bar{p}_k)^{m_k+n}\Gamma(m_k)}$

Next, we will derive closed-form formulas for the probability outage (2) by assuming that the amplitudes of the interferers are Nakagami-m distributed with positive integer fading severity index. In this case, no restrictions were made on the desired signal statistic and the pdf of I can be expressed in a closed form

$$f_I(x) = \sum_{k=1}^{L}\sum_{l=1}^{m_k} \eta_k^{(m_k-l)} \frac{x^{l-1}}{(l-1)!}\delta_k^l \exp(-\delta_k x) \tag{27}$$

where $\delta_k = m_k/\bar{p_k}$ and the coefficients of the partial fractions are given by

$$\eta_k^{(m_k-l)} = \frac{1}{(m_k-l)!\delta_k^l}\left[\prod_{i=1}^{L}\delta_i^{m_i}\right]\frac{d^{(m_k-l)}}{ds^{(m_k-l)}} \times \left[\prod_{i=1, i\neq k}^{L}(s+\delta_i)^{-m_i}\right]\Bigg|_{s=-\delta_k}.$$

Then, the probability of outage (2) can be evaluated using

$$\begin{aligned}P_{\text{out}} &= 1 - \int_0^\infty f_{p_0}(p_0) F_I\left(\frac{p_0-\Lambda}{q}\right)dp_0 \\ &= \sum_{k=1}^{L}\sum_{l=1}^{m_k}\eta_k^{(m_k-l)}\sum_{i=0}^{l-1}\frac{1}{i!}\sum_{\nu=0}^{i} \\ &\quad\times \exp\left(\frac{\Lambda\delta_k}{q}\right)\binom{i}{\nu}\left(\frac{-\Lambda\delta_k}{q}\right)^{i-\nu} \\ &\quad\times \int_0^\infty\left(\frac{p_0\delta_k}{q}\right)^{\nu}\exp\left(\frac{-p_0\delta_k}{q}\right) f_{p_0}(p_0)\,dp_0 \\ &= \sum_{k=1}^{L}\sum_{l=1}^{m_k}\eta_k^{(m_k-l)}\sum_{i=0}^{l-1}\sum_{\nu=0}^{i}\frac{1}{i!\Lambda^\nu} \\ &\quad\times \exp\left(\frac{\Lambda\delta_k}{q}\right)\binom{i}{\nu}\left(\frac{-\Lambda\delta_k}{q}\right)^{i}\phi_0^{(\nu)}(\delta_k/q)\end{aligned} \tag{28}$$

where the derivatives $\phi_0^{(i)}(\cdot)$ for Rayleigh, Rician, Nakagami-m, and Nakagami-q fading models are summarized in Table II. It is straightforward to extend (28) to include the effect of correlated interferers in Nakagami-m (positive integer m) or Rayleigh fading environments (i.e., by replacing δ_k with $1/\lambda_k$ defined in (19) and also letting $m_1 = \cdots = m_L = m$). To the best of the authors' knowledge, all the results presented in this section has not been reported previously.

B. Minimum Signal Power Constraint

If we assume that all the interferers are Rayleigh distributed with nonidentical fading, then $F_I(\cdot)$ in (7) can be expressed as weighted sum of exponential terms. Then, recognizing the resultant integral is essentially a marginal MGF, we obtain closed-form solutions for several important cases

$$P_{\text{out}} = F_0(\Lambda) + \sum_{k=1}^{L}\left[\prod_{i=1, i\neq k}^{L}\frac{\bar{p_k}}{\bar{p_k}-\bar{p_i}}\right]G_0(1/\bar{p_k}) \tag{29}$$

where $G_0(\cdot)$ for Rician, Nakagami-m, Nakagami-q, and Rayleigh faded signal amplitudes are given in (10)–(14). Using this idea, it is straightforward to extend (29) to consider correlated Rayleigh interferers with dissimilar signal powers. In this case, we replace the mean signal powers $\bar{p_k}$ and $\bar{p_i}$ in (29) with the corresponding eigenvalues of matrix $R\xi$ [as defined in (19)]. To the best of our knowledge, these results are new.

It is also possible to derive a closed-form formula for the minimum signal power constraint case when the desired signal amplitude is Nakagami-m faded (arbitrary m) and by restricting the fading severity index for the interfering signals to any positive integer values. Then, using (27), (7) reduces to

$$\begin{aligned}P_{\text{out}} &= F_0(\Lambda) + \frac{1}{\Gamma(m_0)}\left(\frac{m_0}{\bar{p}_0}\right)^{m_0}\sum_{k=1}^{L}\sum_{l=1}^{m_k}\eta_k^{(m_k-l)} \\ &\quad\times\sum_{i=0}^{l-1}\frac{1}{i!}\left(\frac{\delta_k}{q}\right)^i\int_\Lambda^\infty p_0^{m_0-1+i} \\ &\quad\times\exp\left[\frac{-p_0(m_0+\delta_k)}{\bar{p}_0}\right]dp_0\end{aligned}$$

$$= 1 - \frac{1}{\Gamma(m_0)}\Gamma\left[m_0, \frac{\Lambda m_0}{\bar{p}_0}\right] + \frac{1}{\Gamma(m_0)} \times \sum_{k=1}^{L}\sum_{l=1}^{m_k}\eta_k^{(m_k-l)}\sum_{i=0}^{l-1}\frac{1}{i!} \times \left(\frac{\bar{p}_0\delta_k}{qm_0}\right)^i\left(\frac{m_0}{m_0+\delta_k}\right)^{m_0+i} \times \Gamma\left[m_0+i, \frac{\Lambda(m_0+\delta_k)}{\bar{p}_0}\right]. \quad (30)$$

For the particular case of positive integer m_0, (30) collapses to the result derived in [2], as expected.

C. Interference-Limited Case

Substituting $\Lambda = 0$ in (24) and (25), we immediately get closed-form expressions for the probability of outage given that the desired signal amplitude is Nakagami-m (positive integer m) or Rayleigh distributed. In this case, no restrictions were made on the interfering signals statistics. Similarly, substituting $\Lambda = 0$ in (28) or (7), we get

$$P_{\text{out}} = \sum_{k=1}^{L}\sum_{l=1}^{m_k}\eta_k^{(m_k-l)}\sum_{i=0}^{l-1}\frac{(-\delta_k/q)^i}{i!}\phi_0^{(i)}(\delta_k/q) \quad (31)$$

where $\phi_0^{(i)}(\cdot)$ the derivatives for Rayleigh, Rician, Nakagami-m , and Nakagami-q fading models are summarized in Table II. It is worth mentioning that the partial derivatives involved in the evaluation of $\eta_k^{(m_k-l)}$ can be replaced by finite sum using the procedure highlighted in Section III-A.

It should be emphasized that we do not impose any restrictions on the desired signal statistic but rather on the interfering signals to arrive at (31). This observation is particularly interesting in that the use of Nakagami-m approximation model for the desired user may sometimes lead to misleadingly optimistic result but such an approximation on the interfering signals do not affect the outage performance considerably (a detailed discussion is presented in Section V). As such, these new closed-form solutions can provide very accurate approximations for many cases of practical interest, including correlated interferers scenario. In fact, the outage curves obtained using the approximate closed-form solutions (31) coincide (i.e., cannot be distinguished through visual inspection) with that of computed via an exact integral expression [12, eq. (9)] with arbitrary parameters. As we have pointed out earlier, (31) may also be used to predict the outage probability with correlated interferers. In this case, we replace δ_k in the above expressions with reciprocal of the kth eigenvalue λ_k [as defined in (19)] and letting $m_1 = m_2 = \cdots = m_L = m$.

The above technique may also be employed to analyze another case of Nakagami-m interferers, in which the interfering signals satisfy the constraints $\delta_k = \delta$ for $k = 1, \ldots, L$ and $\sum_{k=1}^{L} m_k = D$ is a positive integer. In this case, the pdf of I is also Gamma distributed. Hence, the final expression will be quite similar to (31).

IV. Extension to Random Number of Interferers

In the preceding sections, the probability of outage is derived by assuming that the number of cochannel interferers L is fixed. Therefore, the analysis corresponds to a fixed channel assignment (FCA) strategy which is typically employed in the first generation macrocellular systems. However, micro- and picocellular systems usually will adopt a certain dynamic channel allocation (DCA) scheme owing to the highly erratic nature of the propagation environment and nonhomogeneous traffic pattern (because the number of handoff attempts has been increased considerably due to small cell sizes and propagation anomalies such as street corner effect) [9]. Outage calculations in this situation can also be handled quite easily by letting L be a random variable (which may take any of the values $\{0, 1, \ldots, N\}$ where N denotes the total number of possible interferers). Thus, the outage probability can be expressed as a weighted sum of the conditional probability of outage

$$P_{\text{out}} = \sum_{L=0}^{N}(P_{\text{out}} \mid L)\Pr\{L\} \quad (32)$$

where $(P_{\text{out}} \mid L)$ is interpreted as the conditional probability of outage given that there are L interferers and $\Pr\{L\}$ is the probability that the number of active interferers is L. The choice of DCA schemes will influence/determine the values for $\Pr\{L\}$. For instance, [2] shows that $\Pr\{L\}$ can be described by a binomial distribution given that each of the N independent and identically distributed cochannel cells have N_S different frequency voice channels with blocking probability B

$$\Pr\{L\} = \binom{N}{L} B^{L/N_s}\left(1 - B^{1/N_s}\right)^{N-L} \quad (33)$$

since the status of whether a cochannel interferer in a given cell is active or not is given by a Bernoulli pdf. The effect of nonidentical blocking probability for different cochannel cells (to reflect the nonuniform traffic loading conditions) can also be treated by replacing the standard binomial pdf in (33) with a generalized binomial pdf.

V. Computational Results and Remarks

In the preceding sections, we have developed general numerical methods for calculating the refined outage probability with arbitrary parameters as well as derive several new closed-form solutions for special cases of the desired user and/or interfering signals statistics (including generically correlated interferers case). Also, the accuracy of (17) for outage probability calculations in interference-limited systems with independent interferers and the influence of coefficient c

TABLE III
COMPARISON BETWEEN THE EXACT OUTAGE PROBABILITY OBTAINED USING (3) AND THE TRUNCATED SERIES EXPRESSION (17) FOR DIFFERENT σ_0 AND VARIOUS n VALUES. WE HAVE ASSUMED $K_0 = 7$, $\mathrm{SIR}/q = 20$ dB, $K_k = [0.2, 3.3, 5.8, 2.7, 0.9, 4.4]$, $\bar{p}_k = [0.2, 0.6, 1.3, 0.7, 0.4, 1]$, $\Lambda = 0$, AND $c = a_{\min}/2$

σ_0	P_{out}	n	$\hat{P}_{out}$
0.5 dB	5.842568875 x 10^{-4}	15	4.919110147 x 10^{-4}
		20	5.842568852 x 10^{-4}
		25	5.842568875 x 10^{-4}
2.0 dB	4.596188514 x 10^{-3}	15	4.111391849 x 10^{-3}
		20	4.596188503 x 10^{-3}
		25	4.596188514 x 10^{-3}
3.5 dB	2.555192659 x 10^{-2}	15	1.593690332 x 10^{-2}
		20	2.555192636 x 10^{-2}
		25	2.555192659 x 10^{-2}

on the convergence rate[2] of the series have been investigated in [12]. Since the applications of our analytical expressions for computing the outage probability of complicated mobile radio scenarios is straightforward, here we will focus on assessing the suitability of Nakagami-m approximation for a Rician RV (i.e., whether it can yield accurate outage predictions) and examining the compatibility between two approaches to deal with receiver background noise in outage calculations, in addition to providing additional numerical results and discussions not elaborated in [12].

In Table III, we assess the accuracy of (17) for a shadowed Rician (desired)/Rician (interferer) mixed fading channel model with six interferers at different values of shadowing spread for the desired user signal. The notation $\hat{P}_{\mathrm{out}}$ is the sum in the right hand side of (17) excluding the remainder term and the exact P_{out} is obtained by computing (3) using Matlab's *quad8* function with an absolute tolerance of 10^{-15}. Our numerical results reveal that the rate of convergence of the series in the shadowing cases is also quite good. Hence, a high accuracy can be easily attained with only a reasonable number of MGF samples (i.e., less than 25 samples).

In some of the previous studies (e.g., [3]), the authors have suggested approximating a Rice RV by a Nakagami-m RV with positive integer m to examine the outage performance of cellular mobile radio networks subject to Rician fading with nonidentical CCI statistics. This was done to circumvent the difficulty in performing the L-fold convolution integral to get $f_I(\cdot)$. Hence, in Figs. 1 and 2, we investigate whether this is a valid (realistic) approximation. In Fig. 1, we compare the outage performance of a cellular network where both the wanted signal and the four interfering signals are Rician-faded with its corresponding Nakagami-m approximation fading model. The fading severity index m (also known as fading figure) is obtained using the relationship $m = (K+1)^2/(2K+1)$ [21]. The interference signal statistics are given by $\bar{p_k} = [1.1, 0.6, 0.9, 2.2]$ and $K_k = [2.5, 1.3, 3.2, 4.7]$. It is apparent from Fig. 1 that the Nakagami[3] model for the desired signal yields misleadingly

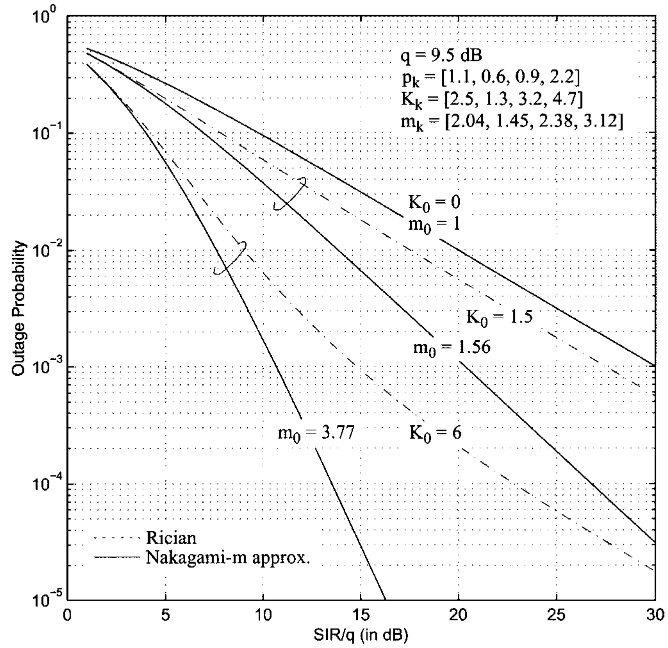

Fig. 1. Comparison of the outage probability between the Rician and its Nakagami-m approximation model.

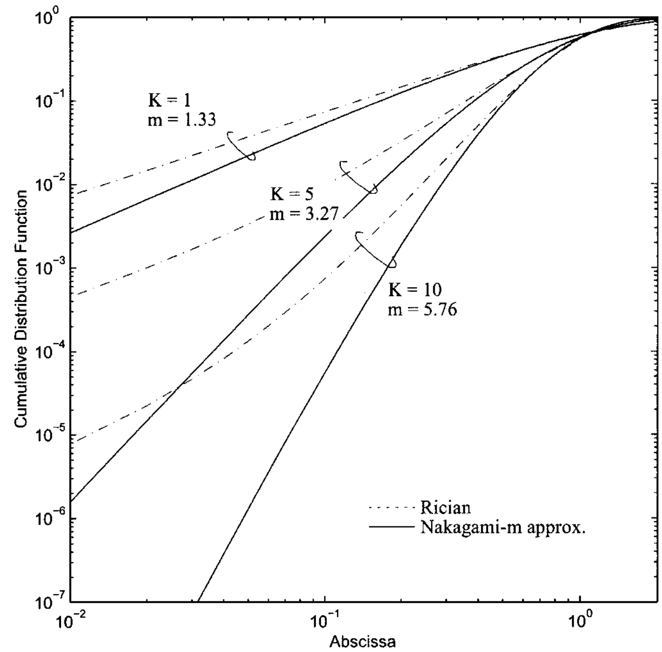

Fig. 2. Comparison between the cdfs of the normalized power for the Rician distribution and their corresponding Nakagami-m RVs.

[2]Ideally, c should be chosen such that $|\phi_\gamma(c + j\omega)|$ decays rapidly as $\omega \to \infty$. Therefore, highest rate of decay is ensured if $s = c$ is a saddle point because at $s = c$, $\phi_\gamma(s)/s$ achieves its minimum on the real axis. To avoid numerical search for the optimal c, it is sufficient in practice to employ a rule of thumb $c = a_{\min}/2$.

[3]It is noted that the real importance of the Nakagami-m fading model lies in the fact that it can often be used to fit experimental data and offers features of analytical convenience in comparison to the Rician distribution. However, the goodness-of-fit tests used by ionospheric physicists to match measured scintillation data to a Nakagami-m do not give special weighting to the deep-fading tail of the distribution [18]. As a result, we sometimes have better fit near the median of the distribution than in the tail region, although the tail behavior is of greater significance to communication system performance analysis.

optimistic results. This observation may be attributed to the fact that the tails of the Rician and its Nakagami-m approximation distributions do not fit very closely. The difference in the behavior below the 1% point of the cdf of the power for these two distributions (see Fig. 2) may have led to a large difference in the required SIR/q to achieve a prescribed value of P_{out}. Comparison between these cdf curves also suggests that some basic results on Nakagami-m approximation model have been overlooked by some researchers previously. In fact, the performance predicted using the approach suggested in [3] will be worse then what is already shown in Fig. 1 because m should be further rounded up to the closest integer value to facilitate the mathematical analysis.

Diesta and Linnartz [4] also obtained a similar result by studying the behavior of the Laplace transform expressions for small arguments (i.e., corresponding to large SIR) by expanding into MacLaurin series. Further, they suggested that Nakagami pdf with $m > 1$ does not model deep fades, so it predicts much better performance than Rician fading. But the Nakagami-m model predicts the same exact performance as that of Rician distribution when $K_0 = 0$ because both these distributions revert to the Rayleigh fading case. From Fig. 1, we can see that the difference in required SIR/q to achieve $P_{\mathrm{out}} = 10^{-3}$ between the actual and approximation model is 7.2 and 4.1 dB when $K_0 = 1.5$ and $K_0 = 6$, respectively. But the Nakagami-m model predicts the exact performance that of Rician distribution when $K = 0$. We conclude that approximating a Rician RV of the desired signal by a Nakagami-m is only accurate for its extreme limiting cases—Rayleigh fading $(K = 0)$ and nonfading $(K = \infty)$. The discrepancy becomes larger as SIR/q increases. The approximation is inaccurate even at moderate SIR/q values. This is particularly interesting because our unified expression handles the Rician-faded desired signal as well as the mixed-fading situation very well.

In Fig. 3, we confirm that one may accurately model the Rician-faded interfering signal by a Nakagami-m signal. Different from Fig. 1, we only model the unwanted Rician-faded signals using its equivalent Nakagami-m approximation and the desired signal is still modeled as Rician-faded (i.e., a mixed Rician/Nakagami fading case). Specifically, we show that the discrepancy observed in our Fig. 1 is attributed to the inaccurate modeling of the desired signal rather than the interfering signal statistics. The curves obtained by modeling the interferers as Nakagami-faded signals yields virtually identical results with that evaluated directly as Rician-faded interfering signals (we cannot distinguish their difference from Fig. 3 because the error due to the approximation is negligible). Therefore, it is expected that our new closed-form formulas derived in Sections III-B and III-C will have broader range of applications since they can closely approximate (i.e., predict accurately) the outage performance of mobile radio systems with arbitrary parameters. Fig. 3 also illustrates the effect of increasing the number of interferers on the outage performance. Note that the abscissa is the average single interferer power. When $L = 1$ the interference statistics $\bar{p_1} = 1.0$ and $K_1 = 2.2$, which also correspond to the first entry of vectors $\bar{p}_k$ and K_k, respectively. The penalty in the required SIR/q to achieve a prescribed outage gets larger with the CCI (i.e., higher number of interferers), as anticipated. As well, when

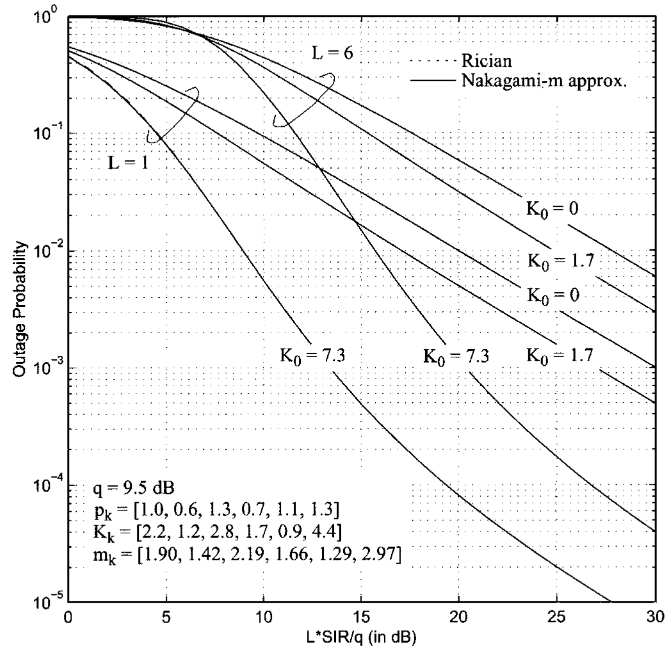

Fig. 3. Comparison of the outage between the Rician and its equivalent Nakagami-m approximation model of the interfering signals (the desired signal is modeled as Rician-faded for both cases) for different K_0 as well as number of interferers L.

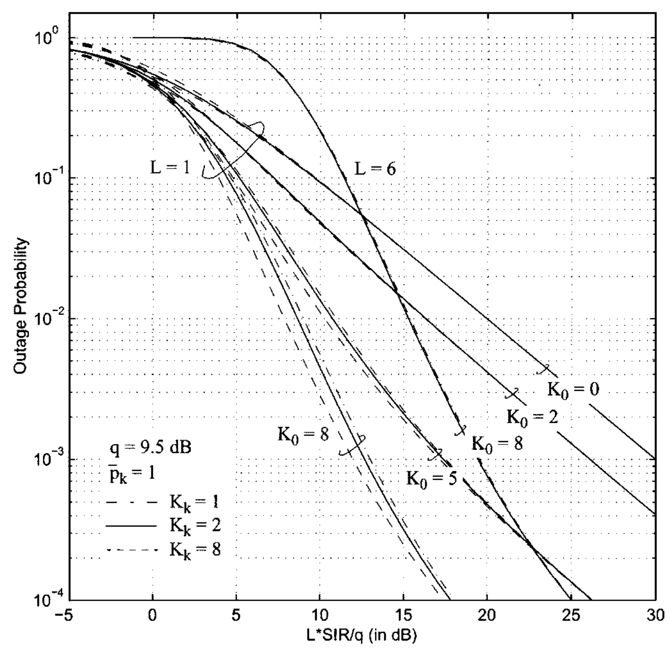

Fig. 4. Outage probability versus average signal power to a single interferer power ratio in a Rician (desired)/Rician (interferer) fading channel.

the Rice factor of the wanted signal increases, a desired outage is achieved at smaller SIR/q because severity of the signal fading decreases.

In Fig. 4, we further substantiate the trends as well as the conclusions drawn from Fig. 3. In particular, we show that the outage performance is not too sensitive to the variations in the fading severity index of the interfering users. For simplicity, let us assume that all fading statistics of the interfering signals are identical. We see that the outage probability does not vary too much despite a considerable perturbation in the Rice factor of the interfering signals if K_0 is small or if L is large. This observation in turn explains why there is negligible amount of error of the outage performance by modeling a Rician RV by a Nakagami-m RV for the interferer signals. However, note that the outage probability becomes more susceptible to the inaccuracies in modeling the interfering signals as K_0 increases when there are only a few number of interferers (i.e., single interferer case) since the spread between the curve gets larger. It can also be seen from Fig. 4 that as K_k $(k = 1, \ldots, L)$ decreases, the outage probability gets larger for a given SIR/q in the moderate range (say 5–25 dB) if $K_0 > 0$. This can be explained by realizing that for a given SIR/q, K_0 and $\bar{p}_k$, $\bar{p}_0$ must be increased if K_k decreases to keep $L \times \mathrm{SIR}/q$ the same because

$$\mathrm{SIR} = \frac{\bar{p}_0}{\sum_k \bar{p}_k} = \frac{(K_0+1)\bar{p}_{0(\text{diffused})}}{\sum_k (K_k+1)\bar{p}_{k(\text{diffused})}} \qquad (34)$$

where $\bar{p}_{i(\text{diffused})}$ denotes the average power in the diffused path. If $K_0 \gg 0$, then the outage probability declines very rapidly even with a slight increase in $\bar{p}_0$ (i.e., 'waterfall curve') in the range of interest, which explains the above phenomena. At the other extreme (i.e., when the desired signal is Rayleigh-faded), the outage probability gets larger as K_k increases. If we change the abscissa to $\bar{p}_0$, then we will find that the outage will always be higher for a larger (K_k), as anticipated.

In Fig. 5, we investigate the accuracy of modeling the shadowed Rician-faded desired signal pdf by its corresponding shadowed-Nakagami approximation. Here we assume that all the six interfering signals envelope follow the Nakagami distribution with the following statistics: $m_k = [1.0, 0.5, 0.75, 0.6, 0.65, 0.9]$ and $\bar{p}_k = [0.1, 0.6, 0.3, 0.7, 0.2, 0.5]$. The Nakagami distribution with fading figure $m < 1$ models a fading channel condition that is more severe than the Rayleigh distribution (which cannot be represented using a Rician distribution). The Rice factor of the desired signal is assumed to be $K_0 = 6.1$. Hence, the fading severity parameter of the Nakagami approximation model is $m_0 = 3.819$. From Fig. 5, we see that the outage probabilities for the shadowed Nakagami-m channel and the shadowed Rician channel have good agreements for small SIR/q; otherwise the deviation can be substantial, in particular when the shadowing is very light. The shadowing has the net effect of shifting the abscissa (i.e., SIR/q) when the departure begins to the right. Our results reveal that this approximation is plausible if the desired signal experience moderate or heavy shadowing (i.e., large σ_0) or in the limiting case (i.e., exact for the Suzuki fading environment). It is pointed that in order to apply the results in [8], m_0 should be rounded up to 4, inducing additional error to the approximation model. As such, the approximation method may be inadequate in some cases and cannot be justified for a general fading environment. However, the intuitive claim by the authors that the accuracy of approximating the shadowed Rician-faded desired signal by a shadowed-Nakagami improves when a strong specular path is available is correct.

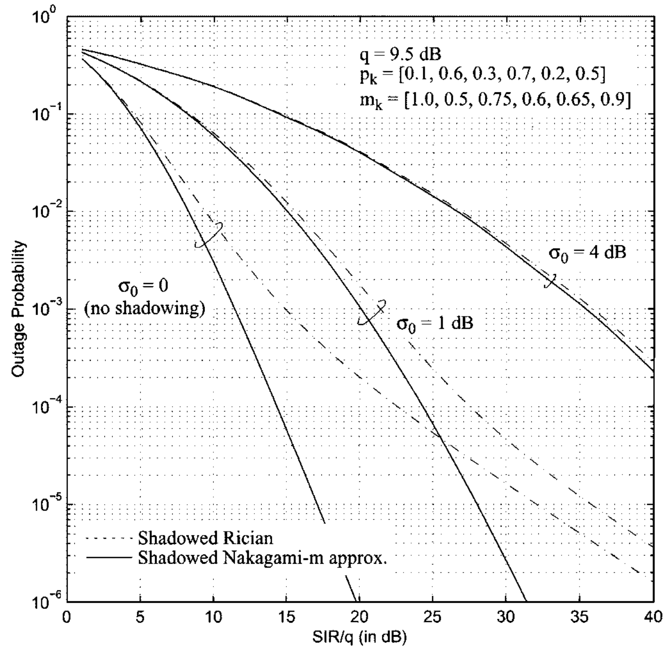

Fig. 5. Comparison of the outage between the shadowed Rician-faded desired signal case ($K_0 = 6.1$) and its equivalent shadowed Nakagami-m approximation model for different shadowing spreads σ_0 of the desired user signal.

In Table IV, we provide an example to show the application of (18) to derive an upper bound on the remainder term. By taking the magnitude of R_n, we can rewrite (18) as

$$R_n^{(\max)} \le \frac{\pi^2}{6n^2} \times \max|\breve{\phi}''(\zeta)|, \qquad \text{for } 0 < \zeta < \pi. \qquad (35)$$

For the sake of illustration, consider the following interferers signal statistics: $K_k = [1.2, 0, 1.5]$ and $P_k = [0.7, 0.3, 0.5]$ for $k = 1, \ldots, L$, and $\mathrm{SIR}/q = 20$ dB. From Table IV, we observe that the remainder term diminishes very rapidly with the increasing number of MGF samples, as anticipated. R_n also decreases as K_0 increases, for a given moderate value of n. Comparison between the entries in the second and third column and/or fourth and fifth column of Table IV reveal that the upper bound derived using (18) is quite loose. Consequently, if (35) is used to estimate the number of MGF samples required to achieve a specified accuracy, it will yield a rather pessimistic result. Nevertheless, this formula provides a useful information about the rate of convergence of the series given in (17), i.e., R_n diminishes by at least the reciprocal of n^2 with increasing n. Similar conclusions cannot be drawn from the expression for the remainder term found in ([14], pp. 889) for GCQ formula.

Next, in Fig. 6, we investigate the effect of correlated interferers on the outage performance. To generate this plot, we have assumed that there are four identical Nakagami-m cochannel interferers with fading severity index ($m = 0.5$ or $m = 0.8$) and signal strength $\bar{p} = 1.2$. The background noise is assumed to be negligible. From this figure, it is apparent that correlated interferers tends to be less harmful. One may also arrive to this conclusion by substituting (20) into (25) (i.e., the desired signal is assumed to be Rayleigh-faded) with $\Lambda = 0$, and then considering the two limiting cases of the correlation coefficient: $\rho = 0$ (uncorrelated case) and $\rho = 1$ (fully correlated case). Since

TABLE IV
COMPARISON BETWEEN THE UPPER BOUND (35) FOR R_n AND ACTUAL $R_n = P_{\rm out} - \hat{P}_{\rm out}$ [OBTAINED USING (3) AND (17)] AT DIFFERENT K_0

Rice Factor K_0	Remainder term R_n			
	$n = 5$		$n = 10$	
	Upper bound	Exact	Upper bound	Exact
0	5.53×10^{-2}	3.877×10^{-7}	1.38×10^{-2}	3.278×10^{-13}
2.1	2.13×10^{-2}	1.487×10^{-7}	5.32×10^{-3}	1.254×10^{-13}
4.7	3.07×10^{-3}	2.119×10^{-8}	7.67×10^{-4}	1.763×10^{-14}
6.8	5.54×10^{-4}	3.758×10^{-9}	1.39×10^{-4}	3.075×10^{-15}

TABLE V
COMPARISON BETWEEN THE LAPLACE INVERSION METHOD (17) AND GIL-PELAEZ FOURIER INVERSION METHOD (36) FOR OUTAGE ANALYSIS IN TERMS OF THE NUMBER OF GCQ SAMPLES REQUIRED TO ACHIEVE A RELATIVE ACCURACY BETTER THAN 1%. INTERFERERS STATISTICS: $\mathrm{SIR}/q = 15$ dB, $K_k = [0.4, 1.3, 5.0, 2.7]$, $\bar{p}_k = [1.1, 0.9, 1.8, 1.2]$, AND $c = a_{\min}/2$

Rice factor K_0	Exact Outage Probability P_{out}	Number of GCQ samples required to achieve an accuracy of $\varepsilon = \left\vert \frac{P_{out} - \hat{P}_{out}}{P_{out}} \right\vert \times 100 < 1\%$	
		Laplace Inversion Method	Gil-Pelaez Inversion Method
0	3.106373×10^{-2}	5	180
2.8	8.184924×10^{-3}	5	350
5.2	1.625258×10^{-3}	5	786
8.6	1.569834×10^{-4}	4	2527

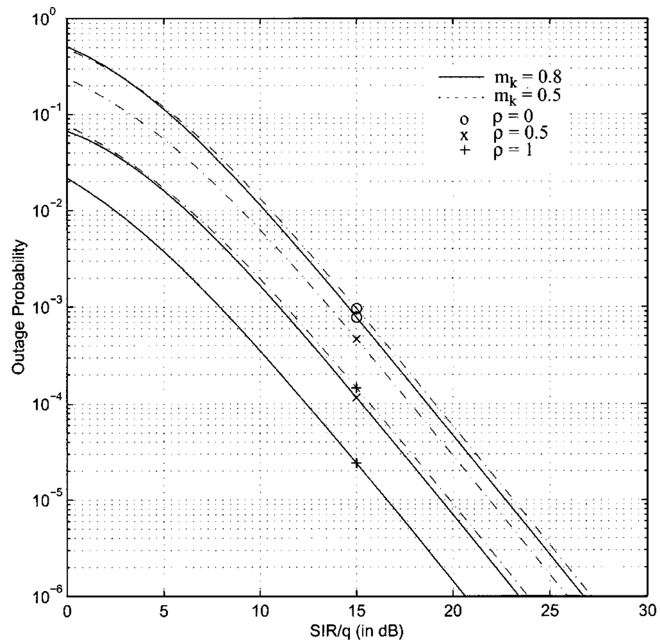

Fig. 6. Effect of correlated interferers (constant correlation model) on the outage performance with $m_0 = 2.5$.

$(m/L\bar{p}s + m) > (m/m + \bar{p}s)^L$ for practical values of m and $\bar{p}$, the outage probability will be smaller as ρ gets larger. Besides, the fading figure of the interferer signals plays an important role in the determination of the outage performance when the correlation is considered (because the spread between the curves for the two limiting cases of ρ becomes wider as m increases). Similar trends were also observed for other correlation models (not shown in the figure) using the MGF for generically correlated Nakagami-m interferers [see (19)] with different eigenvalues.

Fig. 7 reveals that the coarse approximation of the refined outage probability predicted by treating noise as CCI tends to yield slightly pessimistic result compared to the minimum power constraint approach. In fact, this upper bound gets tighter for both very small and very large SIR/q regions. However, the discrepancy between these two curves (at moderate SIR/q values) gets worse if the desired signal does not experience severe fading. When both the methods yield comparable performance and for small Λ, (3) or (5) will be preferred for rapid assessment of the system performance in a myriad of fading scenarios compared to (9) or (16) since they require less programming effort. If the desired signal amplitude is Nakagami-faded (with integer m), then (24) is recommended. However, (9) should be used if accurate outage performance measure is desired (instead of an upper bound). For nonidentical Rayleigh-faded interfering signals, (29) is suggested to improve the computational efficiency.

Finally, in Table V, we show that it is far more efficient to compute the outage probability via a Laplace inversion method instead of the Gil-Pelaez inversion formula (based on Fourier inversion method) specifically at lower outage values. Using

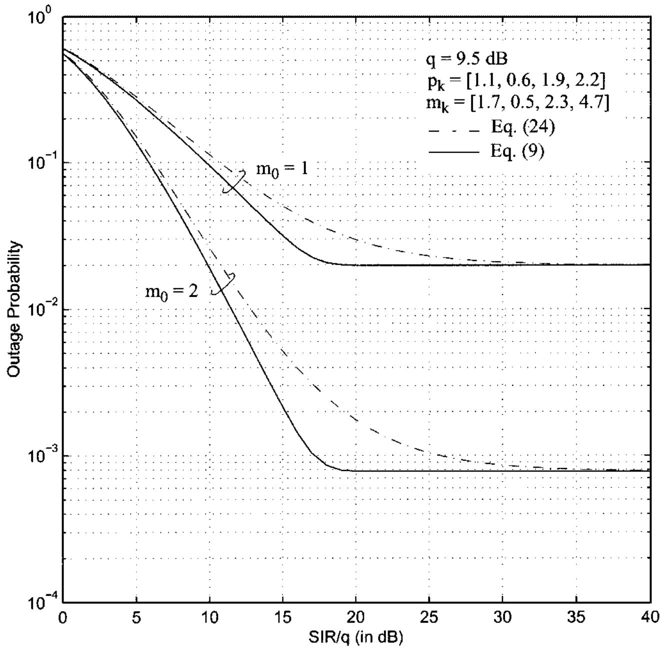

Fig. 7. Assessment of the compatibility and applicability of the two approaches that either treat noise as CCI or consider a minimum detectable receiver signal threshold in the presence of receiver noise in a Nakagami-m fading channel.

variable substitution $t = (1-x)/(1+x)$ in (5) and then applying GCQ formula, we get a series expression for evaluating the outage probability in an interference-limited mobile radio system $(\Lambda = 0)$

$$P_{\text{out}} = \frac{1}{2} - \frac{2}{n}\sum_{i=1}^{n} \frac{\text{Imag}\left[\phi_\gamma\left(-j\tan^2\left(\frac{(2i-1)\pi}{4n}\right)\right)\right]}{\sin\left(\frac{(2i-1)\pi}{2n}\right)} + R_n. \tag{36}$$

Notice that formulas like (5) when evaluated numerically have the form of 0.5 minus a sum [see (36)]. When the tails of the distribution are sought, the sum is also close to 0.5. As a consequence, many steps of numerical integration of the oscillatory integrand are needed to determine the sum accurately enough so that significant figures are not lost by roundoff errors, particularly for very small outage values. Since the Laplace inversion integral method circumvents this issue, then it is expected that the rate of convergence of series shown in (17) to be much faster than (36). This has been validated in Table V. Even if we reduce ϵ to 0.01%, the number of samples required is less than 10 for the Laplace inversion method. However, the corresponding n for (36) grows too large and becomes unmanageable for practical use. This observation in turn suggests that other numerical techniques (e.g., trapezoidal rule) may be more appropriate to approximate (5) or (16) instead of using the GCQ formula. It is also important to highlight at this point that the number of samples required by the Laplace inversion method to achieve a prescribed accuracy is actually smaller than the value tabulated in Table V because we can further optimize parameter c using the procedure suggested in [12].

VI. Conclusion

Despite many years of research, closed-form expressions for outage in certain fading environments are not available. In this paper, however, we have developed new unified outage expressions for all common fading distributions and the outage can be defined as noise-limited or minimum signal power constrained. These exact outage expressions require the knowledge of the MGF (noise-limited case) or marginal MGF and the pdf of the signal power (for minimum signal power constrained model), and offers a convenient method to evaluate the outage with arbitrary parameters. It is a powerful tool for outage analysis—not imposing any restrictions while being easy to program. Moreover, we have shown that, in contrast to common belief, it is not very accurate to approximate the Rician or the shadowed Rician-faded wanted signal statistics by a Nakagami-m or the shadowed Nakagami-m distribution even at moderate SIR/q specifically if the desired signal experience light shadowing. Fortunately, the use of this approximation is unnecessary because our solution can handle Rice fading scenarios quite easily. It is also shown that the outage performance predicted by treating receiver noise as interference tends to be pessimistic compared to the case when a minimum signal power requirement is imposed. Their discrepancy gets larger if the desired signal experiences less severe fading.

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