A Simple and Accurate Analysis of Digital Communication Systems with Diversity Reception in Different Fading Environments[1]

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ABSTRACT

This paper briefly surveys some of the diversity techniques commonly used in cellular radio and satellite communication systems to mitigate the detrimental effects of signal fading. Subsequently, an analytical technique well suited to numerical analysis is presented for computing the average symbol error probability (SER) of a wide class of coherent, differentially coherent and noncoherent communication systems with microdiversity reception under a myriad of fading scenarios. We restrict our analysis to predetection maximal-ratio combining (MRC) scheme, although this method applies to other diversity combining techniques as well. Our novel derivation relies upon the properties of the moment generating function (MGF) of the fading channels, the use of an alternative exponential form of the complementary error function, and the application of Gauss-Chebychev quadrature (GCQ) rules. The closed-form expressions obtained are sufficiently general to allow for arbitrary fading parameters as well as dissimilar mean signal strengths across the diversity branches. Moreover, this method is computationally stable and approximates the true value of average SER within any degree of accuracy.

I. INTRODUCTION

Diversity is a powerful communication receiver technique that provides wireless link improvement at relatively low cost. The underlying premise is that if two or more statistically independent (or at least highly uncorrelated) replicas of a signal received over multiple diversity branches with comparable strengths, then it is improbable that all these signals will be in a fade at any given instant in time. Besides mitigating the deep fades experienced in wireless channels, diversity methods play a crucial role in minimizing the transmit power requirements, particularly in the reverse link, because the battery capacity of handheld subscriber units is usually limited. As well, it reduces the penalty in signal-to-noise ratio (SNR) due to co-channel interference [1].

To take advantage of the improvement in signal statistics due to diversity, several combining techniques have been proposed in literature, and they can be classified into two groups, namely switched combining and gain combining [2, 3]. Among these approaches, maximal-ratio combining is known to be optimum in the sense that it yields the best statistical reduction of fading of any linear diversity combiner as well as provides

the highest average output SNR. In this technique, all the L independent diversity branches are first co-phased, and then weighted in proportion to their signal level before summing. Recent advances in digital signal processing has resulted in widespread usage of MRC diversity techniques in current systems (e.g., in rake receivers and antenna arrays).

While there are many excellent papers on the subject of fading channels and diversity reception, with many cases having been thoroughly analyzed, the approach adopted in this paper results in "clean" derivations for the error probability expressions which are numerically efficient. As in [4, 5], one of the important technique recurring throughout the paper is the use of MGFs to obtain the closed-form SER expressions.

The organization of this paper is as follows. Section II outlines some of the diversity methods commonly used in mobile radio environment. Subsequently in Section III, the MGFs for several typical fading scenarios are derived. The error performance for a wide class of coherent, differentially coherent and noncoherent communication systems with predetection MRC is presented in Section IV. Finally, the main points are summarized in Section V.

II. DIVERSITY TECHNIQUES COMMONLY USED IN WIRELESS COMMUNICATIONS

A. Space Diversity

Historically, space diversity has been the most common form of diversity used in cellular radio networks, owing to its simple implementation and because it does not require additional frequency spectrum resources [6]. The uncorrelated diversity branches are attained from "sufficiently" spaced receiving antennas. The required antenna spacing depends on the multipath angle spread. For the microscopic diversity, antenna separation of the order of 0.5λ–0.8λ is adequate because the multipath signals at the mobile usually arrive from all the directions in the azimuth. On the other hand, a larger coherence distance (typically 10λ–20λ) is required for the macroscopic diversity at the base station due to the small multipath angle spread [1]. Empirical measurements also show that larger antenna heights require wider antenna separation.

Figure 1 illustrates the effect of two branch micro-diversity on the signal fading envelope. This was obtained through simulation [7] with the following parameters: 900 MHz carrier frequency, a mobile speed of 15 kilometers per hour and a Rician factor $K = 2$. It is apparent that the diversity helps to mitigate the effects of the deep fades experienced on wireless channels.

Also notice that the combined signal strength (at the output of MRC) always exceeds that of the best branch.

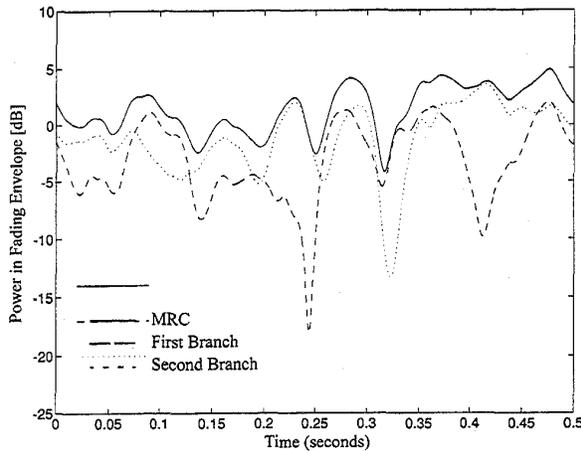
Figure 1. Effects of antenna diversity reception on the received power in a Rician fading channel.

B. Polarization Diversity

The implementation of space diversity at the base station is considerably less practical than at the mobile terminal because the narrow angle of incident fields requires large antenna separation. The comparatively high cost of using space diversity at the base station prompts the consideration of using orthogonal polarization to exploit polarization diversity. While the order of diversity is limited to two, it does allow antenna elements to be collocated. This is an important advantage in the personal communication service (PCS) base stations where low profile antennas (i.e., compact antenna assembly) are needed.

More importantly, since the handset can be held at random orientations (due to hand-tilting) during a call, the use of cross-polarized antennas at the base station is highly desirable because at least one of the two antennas will be well matched to the signal launch polarization.

C. Frequency Diversity

Frequency diversity exploits the fact that if several replicas of a signal are transmitted on different carrier frequencies which are separated by at least the coherence bandwidth (which is related to the reciprocal of the multipath delay spread) of the channel, then it is unlikely that all the signals will experience simultaneous deep fades. However, this technique has the disadvantage that it not only requires additional spectrum (which is proportional to the diversity order), but also requires a higher transmitter power and as many receivers as the number of channels used for frequency diversity. A common form of frequency diversity is multicarrier (or multitone) modulation.

D. Time Diversity

In time diversity, redundant information is transmitted after a specified time spacing that exceeds the coherence time. The coherence time depends on the Doppler spread of the signal, which in turn is a function of the carrier frequency and mobile terminal velocity. The time diversity is usually exploited via interleaving, forward-error correction coding and automatic

repeat-request. One fundamental drawback with this approach is that it introduces unacceptable delay in obtaining time diversity when the mobile terminal is moving very slowly. As well, the increased transmission power requirement makes it less efficient than the space diversity.

E. Angle (Directional) Diversity

Angle diversity is usually obtained by employing directional antennas in situations where the angle spread is very high because it attempts to reduce the Doppler spread. This form of diversity has been extensively utilized in indoor wireless local area networks.

F. Multipath Diversity

Path diversity is achieved by resolving and combining the multipath components of a transmitted signal if the signal bandwidth is much larger than the coherence bandwidth, such as in spread-spectrum systems. Resolvability is ensured if the multipath arrivals are separated by at least one chip duration. Since the diversity branches are created after signal reception, multipath diversity is also referred as implicit diversity. The rake receiver and the adaptive equalizer are two typical examples that exploit path diversity. The most distinct feature of this technique is that no extra antenna, power, or spectrum are necessary to achieve the path diversity. However, the attainable diversity gain is dependent of the delay profile.

III. STATISTICAL REPRESENTATION OF THE FADING CHANNEL

To evaluate and simulate a channel based on the statistical models chosen, the probability density function (PDF), the cumulative distribution function (CDF) and the moment generating function (MGF) for each channel model are required. These functions are briefly described below.

Given a random variable $x(k)$ and any fixed value of x, the CDF of x is defined as,

$$F(x) = \text{Prob}[x(k) \le x] \qquad (1)$$

Differentiating $F(x)$ with respect to x, the PDF $p(x)$ is obtained,

$$p(x) = \frac{d}{dx}F(x) \qquad (2)$$

The PDF indicates the relative frequency of occurrence of any fixed value of x. Finally, the MGF is defined as,

$$m(z) = E[e^{-zx}] = \int_{-\infty}^{\infty} e^{-zx} p(x) dx \qquad (3)$$

where $E[x]$ is the expected value (also called mean or average value) of $x(k)$. (Note, the MGF is closely related to another statistical function, the characteristic function and is easily translated by the variable substitution $z = -jv$). By definition, Eq. (3) is the Laplace transform of the PDF $p(x)$. Thus, using the Laplace transform of a derivative property, the MGF may alternatively be written in terms of the CDF,

$$m(z) = \int_{-\infty}^{\infty} e^{-zx} F'(x) dx = z \int_{-\infty}^{\infty} e^{-zx} F(x) dx - F(0) \qquad (4)$$

where $F'(x)$ is the first derivative of $F(x)$.

The random variable of interest here is the power in the fading envelope of the received signal. Assuming the channel con-

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ditions remain near constant over the duration of any one symbol period (this slow fading assumption is valid for non-frequency selective fading channels), the received signal corresponding to the k-th transmitted symbol may be expressed as,

$$y_k = s_k x_k + n_k \tag{5}$$

where x_k is the transmitted signal corresponding to the k-th information symbol, n_k is additive Gaussian noise with a single-sided spectral density $N_0/2$ and s_k is the complex channel gain (equal to unity in the absence of fading). In complex notation, $s_k = s_{k,I} + js_{k,Q}$ where $s_{k,I}$ and $s_{k,Q}$ are the in-phase and quadrature components of the fading envelope respectively. The received power in the faded envelope is then $\gamma = |s|^2 = s_I^2 + s_Q^2$.

Since our new approach for computing the error performance only requires the knowledge of the MGF of the fading channels, in the following we will summarize the MGFs for some of the commonly used fading channel models.

A. Rician Distribution

The MGF for the non-centralized chi-squared distribution is well known, and is given by [8],

$$m_l(z) = \frac{1+K_l}{1+K_l+z\Omega_l} \exp\left(\frac{-zK_l}{1+K_l+z\Omega_l}\right) \equiv \phi(z\Omega_l, K_l) \tag{6}$$

where $\Omega_l = E[\gamma_l]$, and K_l is the Rice factor of the lth diversity branch, defined as the ratio of the power in the line-of-sight (LOS) path to the power in the multipaths.

In a limiting case when the power in LOS path approaches zero, then $K \to 0$ and the channel reverts to the Rayleigh fading channel. Then the corresponding MGF of the centralized chi-squared distribution is,

$$m_l(z) = \frac{1}{1+z\Omega_l} \tag{7}$$

B. Nakagami Distribution

The Nakagami distribution (m-distribution) is a versatile statistical distribution because it can accurately model a variety of fading environments. It has greater flexibility in matching some empirical data than the Rayleigh, lognormal or Rice distributions owing to its characterization of the received signal as the sum of vectors with random moduli and random phases. As well, this statistical model includes the Rayleigh and the one-sided Gaussian distributions as special cases for the fading figure $m = 1$ and $m = 0.5$, respectively. Moreover, the m-distribution can closely approximate the Rician distribution via relationship $m = (K+1)^2/(2K+1)$ [9].
The MGF for this fading channel is given by,

$$m_l(z) = \left(\frac{\lambda_l}{\lambda_l + z}\right)^{m_l} \tag{8}$$

where $\lambda_l = m_l/\Omega_l$. It is evident that (8) reduces to (7) when $m = 1$ (i.e., Rayleigh fading).

C. Lognormal Rice Distribution

The Rician distribution assumes a constant K. In reality, this may not be the case since as the mobile terminal moves through the cell, a variety of topographical surrounding are encountered. This is particularly evident in terrestrial environment where shadowing (physical obstruction of the signal path)

is more severe. Hence, it is plausible to consider a combined distribution incorporating the effects of shadowing into the Rician distribution.

Expressing the received fading envelope as the product of independent Rice and lognormal distributions, the MGF can be shown to be,

$$m_l(z) = \frac{1}{\sqrt{\pi}} \int_0^\infty \phi\left(z\mu_l \exp\left(\sqrt{2}\sigma_l x\right), K_l\right) \exp\left(-x^2\right) dx \tag{9}$$

where $x = \ln(\Omega/\mu) / (\sqrt{2}\sigma)$, σ is the logarithmic standard deviation of shadowing, and μ is the local mean power. Now applying Hermitian integration, a closed-form expression for (9) is obtained,

$$m_l(z) = \frac{1}{\sqrt{\pi}} \sum_{i=1}^n w_i \phi\left(z\mu_l \exp\left(\sqrt{2}\sigma_l x_i\right), K_l\right) + R_n \tag{10}$$

where x_i and w_i are tabulated in [10] for $n \le 20$ and R_n is a remainder term.

D. Suzuki Distribution

Suzuki fading characterizes the joint effects of Rayleigh fading and lognormal shadowing and models a shadowed multipath channel without a LOS path. Since Suzuki distribution is a special case of the lognormal Rician distribution, its MGF is readily obtained by setting $K = 0$ in (10),

$$m(z) = \frac{1}{\sqrt{\pi}} \sum_{i=1}^n \frac{w_i}{1+za_i} + R_n \tag{11}$$

where a_i is equal to $\mu \exp[\sqrt{2}\sigma x_i]$.

E. Mixed Fading

Owing to the time-varying nature of the wireless channels, a practical wireless channel may be more realistically modelled as combination of different statistical distributions. The lognormal Rice or the lognormal Nakagami distribution may be further refined if additional information of the channel condition is available. For instance, in [11] Lutz presented a two-state land mobile satellite channel model (see Figure 2) based on channel measurements.
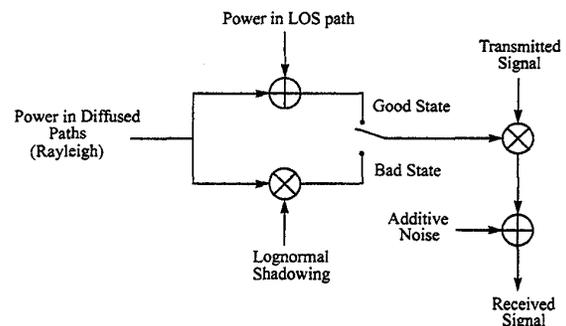

Figure 2. Functional diagram of the satellite channel model.

In this model, the channel is in the good state for a fraction of the time $1 - A$, and modelled as a Rician random process. For the remaining fraction of the time A, the channel is in the bad state modelled as a lognormally shadowed Rayleigh random process, or equivalently, a Suzuki random process. The net PDF of the received power is thus the weighted sum of the Rician

and Suzuki PDFs, $p_{\text{Rice}}(\gamma)$ and $p_{\text{Suzuki}}(\gamma)$ respectively,

$$p(\gamma) = (1-A)p_{\text{Rice}}(\gamma) + Ap_{\text{Suzuki}}(\gamma) \tag{12}$$

Similarly, using the definition of the MGF from (4), the net MGF is [4],

$$m(z) = \int_0^\infty e^{-zs}p(\gamma)\,d\gamma = (1-A)m_{\text{Rice}}(z) + Am_{\text{Suzuki}}(z) \tag{13}$$

where $m_{\text{Rice}}(z)$ and $m_{\text{Suzuki}}(z)$ are the MGFs for the Rician and Suzuki fading states given in (6) and (11), respectively.

IV. ERROR PERFORMANCE OF DIVERSITY SYSTEMS

In this section, numerically efficient solutions for calculating the SER and/or ABER for coherent, differentially coherent and noncoherent communications systems employing predetection MRC are derived. It is shown that for a wide class of modulation schemes, the SER can be obtained directly from the MGF of the fading channel.

Before proceeding further, we will derive the MGF of the combined received signal power at the output of maximal-ratio combiner. Using the definition in (3), we have [12],

$$
\begin{aligned}
\varphi_\gamma(s) &= E[\exp[-s(\gamma_1 + \gamma_2 + \dots + \gamma_L)]] \\
&= \int_0^\infty \dots \int_0^\infty e^{-s(\gamma_1 + \gamma_2 + \dots + \gamma_L)}p(\gamma_1)\dots p(\gamma_L)\,d\gamma_1\dots d\gamma_L \\
&= \prod_{l=1}^{L} m_l(s)
\end{aligned} \tag{14}
$$

where L denotes the order of diversity. In other words, the MGF of the signal at the output of MRC is simply the product of the MGF of received signal at each of the statistically independent diversity branches.

Table 1. Instantaneous SER of several common modulation schemes.

Modulation Scheme	Conditional Error Probability $P_S(\varepsilon\mid\gamma_b)$
Coherent binary signalling:	
(a) Coherent PSK	$0.5\,\text{erfc}(\sqrt{\gamma_b})$
(b) Coherent detection of differentially encoded PSK	$\text{erfc}(\sqrt{\gamma_b}) - \frac{1}{2}\text{erfc}^2(\sqrt{\gamma_b})$
(c) Coherent FSK	$0.5\,\text{erfc}(\sqrt{\gamma_b/2})$
Noncoherent binary signalling:	
(a) DPSK	$0.5\exp(-\gamma_b)$
(b) Noncoherent FSK	$0.5\exp(-\gamma_b/2)$
Coherent quadrature signalling:	
(a) QPSK	$\text{erfc}(\sqrt{\gamma_b}) - 0.25\,\text{erfc}^2(\sqrt{\gamma_b})$
(b) MSK	$\text{erfc}(\sqrt{\gamma_b}) - 0.25\,\text{erfc}^2(\sqrt{\gamma_b})$
Coherent Multilevel signalling:	
(a) Square QAM	$2q\,\text{erfc}(\sqrt{p\gamma_b}) - q^2\text{erfc}^2(\sqrt{p\gamma_b})$ where $q = 1 - 1/\sqrt{M}$ and $p = 1.5\log_2 M/(M-1)$
(b) MPSK	$\text{erfc}[\sqrt{\gamma_b\log_2 M}\sin(\pi/M)]$ (approx.)

Table 1 summarizes the instantaneous SER for several common modulation schemes in an AWGN channel. One immediately recognizes that these expressions can be categorized into one of the three general forms, namely those which are functions of the complementary error function, the two-dimension Gaussian probability integral, and those which are of exponential form. The average SER in the fading channels with diver-

sity may be derived by averaging the conditional error probability (i.e., error rates for the AWGN channel) over the PDF of SNR of the combined received signal envelope in the specified fading environment [8], i.e.,

$$P_S(\varepsilon) = \int_0^\infty P_S(\varepsilon\mid\gamma_b)p_{\gamma_b}(\gamma_b)\,d\gamma_b \tag{15}$$

where $p_{\gamma_b}(\gamma_b)$ denotes the PDF of the instantaneous SNR per bit with L-fold MRC diversity reception.

Consider the first form, where the instantaneous SER is,

$$P_S(\varepsilon\mid\gamma_b) = a\,\text{erfc}(\sqrt{b\gamma_b}) \tag{16}$$

where a and b are selected according to Table 1. In order to obtain an expression which is a function of the channel MGF, an alternate exponential form for $\text{erfc}(.)$ is required. Consider the definite integral and its solution given in [eq. 7.4.11, 10],

$$\int_0^\infty \frac{\exp(\alpha^2 s^2)}{s^2 + z^2}dt = \frac{\pi}{2z}\exp(\alpha^2 z^2)\text{erfc}(\alpha z), \quad \alpha > 0, z > 0 \tag{17}$$

Then the complimentary error function can be expressed in the desired form [12],

$$\text{erfc}(\sqrt{b\gamma}) = \frac{2\sqrt{b}}{\pi}\int_0^\infty \frac{\exp[-\gamma(s^2 + b)]}{s^2 + b}ds, \quad b > 0, \gamma > 0 \tag{18}$$

Making the variable substitution $\tan\theta = s/\sqrt{b}$ and using the trigonometric property $1 + \tan^2\theta = \sec^2\theta$, we arrive to,

$$\text{erfc}(\sqrt{b\gamma}) = \frac{2}{\pi}\int_0^{\pi/2}\exp(-b\gamma\sec^2\theta)\,d\theta \tag{19}$$

This form is both easily evaluated and well suited to numerical integration since the integrand is well behaved over the range of the integral. Substituting this alternative form (i.e., (19)) into (15), and then interchanging the order of integration and recognizing the integral with respect to γ is equal to the MGF of the fading channel evaluated at $b\sec^2\theta$, (15) reduces to,

$$
\begin{aligned}
P_S(\varepsilon) &= \frac{2a}{\pi}\int_0^\infty d\theta \int_0^{\pi/2}\exp(-b\gamma\sec^2\theta)p_\gamma(\gamma)\,d\gamma \\
&= \frac{2a}{\pi}\int_0^{\pi/2}m(b\sec^2\theta)\,d\theta
\end{aligned} \tag{20}
$$

The second form in Table 1 involves both the single and two-dimension Gauss probability integrals,

$$P_S(\varepsilon\mid\gamma) = a\,\text{erfc}(\sqrt{b\gamma_b}) + c\,\text{erfc}^2(\sqrt{d\gamma}) \tag{21}$$

Exploiting Simon's results in [13], we can express $\text{erfc}^2(.)$ as,

$$\text{erfc}^2(\sqrt{d\gamma}) = \frac{4}{\pi}\int_0^{\pi/4}\exp(-d\gamma\csc^2\theta)\,d\theta \tag{22}$$

and the corresponding SER is given by,

$$P_S(\varepsilon) = \frac{2a}{\pi}\int_0^{\pi/2}m(b\sec^2\theta)\,d\theta + \frac{4c}{\pi}\int_0^{\pi/4}m(d\csc^2\theta)\,d\theta \tag{23}$$

The third form in Table 1 is the exponential form. The general expression for the instantaneous SER is,

$$P_S(\varepsilon\mid\gamma) = ae^{-b\gamma} \tag{24}$$

where again a and b are obtained from Table 1. Substituting (24) into (15) and interchanging the order of integration, the average SER can be written in terms of the MGF,

$$P_S(\varepsilon) = am(b\gamma) \tag{25}$$

Thus, the SER for all the modulation schemes listed in Table 1

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can be obtained for different fading environments by simply evaluating (20), (23) and (25), with the appropriate channel MGF presented in Section III.

In general, a closed-form expression for (20) cannot be obtained. However, if the channel's MGF is of the form $m(z) = (1 + za)^{-L}$, one can directly solve (15) to obtain a closed form expression [4].

Next, we will describe the use of GCQ formulas to approximate the average SER of different data transmission schemes in different propagation environments. For the sake of illustration, let us consider QAM data transmission over Nakagami fading channel. By substituting (18) into (15), and then simplifying the integral $\int_0^\infty \exp(-\gamma_b s) p_{\gamma_b}(\gamma_b) \, d\gamma_b$ as $\varphi(s)$, we obtain [14],

$$P_s(\varepsilon) = \frac{4q\sqrt{p}}{\pi} \int_0^\infty \frac{\varphi(s^2 + p)}{s^2 + p} ds$$
$$- \frac{4q^2 p}{\pi^2} \int_0^\infty \frac{1}{(t^2 + p)} dt \int_0^\infty \frac{\varphi(s^2 + t^2 + 2p)}{(s^2 + p)} ds \qquad (26)$$

Equation (26) can be manipulated into a desired form (so that one can apply GCQ formulas directly) using variable transformations $s^2 + p = 2p/(x+1)$ and $t^2 + p = 2p/(y+1)$,

$$P_s(\varepsilon) = \frac{2q}{\pi} \int_{-1}^1 \frac{\varphi(2p/(1+x))}{\sqrt{1-x^2}} dx$$
$$- \frac{q^2}{\pi^2} \int_{-1}^1 \frac{1}{\sqrt{1-y^2}} dy \int_{-1}^1 \frac{\varphi(2p/(1+x) + 2p/(1+y))}{\sqrt{1-x^2}} dx \qquad (27)$$

Then using the GCQ approximation [10, (25.4.38)], leads (27) directly to a closed-form expression for the average SER of MQAM in a slow and flat Nakagami fading channel,

$$P_s(\varepsilon) = \frac{2q}{n} \sum_{j=1}^n \prod_{l=1}^L \left[1 + \frac{\sec^2(\theta_j)}{\lambda_l / p} \right]^{-m_l}$$
$$- \left[\frac{q}{n} \right]^2 \sum_{i=1}^n \sum_{j=1}^n \prod_{l=1}^L \left[1 + \frac{\sec^2(\theta_i) + \sec^2(\theta_j)}{\lambda_l / p} \right]^{-m_l} + R_\varepsilon \qquad (28)$$

The parameter n in (28) is a small positive integer, $\theta_j = (2j-1)\pi/4n$, $\theta_i = (2i-1)\pi/4n$ and R_ε denotes the remainder term (which becomes negligible as n increases). In particular, the remainder term can be bounded using the results of Appendix A in [15]. However, this is not necessary in practice, since one simply computes (28) for several increasing values of n, and stop when the result converges to a prescribed accuracy. Since (28) can approximate the true SER within any degree of accuracy, it can be viewed as an *exact* closed-form solution. Moreover, to the best of the authors knowledge, performance analysis of MQAM on Nakagami fading channels with or without diversity combining are not available in the literature. The generality and computational efficiency of our expression renders itself as a powerful tool for SER analysis under a myriad of fading scenarios.

Note the implications of (28): we are simply sampling the MGF at n^2 points. So as long as the MGF exists and computable, this method can work very effectively. In fact, its accuracy will be high if the high-order derivatives of the MGF vanishes rapidly. In [14], we present an alternative method for computing the second term in (26) involving erfc$^2(.)$ function. Instead of applying a two-dimension GCQ approximation, we resort to Gauss-Lobatto quadrature (GLQ) integration method which requires significantly fewer samples of MGF to evaluate the SER than the former. As well, the 2-dimension GCQ formula can be transformed into a one-dimension GCQ approximation.
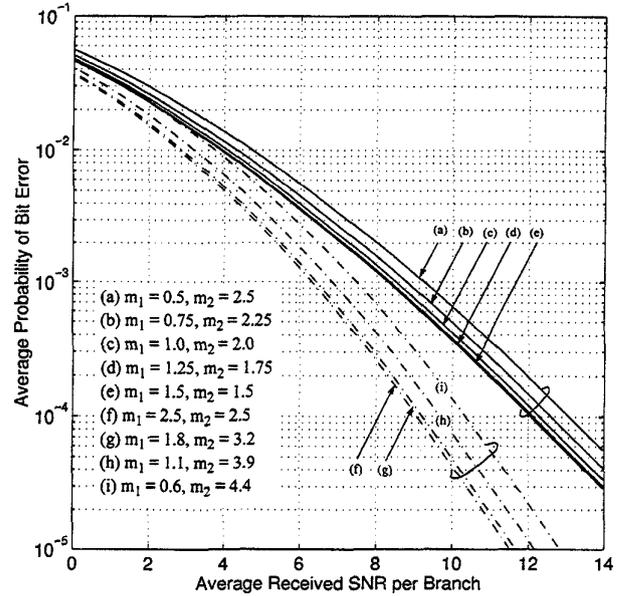

Figure 3. Effect of nonidentical fading parameters on the average BER performance of a dual-branch MRC diversity system.

Figure 3 illustrates the effect of noninteger and/or nonidentical fading parameters on the error performance of a dual diversity MRC system (for coherent binary PSK) with different combinations of fading figures. It is observed that the performance curves for these cases vary considerably. This suggests that the effect of noninteger and/or nonidentical fading parameters cannot be ignored in the analysis of a diversity receiver operating in Nakagami fading environments. As well, it is evident that the best error performance is attained when the statistically independent diversity branches undergo the same level of fading. Therefore, the assumption of identical fading figure tends to yield a rather optimistic result.

The growing demand for high-speed wireless data services despite the limited radio spectrum and the hostile nature of the wireless environment, warrants a consideration of spectrally efficient (i.e., modulation methods with a larger constellation sizes) and robust communication techniques for flat-fading channels. Multilevel modulation schemes may be employed to increase the bandwidth efficiency, while antenna diversity is usually used to ameliorate the effects of deep fades experienced on wireless links as well as to reduce the penalty in SNR due to co-channel interference. For instance, using 16-QAM modulation scheme coupled with pilot symbol assisted fading compensation technique and two antenna diversity reception, it is possible to facilitate 64 kbps transmission with almost same channel spacing as that of present analog systems [19]. These specifications are currently used in TDMA private mobile radio networks, such as the Japanese Multichannel Access system and the Extended Specialized Mobile Radio in the United States, which support integrated services of voice dispatch, cellular phone and data services with a bit rate upto 64 kbps. More recently, 64-QAM has been proposed as the modulation scheme for the digital terrestrial video broadcasting (DVTB) applica-

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tion. Multilevel modulation schemes are also often used in terrestrial microwave and satellite communications links. Hence, in the following we shall examine the error performance of different M-ary QAM over a frequency nonselective Nakagami fading channel. The analysis for MPSK and MDPSK in conjunction with MRC diversity in Nakagami fading channel with arbitrary parameters can be found in [20].
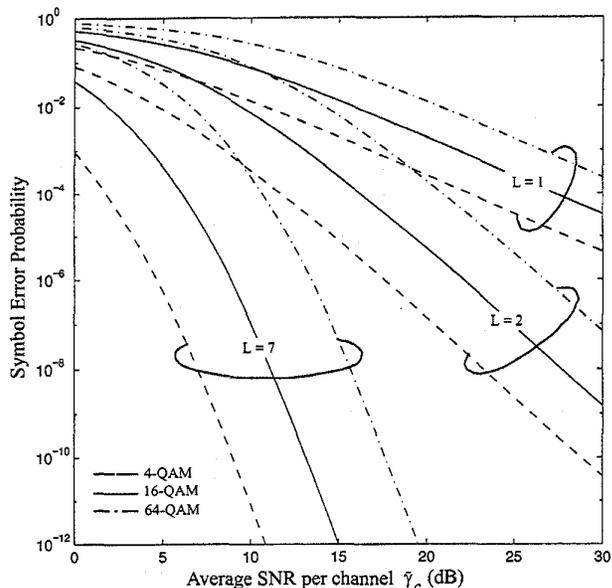

Figure 4. Symbol error probability for MQAM with MRC diversity reception in Nakagami fading with fading figure $m = 1.8$.

Figure 4 depicts the SER performance curves for 4-QAM, 16-QAM and 64-QAM with the assumption that all the maximal-ratio combining space diversity branches undergo identical Nakagami fading with $m = 1.8$. This fading figure corresponds to a Rician channel with Rice factor $K = 2$. From this figure, it is apparent that diversity reception is an effective technique for combatting the detrimental effects of deep fades experienced in wireless channels. It is also observed that the penalty in SNR to achieve a given SER of MQAM system with a larger signal constellation size declines more rapidly than that of a smaller signal set, as the diversity order increases. In other words, the diversity improvement is greater as the size M increases.

V. CONCLUSIONS

Simple yet very accurate SER analytical expressions have been derived for a wide class of coherent, differentially coherent and noncoherent communication systems with predetection MRC in a variety of propagation environments. These closed-form formulas (based on the GCQ approximation) can be easily programmed and evaluated efficiently. The results presented in this paper are sufficiently general to allow for arbitrary fading parameters as well as dissimilar mean signal strengths across the diversity branches. The generality and computational efficiency of the new results render themselves as powerful means for both theoretical analysis and practical applications.

ACKNOWLEDGMENT

The authors thank Mr. A. J. Mueller of 3Com/Primary Access, San Diego, for his contribution to this paper.

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