Multicarrier transmission peak-to-average power reduction using simple block code

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Introduction: Consider a BPSK modulated multicarrier transmission system. The complex envelope can be written as

\[ e(t) = \frac{1}{\sqrt{N}} \sum b_n e^{j2\pi f_n t} \]  

where \( b_n = \pm 1 \) corresponding to the information bits \( b_n = 0 \) or \( b_n = 1 \). \( f_n \) is the frequency of the \( n \)th carrier, \( f_0 = 1 \), and \( N \) is the number of the carriers. The peak-to-average power ratio (PAPR) of codeword \( \mathbf{b} = (b_1, ..., b_N) \) is defined as \( \xi(\mathbf{b}) = \max(i(t)) \). For all possible \( 2^N \) codewords, \( 1 < \xi(\mathbf{b}) \leq \eta \). The peak amplitudes of high PAPR codewords can cause severe distortion in the transmitter. One possible solution is therefore to eliminate codewords with high PAPRs [1]. It remains an open problem to find efficient coding schemes (i.e. with a high rate) to do this.

Table 1: PAPR values and PAPR reduction achieved by coding scheme [2]

<table>
<thead>
<tr>
<th>( N )</th>
<th>Uncoded PAPR</th>
<th>Coded [2] PAPR</th>
<th>Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6.99</td>
<td>4.26</td>
<td>2.73</td>
</tr>
<tr>
<td>15</td>
<td>11.76</td>
<td>10.88</td>
<td>0.88</td>
</tr>
<tr>
<td>25</td>
<td>15.98</td>
<td>13.45</td>
<td>0.52</td>
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<tr>
<td>35</td>
<td>15.44</td>
<td>15.08</td>
<td>0.37</td>
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<tr>
<td>50</td>
<td>16.99</td>
<td>16.73</td>
<td>0.26</td>
</tr>
<tr>
<td>100</td>
<td>20.00</td>
<td>19.87</td>
<td>0.13</td>
</tr>
<tr>
<td>200</td>
<td>23.01</td>
<td>22.95</td>
<td>0.06</td>
</tr>
</tbody>
</table>

All values in decibels

In [2], it is proposed that an extra carrier is added with \( b_{N+1} = b_N \). This would eliminate all-zero, all-one, 1010 \((N \leq 1)\) and \( 001 \) \((N \leq 1)\) codewords. While this scheme realises some reduction in the PAPR for small \( N \) (Table 1, first row), its use for large \( N \) only yields a negligible reduction in the PAPR. The reason is as follows.

For all possible \( 2^N \) codewords, \( \xi(\mathbf{b}) \) takes only \((say)\) \( M \) distinct values \((M < 2^N)\). Let \( N = \xi_1 > \xi_2 > ... > \xi_M \), be those. Thus, there are \( M \) equivalence classes; namely, for \( 1 \leq k \leq M \), \( E_k \) contains all \( n \) codewords with a PAPR of \( \xi_k \), and \( \Sigma_n n_k = 2^N \). For instance, \( E_1 \) contains all the codewords with the maximum PAPR \( N \). The coding scheme [2] eliminates codewords in \( E_n \) but not those in \( E_1 \). Hence, the reduction in the PAPR is from \( N \) to \( \xi_2 \). As \( N \) increases, the number of distinct PAPR values \((M)\) increases rapidly, and the difference between the highest and next highest PAPR values, \( \xi_2 \) and \( \xi_3 \), decreases rapidly, as does the reduction in the PAPR.

Table 1 shows this effect. The second column is \( 10 \log_{10} N \), the PAPR for the uncoded case. The third column is obtained by computing the PAPR of codeword 00...001 (length \( N + 1 \)), which is not eliminated by the above scheme and has one of the highest PAPR values. Note that it is therefore not necessary to search the whole code space to estimate the achievable PAPR reduction (column 4 in Table 1). The reduction in the PAPR is \( < 0.26 \) dB for \( N > 30 \). In fact, the complex envelope of this worst-case code is

\[ e(t) = \frac{1}{\sqrt{N+1}} \sum_{n=1}^{N+1} e^{j2\pi f_n t} \]  

where \( \xi(\mathbf{b}) \) occurs at \( t = 0 \), the PAPR is

\[ \zeta(00...01) = \left(\frac{N+1}{N+1}\right)^2 \]  

which asymptotes to \( N \), which is the same as the maximum PAPR for the uncoded system. We therefore conclude that the coding scheme [2] is ineffective for multicarrier transmission systems with a large number of carriers.

References


Performance bounds of continuous and blockwise decoded turbo codes in Rician fading channel

F. Babich, G. Montorsi and F. Vatta

As a powerful coding technique, turbo codes offer great promise for improving the reliability of communication over wireless channels where fading is the main impairment. The performance of turbo codes on the Rice fading channel is explored, considering both the fully-interleaved channels and correlated Rice slow-fading channels.

Introduction: In [1], the performance of continuous and blockwise decoded turbo codes has been evaluated in terms of an average upper bound to the bit error probability, in conjunction with coherent binary phase shift keyed (BPSK) modulation, transmitted over the additive white Gaussian noise (AWGN) channel, and maximum likelihood (ML) decoding. In a number of important applications, however, the AWGN channel is not the appropriate model for the propagation environment; this is the case in the mobile radio environment, in which the received signal component is subject to fading. It is of interest, therefore, to provide a characterisation of turbo code performance on Rician fading channels.

Fading channel characterisation: A waveform \( s(t) \) is transmitted over a channel characterised by Rician fading, in the hypothesis of multiplicative fading. This fading imposes random amplitude \( a(t) \) and phase \( \Phi(t) \) onto \( s(t) \). The received signal \( r(t) \) contains a stable specular (direct) component and a random diffuse (multipath) component [2]. The channel parameter defined by \( K = a^2 e^{\sigma^2} \), which represents the ratio of specular to diffuse energy (Rician factor). For convenience, we impose the following normalisation: \( \gamma = 2^{\sigma^2} \). In terms of \( \gamma \) and \( \sigma^2 \), the distribution of the amplitude process \( a(t) \) can be expressed as

\[ f(a) = \frac{a}{\sigma^2} \exp \left( -\frac{a^2 + \gamma^2}{2\sigma^2} \right) \left( I_0 \left( \frac{\sqrt{a^2 \gamma^2}}{\sigma^2} \right) \right) \]  

where \( I_0(\cdot) \) is the modified Bessel function of the first kind and zero order. The distribution \( f(a) \) is general, because for Rayleigh