The Use of Interleaving to Reduce the Peak-to-Average Power ratio of an OFDM Signal

A. D. S. Jayalath and C. Tellambura

School of Computer Science and Software Engineering

Monash University Clayton, Victoria 3168 Australia

e-mail :chintha@dgs.monash.edu.au

Abstract- **An interleaver based technique for improving the Peak-to-Average Power Ratio (PAP) of an orthogonal frequency division multiplexing (OFDM) signal is presented. For this technique, *K*-1 random interleavers are used to produce *K*-1 permuted sequences from the same information sequence. *PAPs* of the permuted sequences and the original information sequence are then computed using of *K* oversampled FFTs (OFFTs). The sequence with the lowest PAP is chosen for transmission. Complementary cumulative density function (CCDF), of PAP of an interleaved OFDM signal is observed. Results show that for 256 subcarriers and QPSK data symbols, even with $K=2$, the 0.1% CCDF is reduced by 1.3 dB and with K=4, is reduced by 2dB. The 0.1% CCDF can be reduced by 3 dB and 0.01% CCDF by 4dB at a cost of 16 OFFTs and a data rate loss of less than 0.8% with *K=16*. Further statistical improvement for the PAP is obtained by combining this approach and the partial transmit sequence (PTS) approach. This paper also proposes an adaptive approach to reduce the complexity of the interleaving technique.**

I. INTRODUCTION

OFDM is a promising solution for high data rate transmissions in frequency selective fading radio channels [1]. The advances in VLSI technologies have given a new lease of life to this technique for transmitting information through overlapping carriers. Current OFDM applications include digital audio broadcasting, asynchronous digital subscriber lines, digital video broadcasting and wireless LANs. OFDM is commonly implemented using Discrete Fourier Transform (DFT) techniques. A main disadvantage of OFDM is potentially high PAP values. In practice, peak signal levels are constrained by design factors such as battery power (on portable equipment) or regulatory limits that prevent adjacent channel interference. The use of non-liner amplifiers and digital hard limiting causes inefficiency, interference and performance degradation.

These drawbacks have motivated the search for PAP reduction techniques and a large number of solutions have been proposed to date. Both the PTS and the selective mapping (SLM) approaches proposed in [2], are based on the phase shifting of clusters of data symbols and the multiplication of the data frame by random vectors. In PTS, the input data frame of N symbols is partitioned into M clusters. The N/M subcarriers in each cluster (except the first cluster) are phase shifted by a constant factor; M-1 phase factors are then selected such that the resultant PAP is minimized. Unfortunately, finding the best phase factors is a highly complex and non-linear optimization problem. Therefore some attempts have been made to reduce the complexity of the optimization: [3] has presented a suboptimal iterative flipping algorithm while [4] has derived an alternative optimization criterion. Yet even with full optimization, the maximum 0.1%PAP reduction is about 4.2dB for a 256 QPSK-carrier system.

It is noteworthy that the PAP of the continuous-time OFDM signal cannot be computed [4] precisely by the use of the Nyquist sampling rate, which amounts to N samples per symbol. In this case, signal peaks are missed and PAP reduction estimates are unduly optimistic. Oversampling by a factor of 4 is sufficiently accurate and is achieved by simple computing $4N$-point zero-padded IDFT of the data frame. Taking a $4N$ length FFT of N data symbols and $3N$ zeros does this. We assume oversampling by a factor of 4 throughout this paper. We measure the complexity of any scheme by the number of PAP computations required.

Once the oversampling is taken into consideration, the true complexity of PTS is evident. Even if the M-1 phase factors are limited to 0 and π, 2^{M-1} PAP computations are still required in order to find the optimum phase factors. In fact, to achieve the 4.2dB reduction mentioned above, 32768 PAP computations are needed! Of course, this is assuming an exhaustive search; one can just perform a partial search (e.g. in [3] the full search requires 2000 steps and each step is equivalent to one PAP computation). It is nevertheless clear that the PTS approach is highly complex.

In the SLM approach, the input data frame is multiplied by M random sequences and whichever resultant sequence has the lowest PAP is then selected for transmission. To enable the receiver to recover the data, a pointer to the multiplying sequence

0-7803-6451-1/00/$10.00 © 2000 IEEE

can be transmitted as side information. As a result, the computational cost includes both *MN* multiplications and *M* OFFTs.

In this paper, we propose a data randomization technique for improving the PAP of an OFDM system. This technique is based on the heuristic notion that highly correlated data frames have large PAPs, which could thus be reduced, if the long correlation patterns were broken down. We therefore employ *K*-1 interleavers, each of which produces a re-ordering or permutation of the *N* symbols of the input data frame. The frame with the lowest PAP of all the *K* frames (the original plus *K*-1 permutations) is selected for transmission. To recover the data, the receiver need only to know which interleaved sequence has been transmitted; a pointer (integer between 0 and *K*-1) to the selected interleaver is a minimal additional overhead (i.e. data rate loss) during transmission. For an *N* subcarrier QPSK system, the overhead created by additional information is $\log_2(K)/2N$, which is negligibly small for large *N*. Note that either symbol interleaving or bit interleaving can be used and the latter offers more gain.

This paper also proposes an adaptive technique to reduce the complexity of the above scheme. Since many input data frames already have low PAP values, PAP reduction is unnecessary in most cases. So the key idea in adaptive interleaving (*AIL*) is to establish an early terminating threshold. That is, the search is terminated as soon as the PAP reaches below the threshold, rather than searching all the interleaved sequences (*K*-1). Of course, if the threshold is set to a small value, *AIL* will be forced to search all the permutations. Likewise, if the threshold is set to a large value, *AIL* will search only a fraction of the *K*-1 permutations. So the threshold effects a tradeoff between PAP reduction and complexity.

II. Peak to Average Power Ratio

In OFDM, a block of *N* symbols, $\{X_n \ , \ n = 0,1,\dots,N-1\}$, is formed with each symbol modulating one of a set of *N* subcarriers, $\{f_n \ , \ n = 0,1,\dots,N-1\}$. The N subcarriers are chosen to be orthogonal, that is $f_n = n\Delta f$, where $\Delta f = 1/NT$ and *T* is original symbol period. The resulting signal after D/A conversion can be expressed as

$$x(t) = \sum_{n=0}^{N-1} X_n e^{j2\pi f_n t}, \qquad 0 \le t \le nT \tag{1}$$

The PAP of the transmitted signal in (1) can be defined as

$$PAP = \frac{\max|x(t)|^2}{E\left[|x(t)|^2\right]} \tag{2}$$

III. Interleaver Approach

Two interleaver types used in this application are described below.

A. Random Interleavers (RI)

A random interleaver is a block interleaver that operates on to a block of (N) symbols and reorders or permutes them in a pseudo random order; i.e. symbol sequence $(X_0, X_1, \dots X_n, \dots X_{N-1})$ becomes $(X_{\pi(0)}, X_{\pi(1)}, \dots X_{\pi(n)}, \dots X_{\pi(N-1)})$ where $\{n\} \leftrightarrow \{\pi(n)\}$ is a one to one mapping and $\pi(n) \in \{0,1,\dots N-1\}$ for all n. Both the transmitter and the receiver store the permutation indices $\{\pi(n)\}$ in memory. Thus interleaving and deinterleaving are very simple exercises.

Pseudo random permutations can be obtained from linear congruential sequences. In this paper, however, we use a specific algorithm to generate each permutation (interleaver). At the n^{th} step, we generate a large random number *R* repeatedly (if necessary), until *R mod N* is not equal to any one of $\pi(0), \pi(1), \dots \pi(n)$, and set $\pi(n)$ equal to *R mod N*. This step is repeated for *n=0,1…N-1*. The whole procedure is repeated for *K-1* times to generate the required interleavers. *N* data symbols constitute a frame and *K-1* symbol interleavers produce *K-1* permuted frames. The $4 \times$OFFT of each frame is used to compute its PAP. The minimum PAP frame of the *K* frames is selected for transmission. The identity of the corresponding interleaver is also sent to the receiver as side information.

B. Periodic Interleavers

Periodic Interleavers [5] are very simple to implement. A periodic interleaver with a period *C* and block length *N=CR* writes the values X_n into a $C \times R$ matrix column by column and then reads the $X_{\pi(n)}$ values row by row. Equation (3) and (4) show the matrices of *n* for writing and $\pi(n)$ for reading. The deinterleaver inverts this permutation by using $\pi(n)$ for write labeling and *n* for read labeling.

$$n = \begin{bmatrix} 1 & C+1 & \dots & CR-C+1 \\ 2 & C+2 & \dots & CR-C+2 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ C & 2C & & CR \end{bmatrix} \tag{3}$$

$$\pi(n) = \begin{bmatrix} 1 & 2 & \dots & R \\ R+1 & R+2 & \dots & 2R \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ CR-R+1 & CR-R+2 & & CR \end{bmatrix} \quad (4)$$

IV. ADAPTIVE INTERLEAVER APPROACH

An adaptive technique can be used to reduce the complexity of the interleaver approach. As above, we consider K-1 interleavers. As a first step, compute the PAP of the signal without interleaving. If it is less than a set threshold (PAP_T), then PAP minimization stops immediately. If not, interleave the data sequence and recompute the resulting PAP. If it is less than PAP_T, stop the minimization process. The algorithm continues in this fashion until PAP is less than PAP_T or all (K-1) of the interleavers are searched.

The adaptive algorithm can be written as follows:

Compute the PAP of the original signal

If PAP<PAP_T
  Stop minimization of PAP
Else
  Set InterCount = 1
  while PAP<PAP_T or InterCount <K
    Interleave the data sequence using the interleaver #InterCount
    Recompute the PAP of the interleaved signal
    InterCount ++

The maximum number of permutations used in this technique is K-1, and the minimum is 1. The actual number of permutations tried varies from one input frame to another. We characterize the complexity of this scheme by the average number of permutations per input frame because, each permutation requires $4 \times \text{OFFT}$ to compute the associated PAP.

V. RESULTS

A 256-carrier OFDM system with QPSK modulation was simulated and 10^5 OFDM symbols were generated to obtain the CCDF of PAP. For the use of symbol interleaving, the 256 QPSK symbols of a frame were permuted by K-1 RIs. For the use of bit interleaving, the 512 bits of a frame were permuted by K-1 RIs. Fig. 1 shows CCDF of PAP of an OFDM signal. The $K=1$ curve represents the original OFDM signal. The PAP statistics improve with increasing K. For $K=4$, the 0.1% CCDF is reduced by about 2.2dB.

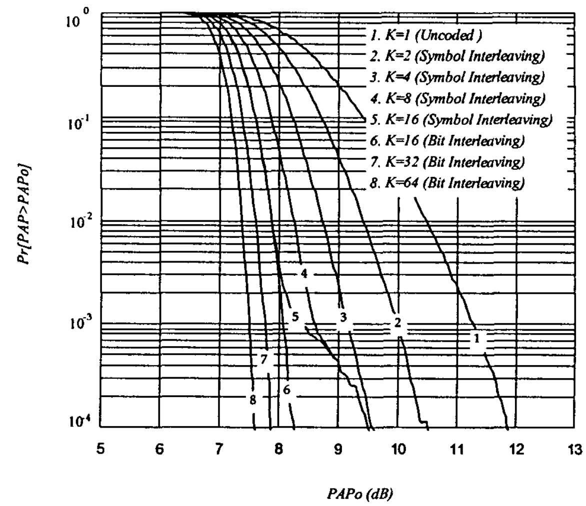

Fig. 1 CCDF of OFDM signal with random K-1 interleavers

For $K=16$, the PAP reduction is about 3dB. This appears to be the maximum PAP reduction achievable with symbol interleaving. For a given K, both bit interleaving and symbol interleaving offer the same PAP reduction in the region $\text{CCDF} > 10^{-3}$ and bit interleaving perform well in the region CCDF<10^{-3}. We should also mention that the Walsh sequences method [3] reduces 0.1% CCDF by 2.8dB at a cost of 16 OFFTs. The RI method also achieves nearly the same result at a cost of 8 OFFTs (see curve 4).

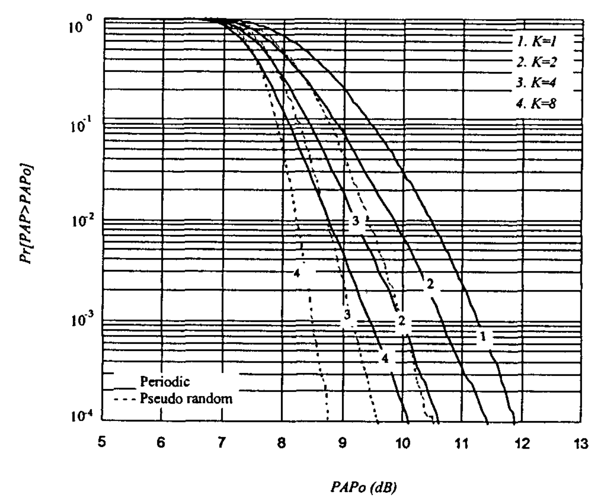

Fig. 2 CCDF of OFDM signal with periodic interleavers (bit)

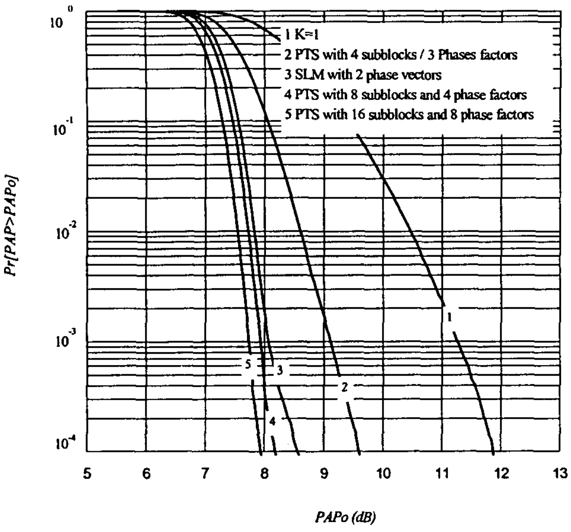

Fig. 3 Random symbol interleavers (RI) combined with other PAP reduction schemes

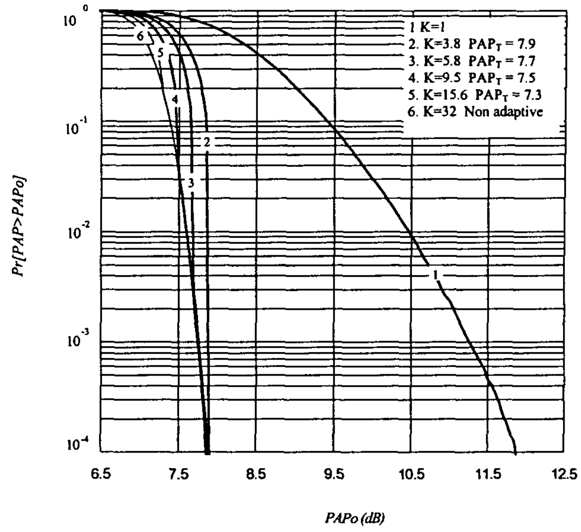

Fig. 4 CCDF of an OFDM signal with adaptive random bit interleavers

Fig. 2 shows a comparison of performance with a simple implementation of periodic interleavers with that of random interleavers. Up to 7 periodic interleavers are used *(K*=8*)*. Results indicate that the use of periodic interleavers does not give satisfactory PAP reduction as in RIs. There is 1.25dB PAP reduction at 10^{-4} CCDF achieved by RI compared to periodic interleavers for *K*=8. There is also a limit in maximum number of periodic interleavers for a given block length. For example, only 8 interleavers are possible for *N*=512.

The RI approach can be incorporated with other PAP reduction techniques to get larger reduction. In particular, clipping or windowing [6] of high peaks may be used with the RI approach. Next we combine the RI approach combined with the PTS and SLM methods. Results are shown in Fig. 3 for interleaving method (*K*=16) combined with PTS and SLM. In this case, PAP of interleaved OFDM signal is further reduced by using simple PTS or SLM scheme. Curve 2 is the PAP distribution for the PTS method with *M=4*. This requires eight OFFTs and the 0.1% CCDF is reduced by 2dB. Curve 3 shows the performance of SLM (2 random vectors) combined with RI approach (*K=16*); the 0.1% CCDF reduces by about 3.2dB. Note that this only requires 18 OFFTs. In curve 4, a somewhat unusual PTS method is tested. 256 carriers are divided into eight clusters and phase shifting is applied only to alternate clusters. As such this requires only four phase factors to be determined. Consequently, the number of OFFTs is 32 (2^4+16). Curve 5 shows the PTS using the flipping algorithm [3] combined with the RI approach. This requires 32 OFFTs and achieves a 4dB PAP reduction.

The 0.01% CCDF of the original OFDM signal is about 11.8dB. Use of non-adaptive interleavers (*K* = 32) improves it by 4dB. Curve 2 in Fig. 4 shows results for *AIL* with a threshold value (PAP_T) of 7.9dB. In the region CCDF <10^{-4}, both the techniques provide identical performance. *K*=32 without adaptation requires 32 permutations per OFDM symbol while *AI* requires only 3.8 (on average) permutations per OFDM symbol. This amounts to an 88% reduction in complexity.

Results are also shown for *AIL* with PAP_T of 7.7dB, 7.5dB and 7.3dB. Those require 5.8, 9.5 and 15.6 iterations (on average) and reduce complexity up to 82%, 70% and 51% respectively. Therefore, it is evident that the complexity can be reduced greatly by limiting the number of iterations by selecting a suitable threshold value. Lower threshold values give out better performances but have higher complexity. Threshold value selection has to be done using the CCDF curve for *K*=32 without adaptation.

Table 1 compares the complexity of the PAP reduction schemes PTS and SLM. Compared to the other two techniques the interleaving approach only needs a *K* number of OFFTs and PAP computations. Both PTS and SLM need extra multiplication by phase factors. Therefore interleaving method is simple to implement and reduces the transmitter complexity when compared with PTS and SLM schemes. If all the *K* , PAP computations are done simultaneously and lowest PAP sequence is selected in one step, the processing delay at the transmitter is significantly reduced. Therefore, it can also be used with high speed data transmissions.

TABLE I

COMPARISON OF THE COMPLEXITY

	No. of IDFTs	No. of PAP computations	No. of Multiplication
PTS	K	K	$N \times K$
SLM	M	2^{M-1}	$\frac{2^{M-1} \times N \times (M-1)}{M}$
Interleaving	K	K	-

VI. CONCLUSION

This paper has presented a data randomization technique for PAP reduction. For this RI approach, K permutations are generated from the input data frame and the permuted frame with the lowest PAP is selected for transmission. The method is less complex than the PTS method, but achieves comparable results. Notably, the RI approach can also be used in conjunction with other methods. Incorporating an adaptive approach a high complexity reduction of the proposed technique is achieved.

While it is well known that interleaving can be used to combat the effect of noise bursts and fading in error correction systems, the use of interleaving in this letter serves a different purpose, one which has not been recognized before. By interleaving a data frame, the peaks in the associated OFDM signal can be compressed. This transformation is most effective for data frames with moderate PAP values (very high PAP values which are nearly N can't be reduced by this method). The maximum PAP reduction of this method occurs at around 0.1% CCDF level. This is most useful because the PAP of practical systems is below the 0.1% CCDF most of the time. Conversely, the worst case PAP value of N for OFDM signals is extremely unlikely for random data. Since the RI approach does not attempt to reduce the worst-case PAP, the data rate loss of this method is negligible. The RI approach naturally lends itself to the use of error correction codes and as such the topic is undergoing further investigation.

REFERENCES

[1] L.J Cimini, Jr., ``Analysis and simulation of a mobile channel using orthogonal frequency division multiplexing", *IEEE Trans. Commun.*, vol. 33, no. 7, pp. 665-675, July 1985.

[2] S. H. Muller and J. B. Huber, "A comparison of peak power reduction schemes for OFDM", *in Proc. GLOBECOM '97*, pp. 1-5, Nov. 1997.

[3] L. J. Cimini Jr. and N. R Sollenberger, "Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences", *in Proc. IEEE Int. Conf. Commun.*, 1999.

[4] C. Tellambura, "Phase optimization criterion for reducing peak-to-average power ratio in OFDM," *Elect. Letts.*, vol. 34, no 2, pp 169-170, Jan 1998.

[5] R. D. Wesel and J. M. Cioffi, "Trellis code design for periodic interleavers" *IEEE Commun. Letts*, vol. 3, no. 4, April 1999.

[6] R.Van Nee and A. D. Wild, "Reducing the peak-to-average power ratio of OFDM", *in Proc. VTC*, pp. 2072-2076, 1998.