A CODING TECHNIQUE FOR REDUCING PEAK-TO-AVERAGE POWER RATIO IN OFDM

C. Tellambura
School of Computer Science and Software Engineering
Monash University
Wellington Road
Clayton, Victoria 3168
Australia
Ph: +61 3 9905 3296
Fax: +61 3 9905 3574
Email: chintha@dgs.monash.edu.au

Abstract

Recent research suggests combining partial transmit sequences (PTS) to reduce the peak-to-average power ratio of orthogonal frequency division multiplexing (OFDM). PTS involves forming several blocks of carriers and multiplying each by a constant phase factor. The phase factors are optimised to minimise the peak signal power. This paper proposes a new optimisation criterion for this purpose and provides simulation results to show the achievable peak factor reduction.

Key Words: Multicarrier Modulation, Orthogonal Frequency Division Multiplexing, Peak Factor

1. Introduction

A promising MCM technique is Orthogonal Frequency Division Modulation (OFDM), which is a method of transmitting data simultaneously over multiple frequency bands. OFDM is commonly implemented using Discrete Fourier Transform (DFT) techniques and has been adopted, or is being investigated, for wireless LANs, wireless ATM, digital audio broadcasting [1], terrestrial digital video broadcasting [2] and the broadband wireless local loop. OFDM offers many advantages such as resistance to the multipath effect and excellent performance under noisy conditions. OFDM, however, has high peak factor (PF) signals.

The OFDM signal comprises N parallel channels where N up to 2048 has been considered for practical systems. The PF problem arises when the N sinusoidal carriers add constructively, resulting in a peak signal power of as much as N times the mean signal power. In practice the peak signal is constrained by design factors such as battery power (portable equipment), or regulatory limits to prevent interference on communications in the adjacent frequency bands. Current attempts to limit the peak signal thus involve the use of non-linear amplifiers and digital hard limiting. But these cause inefficiency, interference and performance degradation.

The limitations of these techniques provide a clear motivation to search for alternative PF reduction techniques. In this vain, Jones *et al* [3] use a block coding technique to transmit across the carriers only those messages with small PF. This entails exhaustive search to identify the best messages and requires large look-up tables for encoding and decoding. Some contributions by other authors on the subject of PF limiting suffer from one of two drawbacks (or both):

- OFDM codec complexity $> O(N \log N)$.
- A vanishingly small code rate as $N \to \infty$.

For instance the schemes of Muller [4-6],Mestadgh [7], and Friese [8] all have coding complexity $> O(N \log N)$, and the scheme of Davis and Jedwab [9] has a rapidly vanishing rate as $N \to \infty$. However, Davis' scheme is based on Reed-Muller codes and the PF can be shown be less than 3 dB. The scheme proposed by Van Eetvelt *et al* [10] is a heuristic scheme, and is by no means optimal in terms of PF reduction. The PTS schemes of Muller *et al* [4-6] make use of the Inverse Discrete Fourier Transform (IDFT), which must be present in some form in the transmitter and receiver anyway. This provides a strong case for the use of such schemes.

PTS [4-6] involves forming several blocks of carriers and multiplying each by a constant phase factor. Fig. 1 illustrates this scheme. The phase optimiser computes the phase factors, so that the maximum amplitude of the IDFT output is minimised. However, this strategy relies on testing for peaks on the DFT bins whereas, in general, the worst-case peak occurs between bins. In other words, this scheme minimises the discrete time PF. This paper however presents a new optimisation criterion to be used for

0-7803-4984-9/98/$10.00 ©1998 IEEE.

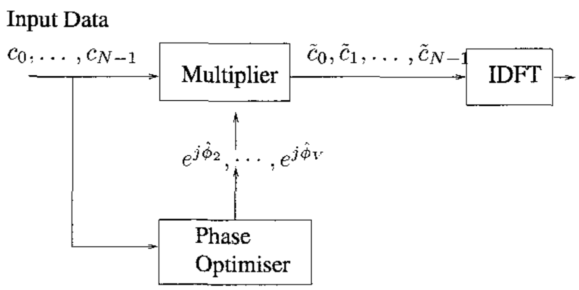

Figure 1: PF reduction with PTS

the computation of the phase factors, and this approach leads to reduction of the true PF. Also, it is shown that the use of discrete signal samples to estimate the peak level can lead to optimistic estimates of the achievable peak factor reduction.

2. PTS Optimisation

The signal consists of N complex carriers and hence

$$s(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} c_n e^{i[n\Delta\omega t]} \qquad (1)$$

where $i = \sqrt{-1}$ and $c_n = e^{i\theta_n}$ ($\theta_n \in \{2\pi k/M \mid k = 0, \dots, M-1\}$ is the M-ary phase shift-keying modulation symbol for the n-th carrier. If $s(t)$ is sampled at a frequency of $1/T$, the OFDM symbol duration is $\tau = NT$ (for orthogonality $\Delta\omega = 2\pi/NT$). In practice, samples of (1) are generated by means of an inverse fast Fourier transform (IFFT), which are fed to a digital-to-analogue converter followed by an anti-aliasing low-pass filter. The peak factor (PF) relating to (1) is defined as

$$\gamma = \max_{0 \leq t < \tau} |s(t)|^2. \qquad (2)$$

An N-point IFFT on $\underline{c} = [c_0, \dots, c_{N-1}]$ only gives samples of $s(t)$ at time instants $t = kT$ for $k = 0, 1, \dots, N-1$. Clearly, the peak amplitude of the IFFT of $\underline{c}$ does not necessarily yield the PF. Therefore, if N samples are used to estimate the PF, this estimate is called the LPF (lower PF), which can also be called the discrete time PF. To get a better estimate, $s(t)$ can be oversampled by a factor of 8, and this estimate is called the TPF. Note that we found the oversampling factor of 8 to be sufficient, and increasing it to 16 hardly changes our results. Since LPF $\leq$ TPF $\leq \gamma$, the use of LPF to estimate the PF reduction of a PTS scheme is prone to error.

The IFFT output can be represented by a matrix multiplication as

$$\underline{x} = A\underline{c} \qquad (3)$$

where $A = 1/\sqrt{N}[\exp i(2\pi lm/N)]$ $(0 \leq l, m < N)$ is the usual Fourier matrix and $\underline{c}$ is the MPSK symbol sequence. It can be seen that ideal low-pass filtering of $\underline{x}$ gives the waveform (1). The PTS approach [4] can be summarised as follows. Divide the symbols c_j to V sub-blocks of N/V symbols. Let $q_k = \{j \mid c_j \in \text{block } k\}$ for $k = 1, \dots, V$. The carriers are numbered from 0 to $N-1$, and $\bigcup q_k = \{0, \dots, N-1\}$. The simplest case is q_k to consist of a block of contiguous carriers (i.e., $q_k = \{j + (k-1)N/V \mid j = 0, \dots, N/V - 1\}$), which is especially suitable for differential detection systems [5]. Blocks may contain non-contiguous carriers for better PF reduction capability at the cost of extra complexity [6]. Let ϕ_k, $k = 1, \dots, V$, be a set of phases with $\phi_1 = 0$. The modified symbols $\tilde{c}_j = c_j e^{i\phi_k}$ for $j \in q_k$ and $\underline{\tilde{c}} = [\tilde{c}_0, \tilde{c}_2, \dots, \tilde{c}_{N-1}]$. For a given information vector $\underline{c}$, the optimisation criterion given in [5] is

$$[\hat{\phi}_2, \hat{\phi}_3, \dots, \hat{\phi}_V] = \underset{[\phi_2, \phi_3, \dots, \phi_V]}{\operatorname{argmin}} \|A\underline{\tilde{c}}\|_\infty \qquad (4)$$

where for vector $\underline{x} = (x_1, x_2, \dots, x_n)$ the norm $\|\underline{x}\|_\infty$ denotes $\max|x_i|$ for $1 \leq i \leq n$. As well, the operator $\operatorname{argmin}(\cdot)$ denotes the argument for which the function is minimised. To make this optimisation problem tractable, the search space can be limited to discrete points. That is, the phase factors are limited so that

$$\phi_2, \phi_3, \dots, \phi_V \in \left\{0, \frac{2\pi}{H}, \dots, \frac{2(H-1)\pi}{H}\right\}. \qquad (5)$$

For QPSK modulated systems, it is highly desirable to have $H = 2$ or 4. This choice enables complex multiplications to be replaced by integer additions, yielding a more efficient implementation.

The actual transmitted sequence is now given by (3) with this set of block phase factors ($\underline{c}$ replaced by $\underline{\tilde{c}}$), which may have to be transmitted by some means (e.g. extra carriers). Therefore, the redundancy needed for PF reduction is

$$\eta = \frac{(V-1)\log_2 H}{N} \quad \text{bits/carrier.} \qquad (6)$$

It is clear that (4) is equivalent to minimising the discrete PF. Therefore, it does not guarantee that the true PF is equally reduced. To elaborate on this point, consider the following modified function:

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=1}^{V} \sum_{n \in q_k} c_n e^{i[n\Delta\omega t + \phi_k]}. \qquad (7)$$

Now if we attempt to minimise the maximum of $|s(kT_s)|$ for $k = 0, \dots, N-1$, since

$$\int |s(t)|^2 dt = \text{constant}$$

it is likely that the true peak of $|s(t)|$ to move away from the sampling points.

A recent letter [11] shows that a bound on the PF can obtained by the sum of the autocorrelation (aperiodic) sidelobe magnitudes of the modulation symbol sequence. Let $\rho(k)$ be the aperiodic autocorrelation of $\underline{\tilde{c}}$:

$$\rho(k) = \sum_{n=1}^{N-k} \tilde{c}_{n+k}\tilde{c}_n^* \quad \text{for} \quad k = 0, \ldots, N-1. \tag{8}$$

The bound on the PF is then given as

$$\gamma \leq 1 + \frac{2}{N} \sum_{k=1}^{N-1} |\rho(k)|. \tag{9}$$

Based on this bound, we propose the following optimisation criterion: for a given information vector $\underline{c}$,

$$[\hat{\phi}_2, \hat{\phi}_3, \ldots, \hat{\phi}_V] = \underset{[\phi_2, \phi_3, \ldots, \phi_V]}{\operatorname{argmin}} \sum_{k=1}^{N-1} |\rho(k)|. \tag{10}$$

This involves computing the sum of the autocorrelation sidelobe amplitudes of the modified sequence ($\underline{\tilde{c}}$) for every possible phase factor combinations (H^{V-1}), and selecting the phase factor combination which minimises this sum.

Both Eqs. (4) and (10) require searching over H^{V-1} points in the phase space. However, computing the IDFT is less complex than computing the sum of the autocorrelation amplitudes. In this sense, (4) is a better choice than (10). On the other hand, the latter is a bound on the true peak factor, and any resultant PF reduction is applicable to the true PF, not just to the discrete PF.

3. Simulation Results

All simulations are performed with $N = 128$, i.e., 128 carriers. Randomly generated data are modulated into QPSK symbols. All PF reduction estimates are based on the true PF, which is obtained by oversampling each OFDM symbol. Note that these PF reduction estimates would be larger if the discrete time PF was used for estimation.

A. Difference between LPF and TPF

To reduce the search complexity here, H is set to 4. Fig. 2 shows the distribution of the PF for the uncoded case (i.e. no attempt to reduce the peak factor) and for PTS with 4 blocks. Several observations can be made:

- For the uncoded case, the LPF follows the Rayleigh distribution. However, the TPF is about 1/2 dB worse than that predicted by the Rayleigh distribution.
- The use of TPF estimates a PF reduction of 1 dB only 1 for the PTS scheme.
- The use of LPF estimates a PF reduction 4 dB for the PTS scheme.

Therefore, at least in this case, the PTS gains reported [5] appear to optimistic. Notice the difference between TPF and LPF is about 3 dB (Fig. 2). The reason seems to be the following. Since (4) is equivalent to suppressing $|s(kT)|$ for $k = 0, \ldots, N-1$ while keeping the area under $|s(t)|^2$ constant, this very fact *causes* the true peak of $|s(t)|$ to move away from the sampling points.

B. PF reduction

Fig. 3 shows the PF distribution with the the use of (10). A PF reduction of about 3 dB can be achieved at $\Pr(\gamma > \alpha) = 10^{-5}$. Note that the difference between TPF and LPF is 0.5 dB in this case (Fig. 3). Since the sum in (10) determines a bound on the true peak, its minimisation leads to a flatter $|s(t)|$ for $0 \leq t < \tau$, not just on the sampling points. In a practical system, implementing (10) can be too complicated. For QPSK, real and imaginary part of each $\rho(k)$ can be obtained without any multiplications (the number of integer additions in the order of N^2). So the total complexity varies as $O(4^{V-1}N^2)$.

C. Varying the number of subblocks

Fig. 4 shows the effect of changing V, the number of subblocks. $V = 1$ depicts the OFDM case without any PF reduction applied. $V = 4$ provides a PF reduction of about 2.5 dB at $\Pr(\gamma > \alpha) = 10^{-4}$. Increasing V to 5 results in an incremental reduction of 0.5 dB. Note that the phase optimiser (Fig. 1) search space has H^{V-1} points. Thus, any increase in V leads to an exponential increase in the overall complexity.

D. Varying the number of phase angles

Fig. 5 shows the effect of increasing the number of phase angles, H. $V = 1$ depicts the OFDM case without any PF reduction applied. $V = 3$ for all the other curves there. $H = 4$ provides a PF reduction of about 2 dB at $\Pr(\gamma > \alpha) = 10^{-4}$. Increasing H to 8 results in an incremental reduction of 0.2 dB. Thus, for $V = 3$, $H = 4$ is a reasonable choice.

E. Complexity reduction

From (8), it can be seen that each $\rho(k)$ requires computing $N - k$ multiplications (although these can be simplified if the symbols are limited to the QPSK signal set). Therefore, computing (10) requires roughly $N^2/2$ complex multiplications. So this is the major source of complexity in this approach, and provides a clear motivation to simplify (10). One possibility is to include only a subset of $|\rho(k)|$ values in the sum, not all $N-1$. Following this idea, only the first 64 values of $|\rho(k)|$ are included in (10). This modified sum requires about 25% fewer complex multiplications. Fig. 6 shows the achievable PF reduction in this case. With $V = 4$ and $H = 4$, this method provides a PF reduction of about 2 dB at $\Pr(\gamma > \alpha) = 10^{-4}$.

4. Conclusions

PTS approach requires determining several block phase factors, for which a new optimisation criterion has been presented. Assuming an ideal anti-aliasing filter, its output closely resembles the multicarrier signal (1). Therefore, it is not sufficient to perform the optimisation on the basis of discrete time PF. The complexity of the search process can be reduced by simplifying the search criterion. This idea needs further investigation.

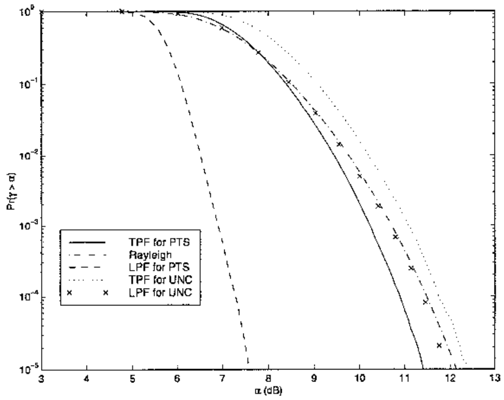

Figure 2: Probability that peak factor, γ, exceeds α for 128 QPSK-modulated careers. TPF -true peak factor, LPF - lower peak factor, UNC - uncoded. Optimisation criterion (4). (10^6 simulation points).

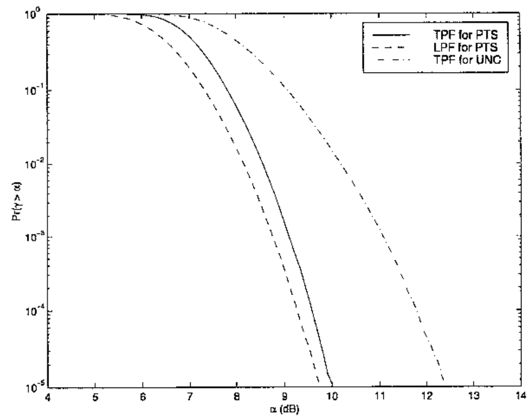

Figure 3: Probability that peak factor, γ, exceeds α for 128 QPSK-modulated careers. Optimisation criterion (10) with $V = 4$ (10^6 simulation points).

References

[1] P. Shelswell, "The COFDM modulation system: the heart of digital audio broadcasting," *Elec. Commun. Eng. J.*, pp. 127–136, June 1995.

[2] G. K. H. Sari and I. Jeanclaude, "Transmission techniques for digital terestrial TV broadcasting," *IEEE Commun. Magazine*, vol. 33, pp. 100–109, Feb. 1995.

[3] A. E. Jones, T. A. Wilkinson and S. K. Barton, "Block coding scheme for reduction of peak to mean envelope power ratio of multicarrier transmission schemes," *Electron. Lett.*, vol. 30, pp. 2098–2099, 1994.

[4] S. H. Müller, R. W. Bäuml, R. F. H. Fisher and J. B. Huber, "OFDM with reduced peak-to-average power ratio by multiple signal representation," *Annals of Telecommunications*, vol. 52, pp. 58–67, Feb. 1997.

[5] S. H. Müller and J. B. Huber, "OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences," *Electron. Lett.*, vol. 33, pp. 368–369, Feb. 1997.

[6] S. H. Müller and J. B. Huber, "A novel peak power reduction scheme for OFDM," in *1997 Int. Sym. on Personal, Indoor and Mobile radio comms. Proc.*, pp. 1090–1094, IEEE, 1997.

[7] D. J. G. Mestdagh and P. M. P. Spruyt, "A method to reduce the probability of clipping in DMT based transceivers," *IEEE Trans. Commun.*, vol. 44, pp. 1234–1238, Oct. 1996.

[8] M. Friese, "Multicarrier modualtion with low peak-to-average power ratio," *Electron. Lett.*, vol. 11, pp. 713–714, Apr. 1996.

[9] J. A. Davis and J. Jedwab, "Peak-to-mean power control and error correction for OFDM transmission using Golay sequences and Reed-Muller codes," *Electron. Lett.*, vol. 33, pp. 267–268, Feb. 1997.

[10] P. Van Eetvelt, G. Wade and M. Tomlinson, "Peak to average power reduction for OFDM schemes by selective scrambling," *Electron. Lett.*, vol. 32, pp. 1963–1964, Oct. 1996.

[11] C. Tellambura, "Upper bound on the peak factor of N-multiple carrires," *Electron. Lett.*, vol. 33, pp. 1608–1609, Sept. 1997.

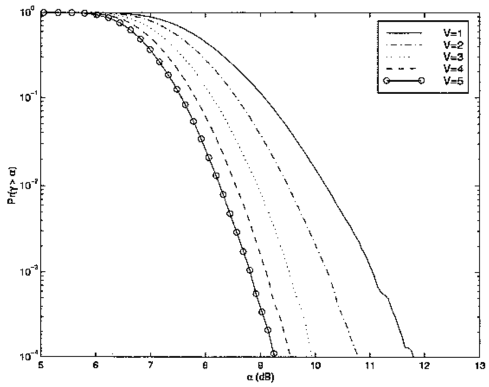

Figure 4: Probability that peak factor, γ, exceeds α for 128 QPSK-modulated careers. Optimisation criterion (10) with $H = 4$ (10^5 simulation points).

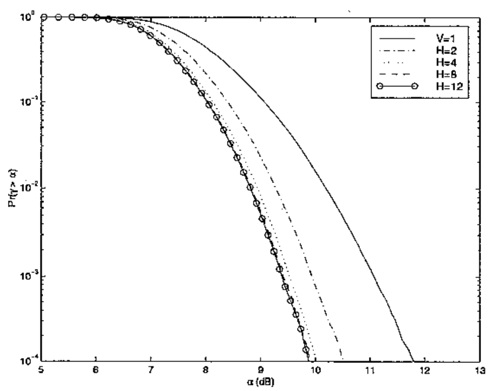

Figure 5: Probability that peak factor, γ, exceeds α for 128 QPSK-modulated careers. Optimisation criterion (10) with $V = 3$ (5×10^5 simulation points).

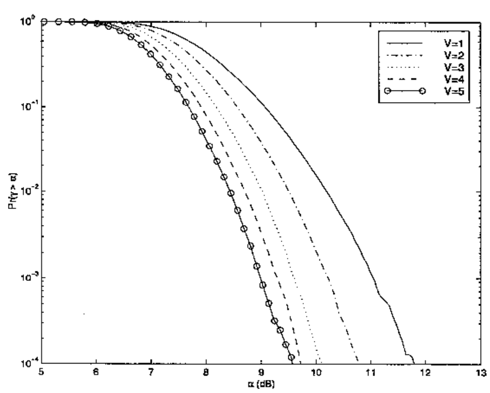

Figure 6: Probability that peak factor, γ, exceeds α for QPSK-modulated 128-career OFDM. Optimisation criterion (10) with $H = 4$ (10^5 simulation points).