Reduced complexity PTS and new phase sequences for SLM to reduce PAP of an OFDM signal

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Abstract - This paper presents a simplified version of partial transmit sequences (*PTS*) and a new scheme for selected mapping sequences to reduce peak-to-average power ratio (*PAP*) of an OFDM signal. Simplification of *PTS* is achieved by having a set of partitions but optimizing phase values only for alternate partitions. This proves to be a promising solution to reduce complexity of *PTS*. It is also proposed to choose the selected mapping sequences are using Newman phase sequences. In this case, albeit with an increase of the complexity, very high *PAR* reduction can be achieved.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a promising transmission scheme for broadband communications over a wireless channel. In OFDM data is transmitted simultaneously through multiple frequency bands. It offers many advantages over single frequency transmission such as resistance to the multipath propagation and impulsive noise. Advances in digital signal processing technologies have made it possible to implement OFDM easily using fast Fourier transform (FFT) chips. OFDM has been adopted or is being investigated for Wireless LAN, digital audio/video broadcasting, broadband wireless loop and wireless ATM. However, it suffers from high PAPs. For an N carrier OFDM system *PAP* can be as high as N.

Proposed methods to reduce the PAP of OFDM include clipping, windowing, and coding [1,2,3]. Partial transmit sequence (*PTS*) and selective mapping (*SLM*) [4] are another two important *PAP* reduction techniques. In these two methods, high *PAP* reduction is achieved at a cost of increased system complexity, which may render *PTS* and *SLM* unsuitable for some applications. Therefore it is important to find ways to reduce the complexity, such that those are suitable for any application. Two recent papers [5,6] investigated possible means for reducing the complexity of the *PTS* technique. In our paper we propose a modified version for *PTS* and *SLM* techniques. Our modified *PTS* technique achieves higher reduction in complexity with little degradation in performance. We also present a new algorithm to find phase sequences for *SLM*.

II. AN OFDM SYSTEM AND PAP

In an OFDM system sub-carrier vector, $A_{\mu,n}$, comprising all modulated input data associated with OFDM symbol interval μ is transformed in to time domain, by a N-point inverse discrete Fourier transform (*IDFT*). The output of the *IDFT* is given by:

$$a_{\mu,k} = \sum_{n=0}^{N-1} A_{\mu,n} e^{j2\pi nk/N} \quad (1)$$

where $k = 0,1..N\text{-}1.$ & $n = 0,1,...,N\text{-}1$

$a_{\mu,k}$ consists of T spaced discrete time samples of the transmitted signal, where T is the sampling time. The samples $a_{\mu,k}$ are transmitted using ordinary T-spaced pulse amplitude modulation (*PAM*). The guard interval, which is an extension of the OFDM signal, is introduced before the transmission. This consists of a partial repetition of the output of the IDFT process ($a_{\mu,k}$).

Because of the statistical independence of carriers, the corresponding time-domain samples in the equivalent complex valued low-pass domain are approximately Gaussian distributed. Resulting high *PAP* is given by:

$$\chi_\mu = \max_k \left|a_{\mu,k}\right|^2 / \varepsilon\left\{\left|a_{\mu,k}\right|^2\right\} \quad (2)$$

Where $\varepsilon\{\}$ denotes expectation. This does not depend on the signal set ($A_{\mu,n}$) used to modulate the signal. Theoretical maximum *PAP* occurs when the data is mapped to a same signal point. The upper bound of the PAP for N number of sub-carriers is *10log(N)* dB. In order to avoid non-linear distortion and spectral spreading of the transmitted signal, highly linear amplifiers operating with a large back off have to be used. Many methods have been proposed to reduce the *PAP*, but when selecting a good scheme, complexity, redundancy and the PAP reduction have to be considered.

III. SYSTEM DESCRIPTION

Modified partial transmit sequences approach

In the normal *PTS* technique, the input complex data vector of *N* symbols is divided into "*v*" sub-blocks or clusters and a *N* point *IFFT* of each cluster is taken. A weighted combination of the "*v*" *IFFTs* forms the final output. The weighting factors are assumed to be pure rotations. Without loss of generality, the first weighting factor can be set to unity. Therefore, the optimization problem is to find "*v-1*" phase values to minimize the *PAR*. For instance, consider an OFDM system with 256 carriers, which is divided into 16 clusters. 15 optimal phase values are to be found for each input data vector. Even if the phase values are discretized to 0 and 180, 2^{15} phase combinations to be searched to find the optimum. Moreover, [5] shows that taking the peak amplitude of the *N*-point *IFFT* output will not give the true peak, and hence over sampling is necessary.

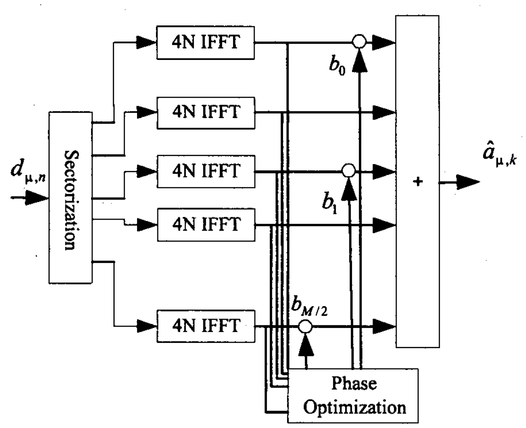

Figure 1 Reduced complexity *PTS* scheme

Our key modification is as follows. As before, the input complex data vector of *N* symbols is divided into "*v*" clusters; however, only "$\frac{v}{2}$" optimized phase values are required. Starting from the first cluster every alternate cluster is kept unchanged and phase values are optimized only for the rest of the blocks. For instance, in the above example, this would mean only 8 phase values to be optimized. If decretized ($0^0 - 180^0$) phase values are used 2^8 total phase combinations are 2^8. Corresponding system block diagram is shown in Figure 1.

New sequences for selected mapping

In *SLM*, the information sequence is multiplied by *M* random sequences of length *N*. In our approach a set of phase sequences for *SLM* is proposed so that the resultant vector is close to the Newman phase sequence [7].

Newman Phase sequence is expressed as:

$$\varphi_n = \frac{(n-1)^2 . \pi}{N} \text{ where } n = 1,2,..N \qquad (3)$$

N is the length of the sequence. The *PAP* of the above sequence is only 2.53dB.

Here our idea is to make the transmit sequence have a phase distribution as close as possible to that of the Newman phase sequence (3). As a first step using a randomly generated set of data sequences *M-1* phase-error sequences are calculated. Let the randomly generated data sequence be expressed as:

$$P_n(m) = A_n(m).e^{j\theta_n(m)} \text{ where m=1,1,...}M\text{-1} \qquad (4)$$

Then the phase error sequence corresponding to this particular data sequence can be found by:

$$\phi_n(m) = \theta_n(m) - \varphi_n \qquad (5)$$

Similarly a set of phase error sequences (*M*) are found for randomly generated data vectors. This phase error vector is then used for the rest of the transmission to generate low *PAP* sequences. For the transmission each data vector is modified using these *M* phase error vectors as follows.

$$d_n = A_n . e^{j\theta_n}$$

$$\bar{\theta}_n(m) = \theta_n - \phi_n(m) \qquad (6)$$

The modified data vector would be

$$d_n(m) = A_n . e^{j\bar{\theta}_n(m)} \quad (7)$$

Then a 4N IFFT is taken and *PAP* is measured. Above procedure is followed for all *M-1* phase error vectors and the one with minimum *PAR* is selected for transmission. Corresponding system block diagram is shown in Figure 2.

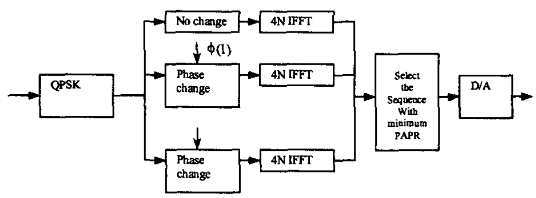

Figure 2. Selected mapping using Newman phase sequence

Side information is also transmitted about the phase-error sequence. Oversampling of factor 4 is used to locate the true peak factor [5]. Very high *PAR* reduction can be observed in this method with the increase of the number of phase sequences but with increasing complexity. These results obtained for 256 carriers by using 256 phase sequences have slightly better performance compared to the optimum performance obtained using ordinary *PTS* with 2000 random phase sequences with 16 partitions [6].

Further reduction of the *PAR* could be obtained by using special signal constellations instead of conventional QPSK or MPSK signal constellations. In this paper, we also investigate how the side information can be transmitted reliably. In most previous publications, this issue is not studied in detail.

IV. RESULTS

Orthogonal frequency division multiplexing system with 256 carriers and QPSK modulation is used in the simulation. For comparison we obtained the results for following: Phase values are descretized to 0 and 180.

1. Uncoded
2. 4 partitions & 3 phases (ordinary *PTS*)
3. 8 partitions & 7 phases (ordinary *PTS*)
4. 8 partitions & 4 phases (modified *PTS*)

Comparison of the above schemes is shown in Figure 3. First complementary cumulative distribution (*CCDF*) of $Pr(PAP > PAP_0)$ for ordinary *PTS* with 4 partitions with 3 phase values and 8 partitions with 7 phase values are shown. Then the same is shown for modified *PTS* with 8 partitions and 4 phases. Results indicate that the modified *PTS* scheme performs 0.75dB better than the ordinary *PTS* scheme with 4 partitions. When it is compared with ordinary *PTS* with 8 partitions, it has a degradation less than 0.5dB at 10^{-4} *CCDF*. The ordinary *PTS* scheme has to evaluate 2^3 and 2^7 phase combinations respectively while modified *PTS* need only 2^4 combinations to be checked. Therefore it can be considered as considerable reduction in complexity with only little degradation of the performance. In other words complexity can be reduced by more than 80% with only 0.5dB degradation of the performance.

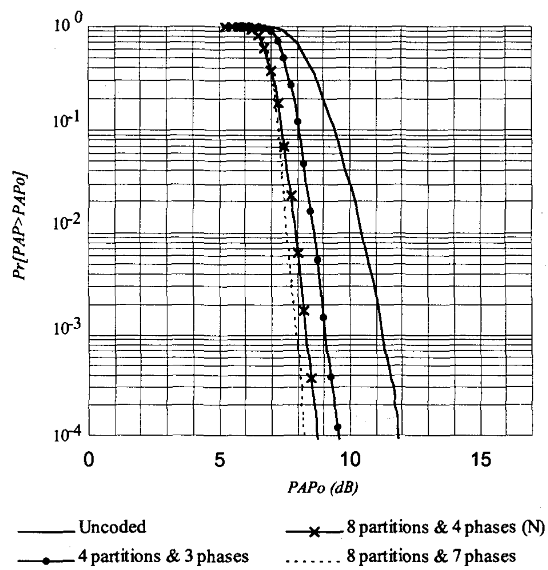

Figure 3 Modified PTS performance

Figure 4 shows the performance of the modified *PTS* including side information. The inclusion degrades the performance by about 0.25 dB degradation. There is no considerable performance degradation with the inclusion of the side information.

Next performance of *SLM* with Newman phase sequences is presented. Performance was observed for different number of phase error sequences, respectively 16, 32, 64 and 256. These sequences have coding gains of 3.5, 4.0, 4.25 and 4.5dB at 10^{-4} *CCDF* respectively.

More initial phase error sequences will increase the performance , but with the increase of the complexity.

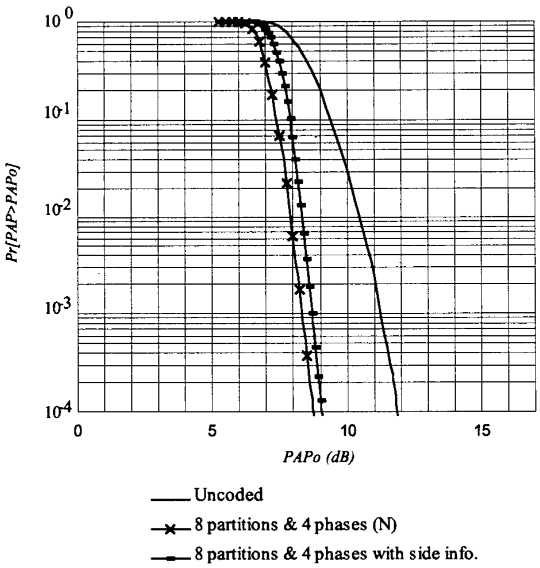

Figure 4 Performance degradation due to the inclusion of side information.

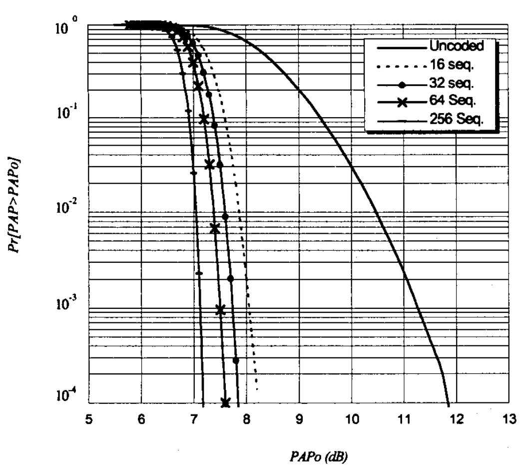

Figure 5 *SLM* performance with Newman phase sequences.

The performance of *PTS* is better than that of *SLM*. *SLM* with 256 sequences is capable of achieving a better performance than optimum performance of *PTS* with 16 partitions. If it is necessary to take an *IFFT* at each step of the *PTS* process (oversampled version to find the true *PAP*), *SLM* is much less complex.

V. CONCLUSION

This paper presents a simplified version of PTS and a new sequences to reduce the PAP of an OFDM signal. Simplification of *PTS* is achieved by having a set of partitions but optimizing phase values only for alternate partitions. This proves to be a promising solution reduce the complexity of *PTS*. It is observed that for *PTS* with 8 partitions complexity can be reduced by more than 80% with only 0.5dB degradation. In the second part the selected mapping sequences are chosen using Newman phase sequences. *SLM* with 256 sequences is capable of achieving a better performance than optimum performance of *PTS* with 16 partitions. In this case with the increase of the complexity very high *PAR* reduction can be achieved. When oversampled IFFT is used to locate the true peak factor, the comparative complexity of *SLM* is less.

REFERENCES

[1]. J. A. Davis and J. Jedwab, "Peak-to-mean Power control and error correction for OFDM transmission using Golay sequences and Reed-Muller codes'" *Electronic. Letters,* Vol. 300, No. 4, pp. 267-268, Feb. 1997.

[2]. R. Van Nee and A De Wild, "Reducing the Peak-to-Average Power ratio of OFDM," *IEEE Conference. Proceedings* VTC '98, pp. 2072-2076, 1998.

[3]. D. Wulich and L. Goldfeld, "Reduction of Peak Factor in Orthogonal Multicarrier Modulation by Amplitude Limiting and Coding," *IEEE Trans. Comm.* Vol. 47, No. 1, pp. 18-21, Jan. 1999.

[4]. S. H. Muller and J.B. Huber, "A Novel Peak Power Reduction Scheme for OFDM," *IEEE Conference. Proceedings* PIMRC '97, pp 1090-1094, 1997.

[5]. C. Tellambura, "Phase Optimization Criterion for Reducing Peak-to-Average Power Ratio in OFDM," *Electronic Letters.*, Vol. 34, No 2, pp 169-170, Jan 1998.

[6]. L. J. Cimini, Jr. and N.R. Sollenberger, "Peak-to-Average Power Ratio Reduction of an OFDM Signal Using Partial Transmit Sequences," *IEEE Conference Proceedings* GLOBECOM '99, 1999.

[7]. S. Boyd, "Multitone signals with low crest factors," *IEEE* Trans. Circuits Syst., Vol. CAS-33, No.10, Oct. 1986.