Contour Integral Representation for Generalized Marcum-Q Function and Its Application to Unified Analysis of Dual-Branch Selection Diversity over Correlated Nakagami-m Fading Channels

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ABSTRACT – Using a circular contour integral representation for the generalized Marcum-Q function, $Q_m(a, b)$, we derive a new closed-form formula for the moment generating function (MGF) of the output signal power of a dual-diversity selection combiner (SC) in bivariate Nakagami-m fading with positive integer fading severity index. This result involves only elementary functions and holds for any value of the ratio a/b in $Q_m(a, b)$. As an aside, we show that previous trigonometric integral representations for $Q_m(a, b)$ can be obtained directly from this contour integral. The MGF is used to unify the evaluation of average error performance and outage performance of a dual-branch SC for coherent, differentially coherent and noncoherent communications systems.

I. Introduction

The evaluation of symbol error rate for several common modulation formats with dual-branch selection diversity in independent and correlated fading channels has been reported in [1]-[5]. The recent work [5] has rekindled the interest in this subject. In [5], the authors first derived an integral expression for the cumulative distribution function (CDF) of the combiner output signal-to-noise ratio (SNR) by exploiting the trigonometric integral representation for $Q_m(a, b)$ [7] and then differentiated it to get the probability density function (PDF). Their resulting expressions depend on the branch power ratios as well as the power correlation coefficient ρ [5, Eq. (3)] (i.e., their result take different forms depending on $a>b$, $a = b$ or $a<b$). Finally, they employed the classical PDF method to obtain the average error rates.

Alternatively, if one derives the SC output MGF first, then the performance of a broad class of modulation formats can be obtained at once using the MGF method [8]! Previously, Okui [2] derived a Gauss hypergeometric series for the MGF of the SC output SNR. However, his result [2, Eq. (7)] holds for the case of equal average SNRs only. Fedele *et. al.* [4] generalize the results in [2] by considering the effect of dissimilar mean received signal strengths. Motivated by Simon and Alouini's work and recognizing the fact that SC output MGF is the key to the performance analysis, we attempted to derive closed-form solutions for the MGF (involving only elementary functions) in correlated Nakagami-m fading channels.

The major results and contributions of this paper include the following: (a) we derive a closed-form expression for the MGF for integer m via a circular contour integral representation for $Q_m(a, b)$. Simon and Alouini only obtained an integral representation for the MGF. Furthermore, in contrary to [5], our final expression applies regardless of the values of the branch power ratios and ρ. For instance, the independent fading case can be treated directly by setting $\rho = 0$ in the expression for the MGF[1]. Therefore, our solution leads to a compact, unified analysis of a broad class of modulation formats for dual-diversity SC in correlated Rayleigh and Nakagami-m fading; (b) we show that the two previously reported alternative integral representation for $Q_m(a, b)$ can be directly obtained from the contour integral representation; (c) the MGF is used to derive error rate expressions for a broad class of modulation formats employing dual branch SC in Nakagami-m and Rayleigh fading channels; and (d) we also derive a new, exact integral expression for the outage rate of error probability using the Fourier inversion formula.

II. Integral Representations for $Q_m(a, b)$

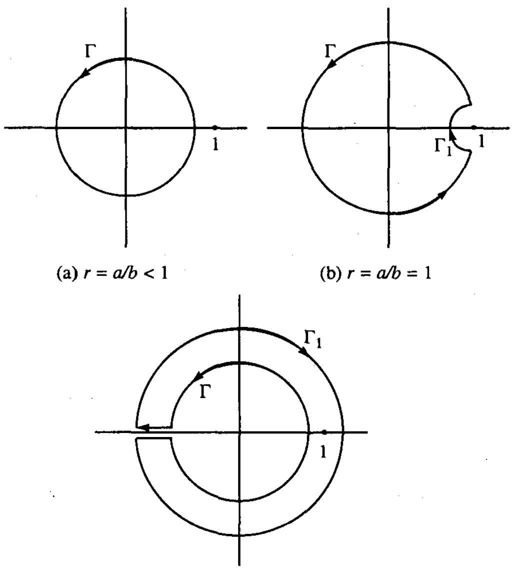
Proakis [10, pp. 885] provides the contour integral representation for the generalized Marcum-Q function,

$$Q_m(a, b) = \frac{1}{2\pi j}\oint_{\Gamma}\frac{e^{g(z)}}{z^m(1-z)}dz \qquad (1)$$

where $g(z) = a^2(1/z-1)/2 + b^2(z-1)/2$ and Γ is a circular contour of radius r that encloses origin. The singularities of the integrand are at $z = 0$ and $z = 1$. Therefore, by Cauchy's theorem we can choose any $0<r<1$. Now if we choose $r = 1$, then we need to remove the singularity at $z = 1$ on Γ by suitably deforming Γ (see Fig. 1b). This representation holds regardless of $a>b$, $a = b$ or $a<b$, and for any positive integer m. In the following, we will show that both Helstrom's [7] and Simon's [6] integral representations

1. In fact, closed-form expression for a broad class of modulation formats (including CPSK, CFSK, MQAM, MDPSK, MPSK, noncoherent MFSK, star-QAM, $\pi/4$-DQPSK and arbitrary two-dimensional M-ary signal constellations) can be derived in a straight-forward fashion for this special case [12].

readily follow from (1) for integer values of m.



(a) $r = a/b < 1$ (b) $r = a/b = 1$

(c) $r = a/b > 1$

Fig. 1 Contours of a line integral for: (a) $r < 1$; (b) $r = 1$; and (c) $r > 1$.

(i) Consider the case $a < b$ (see Fig. 1a) where the circular contour Γ encloses origin with a radius less than unity. Therefore, z in (1) can be written as $z = re^{j\theta}$ with $r < 1$ and $0 \le \theta < 2\pi$. Now select $r = a/b$, so we immediately get

$$Q_m(a, b) = \frac{e^{-a^2/2 - b^2/2}}{2\pi}\left(\frac{b}{a}\right)^m \int_0^{2\pi} \frac{e^{ab\cos\theta - j(m-1)\theta}}{(b/a - e^{j\theta})} d\theta, \quad m \in Z \quad (2)$$

where Z is the set of positive integers. Taking the magnitude of the integrand, we obtain the new bound

$$Q_m(a, b) \le e^{-(a-b)^2/2}\left(\frac{b}{a}\right)^m \frac{a}{\sqrt{a^2 + b^2}} \quad (3)$$

which holds for any integer $m \ge 1$, whereas the bound due to Simon [6, Eq. (12)] holds only for $m = 1$.

(ii) Consider the case $a = b$ (see Fig. 1b). Now $g(z) = a^2(z + 1/z)/2 - a^2$ and Γ as shown in Fig. 1b. Hence,

$$Q_m(a, b) = \frac{e^{-a^2}}{2\pi}\int_{\varepsilon}^{2\pi - \varepsilon} \frac{e^{a^2\cos\theta - j(m-1)\theta}}{(1 - e^{j\theta})} d\theta + \frac{1}{2\pi j}\oint_{\Gamma_1} \frac{e^{g(z)}}{z^m(1 - z)} dz \quad (4)$$

where Γ_1 is the half-circle contour centered on $z = 1$ with radius ε. On Γ_1, $z = 1 - \varepsilon e^{j\alpha}$ and $-\pi/2 \le \alpha \le \pi/2$. Taking the real value of the first integral on the right hand side and letting $\varepsilon \to 0$, we obtain

$$Q_m(a, b) = \frac{1}{2}\int_0^{2\pi} e^{-a^2(1 - \cos\theta)} \frac{\sin[(m - 1/2)\theta]}{\sin(\theta/2)} \frac{d\theta}{2\pi} + \frac{1}{2}, \quad m \in Z \quad (5)$$

This result (i.e., Eq. (5)) is in fact identical to [7, pp. 528] derived by Helstrom.

(iii) If $a > b$, $r = a/b$ is greater than unity. So we need to consider the closed contour shown in Fig. 1c. The inner circle Γ has a radius less than unity, while the outer circle Γ_1 has a radius of a/b. Inside the closed contour, the only singularity of the integrand occurs at $z = 1$. Hence using Cauchy's theorem, we find

$$\frac{1}{2\pi j}\oint_{\Gamma} \frac{e^{g(z)}}{z^m(1 - z)} dz + \frac{1}{2\pi j}\oint_{\Gamma_1} \frac{e^{g(z)}}{z^m(1 - z)} dz = \lim_{z = 1} \frac{e^{g(z)}}{z^m} \quad (6)$$

The first integral is $Q_m(a, b)$ and $z = (a/b)e^{j\theta}$ on Γ_1. Therefore, we get

$$Q_m(a, b) - 1 = \frac{e^{-a^2/2 - b^2/2}}{2\pi}\left(\frac{b}{a}\right)^m \int_0^{2\pi} \frac{e^{ab\cos\theta - j(m-1)\theta}}{(b/a - e^{j\theta})} d\theta, \quad m \in Z \quad (7)$$

Note that (2) and (7) are identical to Helstrom's results [7], except the integrands are in a complex format, and hence are slightly more compact. Since the integrals are real-valued, taking the real parts of the integrands in (2) and (7) gives the exact same integral representations of Helstrom. Similarly, Simon's results [6] are very closely related. For instance, consider the $a < b$ case. As in the derivation of (2), we can select $z = (a/b)je^{j\theta}$. Note that the magnitude of z is still less than unity, i.e., $|z| < 1$. Hence using this new substitution in (1), we find

$$Q_m(a, b) = \frac{e^{-a^2/2 - b^2/2}}{2\pi}\left(\frac{b}{a}\right)^m \int_0^{2\pi} \frac{e^{-ab\sin\theta - j(m-1)[\theta + \pi/2]}}{(b/a - je^{j\theta})} d\theta, \quad m \in Z \quad (8)$$

Again, this representation is very compact, yet it is identical to [6, Eq. (7)] and [6, Eq. (10)].

III. Derivation of the MGF of SNR at the Output of a Dual-Branch SC Combiner

The joint probability density function (PDF) of the bivariate Nakagami-m fading is given by [11],

$$f(x, y) = \frac{4(xy)^m}{\Gamma(m)\Omega_1\Omega_2(1 - \rho)(\sqrt{\Omega_1\Omega_2\rho})^{m-1}} \times \exp\left(\frac{-1}{1 - \rho}\left[\frac{x^2}{\Omega_1} + \frac{y^2}{\Omega_2}\right]\right) I_{m-1}\left(\frac{2\sqrt{\rho}xy}{\sqrt{\Omega_1\Omega_2}(1 - \rho)}\right) \quad (9)$$

where $x \ge 0$, $y \ge 0$, $\Omega_1 = \{\overline{x^2}/m\}$, $\Omega_2 = \{\overline{y^2}/m\}$, $\rho = cov(x^2, y^2)/\sqrt{var(x^2)var(y^2)}$ ($\rho \ne 0, 1$), and m is a positive number greater than $1/2$.

Then the cumulative distribution function (CDF) of signal envelope at the output of SC combiner is

$$F(r) = \int_0^r\int_0^r f(x, y)dxdy \quad (10)$$

Now differentiating (10) with respect to r, we obtain the PDF of the signal envelope,

$$f(r) = \int_0^r f(x, r)dx + \int_0^r f(r, y)dy \quad (11)$$

From the definition of the m-th order generalized Marcum-Q function [10],

$$Q_m(a, b) = 1 - \int_0^b x\left(\frac{x}{a}\right)^{m-1} \exp\left(\frac{-x^2 - a^2}{2}\right) I_{m-1}(ax)dx \quad (12)$$

we can show that

$$\int_0^b x^m \exp(-cx^2) I_{m-1}(ax)dx = \frac{a^{m-1}}{(2c)^m}\exp\left(\frac{a^2}{4c}\right)[1 - Q_m(a/\sqrt{2c}, b\sqrt{2c})] \text{ if } a, b, c > 0 \quad (13)$$

Substituting (9) into (11) and using identity (13), we get

$$f(r) = \psi_2(r)\left[1 - Q_m\left(r\sqrt{\frac{2\rho}{(1-\rho)\Omega_2}}, r\sqrt{\frac{2}{(1-\rho)\Omega_1}}\right)\right] + \psi_1(r)\left[1 - Q_m\left(r\sqrt{\frac{2\rho}{(1-\rho)\Omega_1}}, r\sqrt{\frac{2}{(1-\rho)\Omega_2}}\right)\right] \quad (14)$$

where $\psi_j(r) = \frac{2r^{2m-1}}{\Gamma(m)\Omega_j^m}\exp\left(\frac{-r^2}{\Omega_j}\right), j \in \{1, 2\}$ is introduced for ease of notation.

Now, the desired MGF may be evaluated as

$$\phi(s) = \int_0^\infty \exp(-sr^2) f(r) dr \quad (15)$$

Clearly, the solution to the definite integral (16) is required to solve our problem on hand:

$$I_m(p, a, b) = \int_0^\infty x^{2m-1}\exp(-px^2)[1 - Q_m(ax, bx)]dx \quad (16)$$

where $\mathrm{Real}\{p\} > 0$. From (1), we have

$$Q_m(ax, bx) = \frac{1}{2\pi j}\oint_\Gamma \frac{\exp[x^2 g(z)]}{z^m(1-z)}dz \quad (17)$$

where $g(z) = a^2(1/z - 1)/2 + b^2(z-1)/2$ and Γ is a circular contour of radius less than unity that encloses origin. This representation holds regardless of $a > b$, $a = b$ or $a < b$. Now substituting (17) into (16) and performing the integration with respect to x first (changing the order of integration is valid because both the integrals are convergent), we find

$$I_m(p, a, b) = \frac{\Gamma(m)}{2p^m} - \left(\frac{-2}{b^2}\right)^m \frac{\Gamma(m)}{4\pi j}\oint_\Gamma \frac{dz}{(1-z)[z^2 - \alpha_0 z + \alpha_1]^m} \quad (18)$$

where $\alpha_0 = (2p + a^2 + b^2)/b^2$ and $\alpha_1 = (a/b)^2$. The denominator of the integrand in (18) has two positive roots. Therefore, applying the residue theorem, we get

$$I_m(p, a, b) = \frac{\Gamma(m)}{2p^m} - \frac{\Gamma(m)}{2}\left(\frac{2}{b^2}\right)^m \frac{d^{(m-1)}}{dz}\left\{\frac{1}{(1-z)(r_2 - z)^m}\right\}\Bigg|_{z=r_1} \quad (19)$$

Now invoking the Liebnitz' differentiation rule and after simplifications, we obtain a closed-form solution:

$$I_m(p, a, b) = \frac{\Gamma(m)}{2p^m} - \frac{\Gamma(m)(2b)^{2m}}{2[Y(Y-X)]^m}\sum_{k=0}^{m-1}\frac{\Gamma(m+k)}{k!\Gamma(m)}\left[\frac{1}{2}\left(1 - \frac{X}{Y}\right)\right]^k \quad (20)$$

where $X = 2p + a^2 - b^2$.

Obviously (20) holds only for integer m. Besides, we would like to point out that (20) is equivalent to [2, Eq. (6)] but simpler than the latter. Therefore, the MGF of SNR at the SC combiner output in bivariate Nakagami fading can be conveniently evaluated using

$$\phi(s) = \sum_{i=1, j\neq i}^{2} \frac{2}{\Gamma(m)\Omega_i^m} I_m\left(s + \frac{1}{\Omega_i}, \sqrt{\frac{2\rho}{(1-\rho)\Omega_i}}, \sqrt{\frac{2}{(1-\rho)\Omega_j}}\right) = \sum_{i=1, j\neq i}^{2}\left\{C_i(s) - \frac{(2A_{ij})^{2m}}{\Omega_i^m\Gamma(m)}\sum_{k=0}^{m-1}\frac{\Gamma(m+k)(B_{ij}-1)^{k-m}}{2^k B_{ij}^{k+m} k!}\right\} \quad (21)$$

where $C_i(s) = (1 + s\Omega_i)^{-m}$, $A_{ij} = \frac{\sqrt{\Omega_j(1-\rho)/2}}{\Omega_j/\Omega_i - 1 + s\Omega_j(1-\rho)}$ and $B_{ij} = \frac{\sqrt{[s\Omega_i\Omega_j(1-\rho) + \Omega_i + \Omega_j]^2 - 4\rho\Omega_i\Omega_j}}{s\Omega_i\Omega_j(1-\rho) + \Omega_j - \Omega_i}$.

In [9, Appendix A], we have shown that it is possible to derive yet another closed-form formula (finite series expression) for the MGF with the aid of the integral formula identity [2, Eq. (6)].

IV. Error Rates for Binary and M-ary Signals

A. Average Error Probability

In this subsection, the MGF formula is used to derive error rate expressions for a broad class of modulation formats employing dual branch SC in Nakagami-m and Rayleigh fading channels. Craig outlined a simple method for computing the conditional error probability (CEP) of an arbitrary two dimension signalling constellation. Exploiting his result and some others, we can show that the CEP of a wide range of binary and M-ary signalling schemes (with coherent, differentially coherent and noncoherent detection) as a special case of the following generic form

$$P_S(\varepsilon|\gamma) = \sum_k \int_0^{\eta_k} a_k(\theta)\exp(-\gamma b_k(\theta))d\theta \quad (22)$$

where $a_k(\theta)$ and $b_k(\theta)$ are coefficients independent of γ but may be dependent on θ. Then, the ASER can be expressed in terms of only the MGF of the combiner output SNR (by taking the Laplace transform of the PDF):

$$P_S = \sum_k \int_0^{\eta_k} a_k(\theta)\phi(b_k(\theta))d\theta \quad (23)$$

It is clear that the evaluation of generic ASER expression only involves a single integral with finite integration limits since we have a closed-form solution for the MGF. Unlike the development of [5, Eq. (59)], no further manipulations are necessary. Furthermore, the evaluation for the independent fading case can be directly obtained by substituting $\rho = 0$ in our expressions. More importantly, our expressions (e.g., (21), [9, Eqs. (A.2), (A.4), (A.5)]) are not conditioned on the ratio between the arguments of the Marcum-Q function.

If the conditional error probability is in the exponential form, $P_S(\varepsilon|\gamma) = a\exp(-b\gamma)$, then we also have a closed-form expression for the ASER. For instance, the average bit error rate performance for binary DPSK and noncoherent FSK with dual-branch SC is given by

$$P_S = a\phi(b) \quad (24)$$

where $\{a = 1/2, b = 1\}$ for binary DPSK and $\{a = 1/2, b = 1/2\}$ for binary orthogonal FSK. As well, when $m = 1$ (Rayleigh fading), we get

$$\phi(s) = \sum_{i=1, j\neq i}^{2}\frac{(2A_{ij})^2}{[\Omega_i B_{ij}(1 + B_{ij})]} \quad (25)$$

where A_{ij} and B_{ij} are as defined in (21). Eqs. (31) and (32) in [5] follow at once from (25). Also notice that (25) (unlike Eq. (32) in [5]) is independent of the ratio between the arguments of the generalized Marcum-Q function even when $\rho \neq 0$. Similarly, for integer m and $\rho = 0$, Eq. (57) in [5] follows at once from (21).

For noncoherent MFSK modulation [10, Eq. (5-4-46)], it is straight-forward to show that the corresponding ASER is given by

$$P_S = \sum_{n=1}^{M-1}\frac{(-1)^{n+1}}{n+1}\binom{M-1}{n}\phi\left(\frac{n}{n+1}\right) \quad (26)$$

If the CEP is of the form $P_S(\varepsilon|\gamma) = a\,\mathrm{erfc}(\sqrt{b\gamma})$ (e.g., coherent binary PSK or FSK), then the ASER can be expressed as

$$P_S = \frac{2a}{\pi}\int_0^{\pi/2}\phi(b\csc^2\theta)d\theta \quad (27)$$

Similarly for $P_S(\varepsilon|\gamma) = a\mathrm{erfc}(\sqrt{b\gamma}) - c\mathrm{erfc}^2(\sqrt{b\gamma})$ (e.g., square QAM, quaternary PSK, coherent detection of differentially encoded PSK) the ASER is given by

$$P_S = \frac{2a}{\pi}\int_0^{\pi/2}\phi(b\csc^2\theta)d\theta - \frac{4c}{\pi}\int_0^{\pi/4}\phi(b\csc^2\theta)d\theta \quad (28)$$

Finally, it is also possible to derive closed-form solutions for a broad class of coherent, differentially coherent and noncoherent modulation formats in Nakagami-m channel (positive integer m) for the independent fading case by substituting $\rho = 0$ in (21) and utilizing the solutions to three generic trigonometric integrals derived in [12].

B. Outage Rate of Error Probability

Recognizing that the CDF can be expressed in terms of only the MGF by invoking Fourier inversion formula, the outage rate may be computed as

$$\begin{aligned} P_{out} &= F_\gamma(x^*) \\ &= \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{1}{\omega}\mathrm{Imag}[\phi(-j\omega)\exp(-j\omega x^*)]d\omega \\ &= \frac{1}{2} - \frac{2}{\pi}\int_0^{\pi/2}\mathrm{Imag}\left[\frac{\phi(-j\tan\theta)}{\sin 2\theta}\exp(-jx^*\tan\theta)\right]d\theta \end{aligned} \quad (29)$$

where the MGF $\phi(.)$ (in closed-form) for positive integer m is given in (21). For noninteger m, it is probably more appropriate to use (29) in conjunction with the MGF for the dual-diversity SC combiner output SNR in Nakagami-m fading derived by Okui instead of [9, Eq. (28)]:

$$\phi(s) = \frac{2^{2m}\Gamma(2m)}{\Gamma(m)\Gamma(m+1)}\sum_{i=1, j\neq i}^{2}\frac{(A_{ij})^{2m}}{[\Omega_i B_{ij}(1+B_{ij})]^m} \times {}_2F_1(1-m, m; 1+m; (1-1/B_{ij})/2) \quad (30)$$

Notice that the above formula reduces to a single finite range integral expression when m assumes a positive integer value. This is because the Gauss hypergeometric series reduces to a finite polynomial.

The advantage of our new CDF expression (using (29) and (30)) with respect to [5] is two fold. First of all, it handles all combinations of SNRs and power correlation coefficients in a single expression. Secondly, our expression is also valid for any real value of $m \geq 1/2$.

Comparison between (30) and [9, Eq. (28)] reveals that our new CDF expression can be computed more efficiently than the latter for any non-integer m since [9, Eq. (28)] involves the computation of a triple infinite series expression whereas the computational complexity of (29) is comparable to the evaluation of only two infinite series. For integer m, the computational complexity for both these expressions are almost the same, which is equivalent to the evaluation of a single infinite series.

V. Conclusions

A circular contour integral representation for the generalized Marcum-Q function, $Q_m(a, b)$ has been used to derive a new closed-form formula for the moment generating function (MGF) of the output signal power of a dual-diversity selection combiner (SC) in bivariate Nakagami-m fading with positive integer fading severity index. This result involves only elementary functions and holds for any value of the ratio a/b in $Q_m(a, b)$. As an aside, we showed that previous trigonometric integral representations for $Q_m(a, b)$ can be obtained directly from this contour integral. The MGF was used to unify the evaluation of average error performance and outage performance of a dual-branch SC for coherent, differentially coherent and noncoherent communications systems for both correlated and independent fading cases.

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