94.0% under the same conditions. We believe that this enhancment is due to the global optimisation function of the GA.

Conclusion: A method for using genetic algorithms to train HMMs has been successfully developed. The main contribution of this study is that it presents the idea of searching for the most optimal HMM. The experiments also show that the approach is superior to the classical method.

© IEE 1998 25 May 1998
Electronics Letters Online No: 19980096

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References

1 BAHL, L.R., JELINEK, F., and MERCER, R.L.: 'A maximum likelihood approach to continuous speech recognition', *IEEE Trans. Pattern Anal. Mach. Intell.*, 1983, **PAMI-5**, (2), pp. 179–190
2 KAIFU LEE, and HSIAO-WUEN HON: 'Speaker-independent phone recognition using hidden Markov models', *IEEE Trans. Acoust. Speech Signal Process.*, 1989, **ASSP-37**, (11), pp. 1641–1648
3 BOURLARD, H., and WELLEKENS, C.J.: 'Links between Markov models and multilayer perceptrons', *IEEE Trans. Pattern Anal.*, 1990, **PAMI-12**, (12), pp. 1167–1178
4 HUNG, S.L., and ADELI, H.: 'A parallel genetic/neural Netw., learning algorithm for MIMD shared memory machines', *IEEE Trans. Neural Netw.*, 1994, **5**, (6), pp. 900–909
5 MANIEZZO, V.: 'Genetic evolution of the topology and weight distribution of neural networks', *IEEE Trans. Neural Netw.*, 1994, **5**, (1), pp. 39–53

Unified analysis of MPSK and MDPSK with diversity reception in different fading environments

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Computationally efficient expressions are presented for evaluating the symbol-error rates of coherent and differentially coherent *M*-ary phase-shift-keying signalling in conjunction with maximal-ratio combining diversity receivers in fading scenarios.

Introduction: The evaluation of symbol error rates (SERs) for *M*-phase signals with diversity combining and fading has been studied extensively (see [1] and its references). Most authors place the emphasis on deriving explicit closed-form solutions for the SER and bit error rate (BER). When it is difficult to obtain exact expressions, bounds are derived instead. Proakis [2] provides a comprehensive treatment of the problem for maximal-ratio combining (MRC) under Rayleigh and Ricean fading, and presents a general closed form expression for all signalling with alphabet size M and order of diversity L. However, the evaluation of the general expression requires (L–1)th order derivatives for L channel reception. Alternative expressions have been derived in [1] for the Rayleigh fading channel. The derivation of these closed-form formulas tends to be rather ad hoc, requiring some ingenuity in evaluating the necessary integrals. With the proliferation of standard mathematical software such as Maple and Matlab, it is extremely simple to evaluate (numerically) these integrals with high accuracy, whereas explicit closed-form solutions tend to require much more programming effort. Our approach here is motivated by this consideration. Moreover, the integrals can be approximated by extremely accurate sums requiring knowledge of the moment generating function (MGF) at only a small number of points.

In this Letter we unify and add to the previous results by providing a unified and convenient method to compute the exact SER for *M*-ary phase-shift keying (MPSK) and *M*-ary diversity PSK (MDPSK) signals in conjunction with MRC, and presenting new results for SER in Nakagami and lognormal-Rice fading channels. In contrast to [1, 2], our approach relies upon knowledge of the MGF of the received signal-to-noise ratio (SNR). The resulting expression is an integral over a finite range of integrands containing only elementary functions. This new expression can be further approximated using Gauss-Chebychev quadrature (GCQ) formulas, which lends itself to an efficient method for evaluating the SER via a rapidly converging series.

Statistical representation of fading channel: Given a random variable X, the probability density function (PDF) $p(x)$ indicates the relative frequency of occurrence of any fixed value of x. The MGF can then be defined as

$$\psi(z) = E[e^{-zx}] = \int_{-\infty}^{\infty} e^{-zx} p(x) dx \qquad (1)$$

which is simply the Laplace transform of the PDF. $E[x]$ in eqn. 1 denotes the expected value (mean or average value) of X. Note that the MGF is closely related to another statistical function, the characteristic function, and is easily translated by the variable substitution $z = -jv$.

Since our new approach for computing the error performance only requires knowledge of the MGF of the SNR, in the following we will summarise the MGFs for several commonly used fading channel models.

Ricean and Rayleigh fading: The MGF for the non-centralised chi-squared distribution is given by [3]

$$\psi_l(z) = \frac{1+K_l}{1+K_l+z\Omega_l} \exp\left(\frac{-z\Omega_l K_l}{1+K_l+z\Omega_l}\right) \equiv \phi(z\Omega_l, K_l) \qquad (2)$$

where $\Omega_l = E[\gamma_l]$ and K_l denote the average received SNR and the Rice factor of the lth diversity branch, respectively. In a limiting case when the power in the line-of-sight path approaches zero, $K \to 0$ and the channel reverts to the Rayleigh fading channel. Then the corresponding MGF of the centralised chi-squared distribution is

$$\psi_l(z) = \frac{1}{1+z\Omega_l} \qquad (3)$$

Nakagami-m and Nakagami-q (Hoyt) fading: The MGF of the SNR for the *Nakagami-m* fading channel is given by

$$\psi_l(z) = \left(\frac{m_l}{m_l+z\Omega_l}\right)^{m_l} \qquad m_l \geq 0.5 \qquad (4)$$

where m_l denotes the fading figure. It is evident that eqn. 4 reduces to eqn. 3 when $m = 1$ (i.e. Rayleigh fading).

It can be easily shown that the MGF of the SNR for *Nakagami-q* [4] fading is

$$\psi_l(z) = \frac{1}{\sqrt{[z(1+b_l)\Omega_l+1][z(1-b_l)\Omega_l+1]}} \qquad -1 \leq b_l \leq 1 \qquad (5)$$

where $b_l = [1-q_l^2]/[1+q_l^2]$ and q_l ($0 \leq q_l \leq \infty$) is the fading parameter. The Nakagami-q distribution reverts to the Rayleigh and the normal distribution when $b_l = 0$ and $b_l = 1$, respectively.

Lognormal-Rice and Suzuki fading: Expressing the received fading envelope as the product of independent Rice and lognormal distributions, the MGF can be shown to be

$$\psi_l(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \phi(z\mu_l \exp(\sqrt{2}\sigma_l x), K_l) \exp(-x^2) dx \qquad (6)$$

where $x = \ln(\Omega/\mu)/(\sqrt{2}\sigma)$, σ is the logarithmic standard deviation of shadowing, and μ is the local mean power. By applying Hermitian integration, a closed-form expression for eqn. 6 is obtained:

$$\psi_l(z) = \frac{1}{\sqrt{\pi}} \sum_{i=1}^{n} w_i \phi(z\mu_l \exp(\sqrt{2}\sigma_l x_i), K_l) + R_H \qquad (7)$$

where abscissas x_i (ith root of an nth order Hermite polynomial) and weights w_i are tabulated in [5] for $n \leq 20$ and R_H is a remainder term.

Since the Suzuki distribution is a special case of the lognormal Rician distribution, its MGF is readily obtained by setting $K = 0$ in eqn. 7 [3]:

$$\psi_l(z) = \frac{1}{\sqrt{\pi}} \sum_{i=1}^{n} \frac{w_i}{1+za_i} + R_H \qquad (8)$$

where a_i is equal to $\mu\exp[\sqrt{2}\sigma x_i]$.

Lognormal-Nakagami-m fading: As in our derivation of eqn. 7, the MGF of the SNR in a *Nakagami-m* fading channel with lognormal shadowing can be expressed as

$$\psi_l(z) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \left[1 + z\exp(\sqrt{2}x\sigma)\right]^{-m_l} \exp(-x^2)dx$$
$$= \frac{1}{\sqrt{\pi}} \sum_{i=1}^{n} w_i \left[1 + z\exp(\sqrt{2}x_i\sigma)\right]^{-m_l} + R_H \qquad (9)$$

where x_i and w_i correspond to the abscissas and the weights of the Gauss-Hermite integration formula, respectively.

Assuming independent fading across the diversity branches, the MGF of the combined received signal power at the output of the maximal-ratio combiner can be readily shown to be

$$\varphi(s) = E[\exp[-s(\gamma_1 + \gamma_2 + ... + \gamma_L)]] = \prod_{l=1}^{L} \psi_l(s) \qquad (10)$$

SER analysis in fading environments: MPSK: Our derivation begins from the exact SER expression for MPSK in an AWGN channel provided in [6]:

$$P_S(\varepsilon|\gamma) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left(\frac{-b\gamma}{\sin^2\theta}\right) d\theta \qquad (11)$$

where $b = (\log_2 M)\sin^2(\pi/M)$. The average symbol error probability in a slow and flat fading channel may be derived by averaging the error rate for the AWGN channel over the PDF of the SNR in a given fading channel, i.e.

$$P_S(\varepsilon) = \int_0^{\infty} P_S(\varepsilon|\gamma) p_\gamma(\gamma) d\gamma \qquad (12)$$

Substituting eqn. 11 into eqn. 12, and then interchanging the order of integration and recognising that the integral with respect to γ is equal to the MGF of the SNR evaluated at $b\csc^2\theta$, eqn. 12 reduces to

$$P_S^{(MRC)}(\varepsilon) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} d\theta \int_0^{\infty} \exp(-\gamma b \csc^2\theta) p_\gamma(\gamma) d\gamma$$
$$= \frac{1}{\pi} \int_0^{(M-1)\pi/M} \varphi(b\csc^2\theta) d\theta \qquad (13)$$

This form is both easily evaluated and well suited to numerical integration, since it only involves finite integration limits. The simplicity of this result should be compared with the more complex forms given by Proakis [2] and other references in the literature.

Using variable substitution $t = \cos(M\theta/[M-1])$ in eqn. 13 and applying GCQ approximation [5], we obtain a closed-form expression for the SER of MPSK in the form of a truncated series:

$$P_S^{(MRC)}(\varepsilon)$$
$$= \frac{M-1}{M\pi} \int_{-1}^{1} \varphi\left[b\csc^2\left[\frac{(M-1)}{M}\cos^{-1} t\right]\right] \frac{dt}{\sqrt{1-t^2}}$$
$$= \frac{M-1}{nM} \sum_{i=1}^{n} \varphi\left[b\csc^2\left[\frac{(M-1)(2i-1)\pi}{2nM}\right]\right] + R_n \qquad (14)$$

where n is a small positive integer and the remainder term is

$$R_n = \frac{(M-1)^3\pi^2}{3M^3n^2}\left[\left(\frac{2b\cos\zeta}{\sin^3\zeta}\right)^2 \varphi''\left(\frac{b}{\sin^2\zeta}\right) + \frac{2b(1+2\cos^2\zeta)}{\sin^4\zeta}\varphi'\left(\frac{b}{\sin^2\zeta}\right)\right] \qquad (15)$$

for some $0 < \zeta < (M-1)\pi/M$. Notations $\varphi'(s)$ and $\varphi''(s)$ in eqn. 15 are used to denote the first and second order derivatives of the MGF. When the derivatives are available, bounding on R_n is possible. The derivation of eqn. 15, details of which are omitted for brevity, relies on the fact that the GCQ rule collapses to the mid-point trapezoidal rule for the class of integral considered here.

MDPSK: The conditional SER of MDPSK is given by [6]

$$P_S(\varepsilon|\gamma) = \frac{\sin(\pi/M)}{\pi} \int_0^{\pi/2} \frac{\exp(-\gamma\log_2 M[1-\cos(\pi/M)\cos\theta])}{1-\cos(\pi/M)\cos\theta} d\theta \qquad (16)$$

Following our analysis for MPSK, the unconditional exact SER for MDPSK can be written in the form of a single integral with finite integration limits:

$$P_S^{(MRC)}(\varepsilon) = \frac{\sin(\pi/M)}{\pi} \int_0^{\pi/2} \frac{\varphi(\log_2 M[1-\cos(\pi/M)\cos\theta])}{1-\cos(\pi/M)\cos\theta} d\theta \qquad (17)$$

Applying the GCQ formulas on eqn. 17, we obtain a closed-form expression for the SER of MDPSK in different fading environments:

$$P_S^{(MRC)}(\varepsilon) = \frac{\sin(\pi/M)}{2\pi n} \sum_{i=1}^{n} \frac{\varphi(\log_2 M[1-\cos(\pi/M)\cos\alpha])}{1-\cos(\pi/M)\cos\alpha} + R_n \qquad (18)$$

where $\alpha = (2i-1)\pi/4/n$ and the remainder term is given by

$$R_n = \frac{\pi^2\sin(\pi/M)}{24n^2}\left[\varphi(p\delta)\left(\frac{2z^2\sin^2\zeta - \delta z\cos\zeta}{\delta^3}\right) + \varphi''(p\delta)\frac{(pz\sin\zeta)^2}{\delta} + \varphi'(p\delta)\left(\frac{\delta pz\cos\zeta - 2pz^2\sin^2\zeta}{\delta^2}\right)\right]$$
$$\text{for } 0 < \zeta < \pi/2 \qquad (19)$$

where $p = \log_2 M$, $z = \cos(\pi/M)$ and $\delta = 1 - z\cos\zeta$.

Conclusions: Previously, much effort has been expended to find closed-form SER expressions for different fading models. This ad hoc development has led to various formulas and error bounds. In contrast, we have developed unified SER expressions (in the form of a single finite-range integral) for coherent and differentially coherent MPSK signalling in conjunction with MRC diversity reception under many fading scenarios. These simple yet exact SER expressions require knowledge of the MGF. We also provide new closed-form expressions (based on the GCQ approximation) that offer a convenient method to evaluate the SER for higher orders of diversity and larger alphabet sizes. The results presented are sufficiently general to allow for arbitrary fading parameters as well as dissimilar signal strengths across the diversity branches.

23 June 1998
Electronics Letters Online No: 19981105

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References

1 CHENNAKESHU, S., and ANDERSON, J.B.: 'Error rates for Rayleigh fading multichannel reception of MPSK signals', *IEEE Trans. Commun.*, 1995, **43**, pp. 338–346

2 PROAKIS, J.G.: 'Digital communications' (McGraw-Hill, New York, 1995), 3rd edn.

3 TELLAMBURA, C., *et al.*: 'Analysis of M-ary phase-shift-keying with diversity reception for land-mobile satellite channels', *IEEE Trans. Veh. Technol.*, 1997, **46**, pp. 910–922

4 HOYT, R.S.: 'Probability functions for the modulus and angle of the normal complex variate', *Bell Syst. Tech. J.*, 1947, **26**, pp. 318–359

5 ABRAMOWITZ, M., and STEGUN, I.A.: 'Handbook of mathematical functions'. National Bureau of Standards Applied Mathematics Series 55, 1964

6 PAWULA, R.F., RICE, S.O., and ROBERTS, J.H.: 'Distribution of the phase angle between two vectors perturbed by Gaussian noise', *IEEE Trans. Commun.*, 1982, **30**, pp. 1828–1841