Multicarrier CDMA Systems using PCC-OFDM

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Abstract - Polynomial cancellation coded orthogonal frequency division multiplexing (PCC-OFDM) with overlapping symbol periods is a modulation scheme with properties that make it well suited to high speed data transmission including mobile applications. PCC-OFDM is much less sensitive to frequency offset and Doppler spread than ordinary OFDM. Equalizers to counteract multipath transmission are much simpler for PCC-OFDM than for single carrier systems or for OFDM systems without a cyclic prefix. In this paper PCC-OFDM in combination with code division multiple access (CDMA) for a downlink is considered. It is shown that the proposed system gives lower bit error rate (BER) than OFDM for channels where the frequency offset or Doppler spread is significant, and for channels where the delay spread is more than a fraction of a symbol period. Increasing delay spread has little effect on the performance of the PCC-OFDM system. The upper limit on the delay spread, which can be tolerated, is set by the span of the equalizer which can be a number of symbol periods long without excessive complexity. It is shown that in this application the two-dimensional equalizer can be simplified to a number of one-dimensional equalizers operating in parallel without significant loss in performance.

I. Background on polynomial cancellation coding with overlapping symbol periods

Polynomial cancellation coding (PCC) is a coding technique for orthogonal frequency division multiplexing (OFDM) in which the data to be transmitted is mapped onto weighted groups of subcarriers rather than individual subcarriers. PCC-OFDM has been shown to be much less sensitive to frequency offset and Doppler spread than standard OFDM [1,2,3]. The intercarrier interference (ICI) caused by a given frequency offset is 10-20dB less when PCC is used. This reduced sensitivity is because the subcarrier pairs in PCC-OFDM have a spectrum with much lower sidelobes than the single subcarrier in OFDM. See Figure 1. In the rest of the paper the term 'subchannels' will be used to describe the weighted subcarrier pairs in PCC-OFDM.

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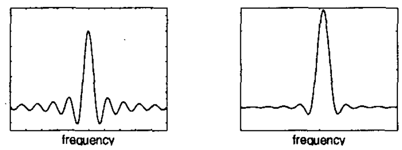

Figure 1: (a) one subcarrier in OFDM, (b) one subchannel in PCC-OFDM

PCC-OFDM has also been shown to have better spectral roll-off and, because of the windowing properties of PCC, reduced sensitivity to multipath propagation [4]. Figure 2 shows the block diagram of a PCC-OFDM communication system.

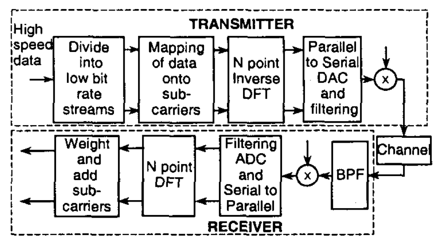

Figure 2. Block diagram of PCC-OFDM system

Despite its many advantages, PCC-OFDM used in its simplest form has one major disadvantage – the spectral efficiency is approximately halved. One way of retaining the advantages of PCC-OFDM, without this loss, is to overlap adjacent symbols. Symbols of duration T are transmitted at intervals of less than T, typically $\sim T/2$. Intersymbol interference (ISI) is deliberately introduced at the transmitter to increase the data rate [9].

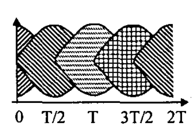

Figure 3: PCC-OFDM with overlapping in time domain

Figure 3 demonstrates the concept of overlapping symbols. For $d = T/2$, where d is the overlap between symbols, the transmitted signal at any instant is the sum of components due to two different symbols. The concept can be generalized to other d. Figure 4 shows the form of the receiver required to recover the information from the overlapped symbols.

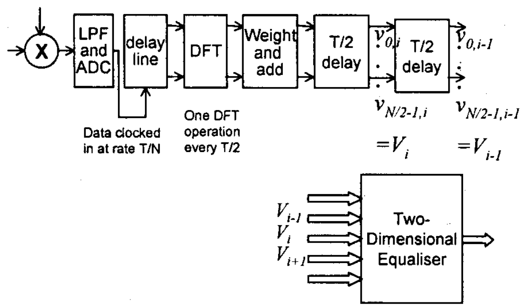

Figure 4: Receiver for PCC with overlapping symbols

The equalizers required to recover the information from the received signal are two-dimensional: interference occurs in the time domain between symbols and in the frequency domain between subchannels within a symbol. Figure 5 shows the two dimensional impulse response for PCC-OFDM with overlapping symbol periods, with $d = T/2$, and a distortionless channel. There are only seven significant terms. Even severe multipath distortion has little effect on the number of significant terms. This is because the low sidelobes of the spectra of the subchannels in PCC-OFDM mean that irrespective of the channel transfer function there is only significant ICI from the immediately adjacent subchannels.

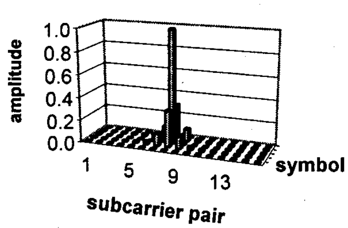

Figure 5: 2-D impulse response for ideal PCC-OFDM with overlapping symbol periods, $d = T/2$

A number of papers have considered the use of two dimensional frequency domain equalizers for windowed OFDM with overlapping symbol periods [5] or for OFDM without a cyclic prefix [6, 7, 8]. However the equalizers required in these applications have many more significant tap values and the location of significant tap values depends on the channel transfer function [8]. All of the equalization techniques used for one dimensional equalization such as zero forcing linear equalization, minimum mean square error (MMSE) equalization, DFEs and maximum likelihood sequence estimation (MLSE) can be extended to two dimensions and applied to OFDM [5-8]. Similar equalizer structures were used in these simulations, but for PCC-OFDM the equalizers operate on the output of the weighting and adding block, rather than on the output of the receiver DFT. This both reduces the complexity of the equalizers and contributes to the cancellation properties of the PCC code [3]. The MMSE formula used to calculate the matrix of equalizer taps used in the simulations in this paper is given by

$$C_{mmse} = \frac{E\{d^2\}}{E\{n^2\}} A H_s^* \left(\frac{E\{d^2\}}{E\{n^2\}} H_s H_s^* + H_n H_n^* \right)^{-1} \qquad (1)$$

where H_S represents the effect of the channel etc on the signal component. H_S is a matrix which depends on the transfer function from the input of the transmitter mapping block to the output of the receiver weighting and adding block. The dimensions of H_S depend on the length of the equalizer. H_N represents the effect of overlapping and of the weighting and adding block on the noise. Because of the overlapping, the noise samples at the input to the equalizer are not independent. $E\{d^2\}$ is variance of the data at the input to the transmitter mapping block. $E\{n^2\}$ is the variance of the noise at the input to the receiver. When there is frequency offset between transmitter and receiver, better performance can be achieved by including the effect of ICI due to frequency error in the calculation of MMSE equalizer taps. This was not done in the simulations presented in this paper.

II. Performance of decision feedback and linear equalizers for PCC-OFDM

In all of the simulations described in this paper 4-QAM modulation was used. Figure 6 shows the performance of a PCC-OFDM system using DFEs of varying complexity, for $d = T/2$ and an additive white Gaussian noise (AWGN) channel. Results are shown for DFEs with one decision feedback stage and for varying m where m is the number of linear stages. That is the equalizer inputs are the estimated data values from the previous symbol, and the outputs of the weighting and adding block for the current and $(m-1)$ following symbols. In this, and subsequent graphs, the solid line indicates the performance of OFDM in AWGN. The equalizer with $m = 4$ gives performance in AWGN that is within a fraction of a dB of OFDM. Results are also shown for a normalized frequency offset between transmitter and receiver, ΔfT of 0.1 and 0.25. The PCC-OFDM system is much more tolerant of frequency error than OFDM.
Figure 6 also shows the results for the PCC-OFDM system for the case of a channel subject to multipath and frequency offset. In this case there are two equal amplitude paths with a delay between the paths, τ, of $T/16$. For comparison, results for an OFDM system for the same channel are presented. The simulations are for the case where the cyclic prefix, cp, is also of length $T/16$.

The PCC-OFDM has a lower BER than the ordinary OFDM system and is also slightly more bandwidth efficient due to the loss in efficiency in OFDM due to the cyclic prefix. The difference between the PCC-OFDM system and an OFDM system increases as the delay spread of the multipath increases (even when the cyclic prefix is longer than this delay) and when the frequency offset increases. In other words, the worse the channel, the greater the benefits in using PCC. The simulation results in this paper are for 64 subcarriers, that is $N = 64$, but the results for other values of N are very similar.

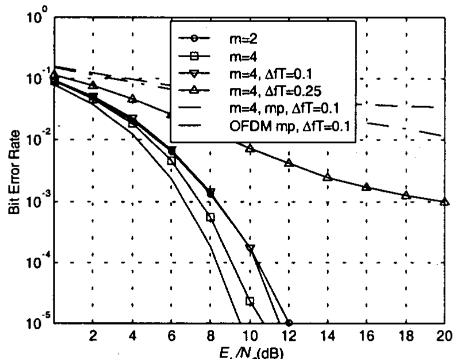

Figure 6: DFEs for PCC-OFDM with $T/2$ overlap

The results in Figure 6 assume that the correct decisions are fed back. This is a realistic assumption in applications such as digital television where coding across the subcarriers in one symbol can be used to minimise the probability of error propagation. Unfortunately this DFE structure cannot readily be adapted to multi-access applications such as mobile telephony where the traffic is time varying. Linear equalizers can be combined with error coding and CDMA in a much more flexible way.

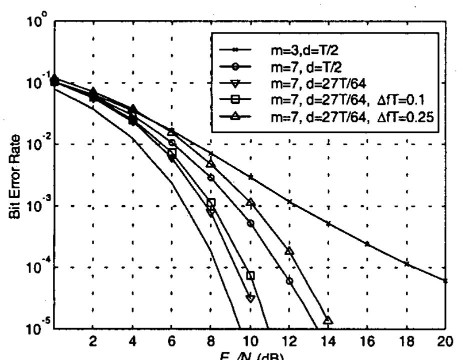

Figure 7: Linear MMSE equalizers for PCC-OFDM

Figure 7 shows the performance of two-dimensional linear MMSE equalizers for PCC-OFDM for a number of cases. For these equalizers, m is the total number of linear stages, equal numbers of preceding and following symbols were used as inputs to the equalizers. For $d = T/2$ linear equalization causes considerable noise enhancement both for the additive noise from the channel and the ICI noise due to the frequency error. Figure 7 shows that by reducing d slightly the performance can be considerably improved.

Figure 8 shows the performance of a PCC-OFDM system in the presence of multipath for the case of two paths. Results are shown for different delays between the two paths and for varying amplitude, a, of the second path relative to the first. In these and other simulations the receiver is synchronized to the first path. Slightly different performance will result for different assumptions about receiver synchronization. Note that increasing the delay between paths has little effect on the performance.

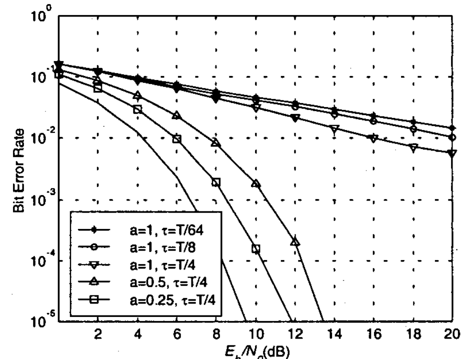

Figure 8: Performance of seven stage linear equalizers for multipath channels, $d = 27T/64$

The effect of the multipath channel is to cause constructive interference for some subchannels and destructive interference for others. The high error rate is almost entirely due to the few 'bad' subchannels. To overcome this some form of diversity is required. This can be achieved by using error correcting coding, as in coded OFDM (COFDM), or by using CDMA with the information for each bit spread across a number of subchannels. In this paper the use of CDMA is considered.

III. CDMA for PCC-OFDM with overlapping symbol periods

Figure 9 shows the structure of the transmitter of the proposed system. Walsh-Hadamard spreading codes are used to spread the data from a given user across a number of PCC subchannels.

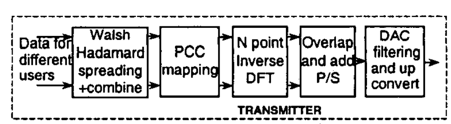

Figure 9: Transmitter for PCC-CDMA with overlapping symbol periods

Figure 10 shows the structure of the receiver. A linear MMSE equalizer follows the weighting and adding block and precedes the Walsh-Hadamard decoding. This means that an equalizer with comparatively few active taps can be used. This would not be the case for an equalizer placed after the Walsh-Hadamard decoding.

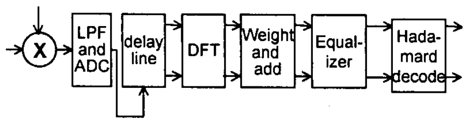

Figure 10: Receiver for PCC-CDMA

All of the simulations in this section show the results for fully loaded CDMA systems. Every subchannel and every Walsh-Hadamard function is used in every symbol. For PCC-CDMA seven stage linear MMSE equalizers were used. The choice of spreading factor for PCC-CDMA is not trivial, as the optimum spreading factor depends on the amplitude and delay spread of the multipath components and the degree of overlap of the PCC symbols. This is because the equalization to recover the data from the overlapped symbol periods causes greater loss in SNR for some Walsh-Hadamard functions than others. In these simulations, the data was spread over all of the subchannels

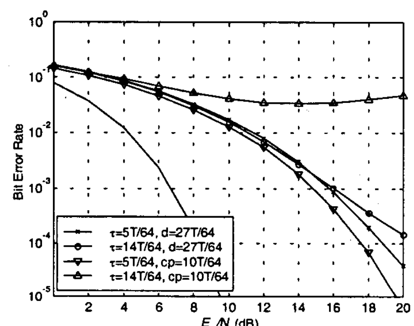

Figure11: BER of PCC-CDMA and MC-CDMA as a function of E_b/N_0

Figure 11 shows the performance of PCC-CDMA systems in multipath channels where there are two equal amplitude paths and for varying delay, τ, between the paths. Unlike OFDM systems which use a cyclic prefix, there is no hard limit on the length of delay which can be tolerated. For comparison some results for a multicarrier CDMA (MC-CDMA) system using MMSE single tap equalizers before the Walsh-Hadamard decoding are given. The symbol overlap for PCC-CDMA and length of the cyclic prefix for MC-CDMA have been chosen so that the systems have the same bandwidth efficiency. For $\tau = 5T/64$ MC-CDMA has a slightly lower BER. For $\tau = 14T/64$ the BER of MC-CDMA is much larger, this is because the delay spread exceeds the length of the cyclic prefix and ISI occurs. The BER increases for increasing SNR because the MMSE equalizer is optimised for the AWGN only, not the ISI.

Figure 12 shows the relationship between BER and τ more clearly for $E_b/N_0 = 15\text{dB}$, $a = 1$, and $\Delta fT = 0$. Results are presented for PCC- CDMA for three values of *d*, and for MC-CDMA for a cyclic prefix of length $6T/64$. For delay spreads which are less than the length of the cyclic prefix the MC-CDMA outperforms the PCC-CDMA system with the same bandwidth efficiency, that is, for $d = 29T/64$. However as the delay spread increases the BER for the PCC-CDMA systems increases only slowly, whereas that of the MC-CDMA system increases rapidly once the delay spread exceeds the cyclic prefix. (For PCC and $a = 1$, there are some peaks in the BER for values of *d* which are simple fractions of *T* because of the nulls for some subchannels. This effect is not significant for other values of *a*.) The amount of delay spread depends only on the length of the equalizer. It will be shown that this can be a number of symbol periods long without undue receiver complexity. The performance also depends on the symbol overlap *d*. For $d = T/2$, linear equalization causes considerable noise enhancement, however this can be reduced by reducing *d*, so there is a simple trade-off between performance and bandwidth efficiency, which can be optimised for a particular application. For applications where DFEs can be used there is no such trade-off required and maximum bandwidth efficiency and performance can be achieved.

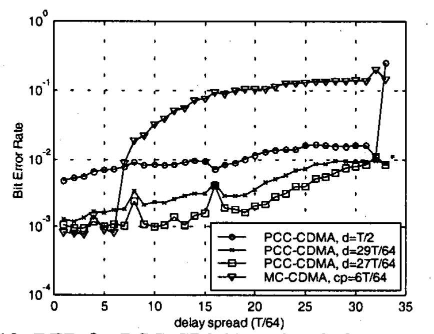

Figure12: BER for PCC-CDMA and MC-CDMA as a function of τ for $E_b/N_0 = 15\text{dB}$, $a = 1$, and $\Delta fT = 0$

Figure 13 shows the effect of frequency offset and of equalizers of varying complexity. That is for varying *l*, where *l* is the number of subchannels used from each symbol in the estimation of each data value. For $l = 1$ only one subchannel is used, all except the main diagonal terms of C_{mmse} are set to zero. Thus the equalizer reduces to number of one dimensional equalizers. For $l = 3$ only three subchannels from each symbol are used. Figure 13 shows that for $d = 27T/64$ there is little loss in performance if the simplest equalizer is used. (Figure 13 shows the simpler equalizer outperforming the more complex in some cases. This is because the equalizer tap calculation ignores the ICI due to frequency offset.) For this frequency offset and multipath, the PCC-CDMA outperforms the MC-CDMA system for all delay spreads. This is because ICI due to frequency offset is much less for

PCC-CDMA than MC-CMDA. Similar improvements can be expected for channels subject to Doppler spread.

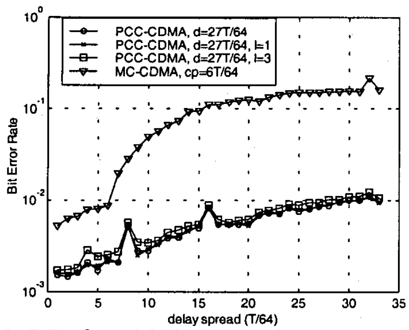

Figure13: BER for PCC-CDMA and MC-CDMA as a function of τ for $E_b/N_0 = 15\text{dB}$ $a=1$, and $\Delta fT = 0.1$

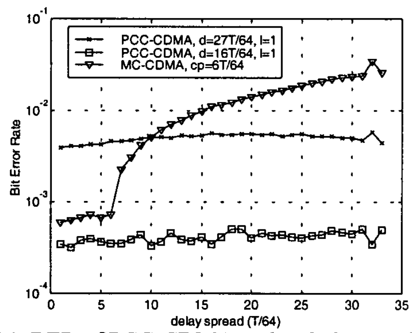

Figure14: BER of PCC-CDMA and MC-CDMA for $E_b/N_0 = 10\text{dB}$, $a = 0.5$, and $\Delta fT = 0.05$

Figure 14 gives results for $E_b/N_0 = 10\text{dB}$, $a = 0.5$, and $\Delta fT = 0.05$. In this case, because the ICI due to frequency offset is smaller compared to the AWGN, the MC-CDMA outperforms the system with $d = 27T/64$ for $\tau \le cp$. However by decreasing the overlap the performance of the PCC-OFDM can be improved, at the cost of bandwidth efficiency.

IV. Discussion and Conclusions

Results of simulations have been presented for a CDMA system based on PCC-OFDM with overlapping symbol periods. It has been shown that PCC-OFDM systems are much less adversely affected by frequency offset and large delay spreads than ordinary OFDM. As a general rule, ordinary OFDM will outperform PCC-OFDM in applications where the ordinary OFDM system can be designed such that the ICI caused by Doppler and frequency offsets is minimal, and where a symbol period that is large compared to the delay spread can be used without the computational load associated with the FFT becoming unreasonable. In applications where these constraints cannot be met, for example, in systems with high data rates, large delay spreads or high carrier frequencies PCC-OFDM systems will outperform OFDM. A number of second order effects, not considered in this paper, also act in favour of PCC-OFDM: channel estimation can be more accurate due to the reduced ICI, simple algorithms exist for frequency and symbol synchronization based on the structure of PCC-OFDM. In cellular applications the effect of inter cell interference is more predictable because even signals with large relative delays do not cause ICI. PCC-OFDM also lends itself to antenna diversity.

It has been shown that because of the properties of PCC, very simple equalizers can be used without significant loss in performance. In many ways PCC-OFDM can be considered as a flexible modulation scheme intermediate between single carrier and multicarrier systems and combining some of the best properties of each.

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