where $\times$ and $*$ denote the correlation and convolution operators, respectively.

To achieve multipath diversity combining, a multipath diversity combiner with coefficient vector $\mathbf{W}_k = [w_k^0, w_k^1, ..., w_k^{L-1}]^T$ is employed. The multipath diversity combiner output is

$$y_k = \mathbf{W}_k^T \mathbf{X}_k \tag{4}$$

where $\mathbf{X}_k = [x_k^0, x_k^1, ..., x_k^{L-1}]^T$ is the input vector of the improved KMA algorithm and x_k^m, $m = 0, 1, ..., L-1$ are taken from the outputs of the arm filters as shown in eqn. 3. Based on the derivation and discussion presented previously [3], the update procedure can be rewritten as

$$\mathbf{W}_{k+1} - \mathbf{W}_k = -\frac{\mu}{\|\mathbf{X}_k\|^2}\mathbf{X}_k^*[\hat{e}_k^R + j\hat{e}_k^I] \tag{5}$$

and the error signal $[\hat{e}_k^R + j\hat{e}_k^I]$, can thus be approximated as

$$\hat{e}_k^R = \begin{cases} \frac{\tilde{e}_k^R}{(y_k^R)^2} & y_k^R \notin Z_k^R \\ y_k^R - \hat{y}_k^R & y_k^R \in Z_k^R \end{cases}$$
$$\hat{e}_k^I = \begin{cases} \frac{\tilde{e}_k^I}{(y_k^I)^2} & y_k^I \notin Z_k^I \\ y_k^I - \hat{y}_k^I & y_k^I \in Z_k^I \end{cases} \tag{6}$$

where

$$\begin{cases} \tilde{e}_k^R = y_k^R((y_k^R)^2 - R_k^R) & R_k^R = \frac{E\{[d_k^R * h_k]^4\}}{E\{[d_k^R * h_k]^2\}} \\ \tilde{e}_k^I = y_k^I((y_k^I)^2 - R_k^I) & R_k^I = \frac{E\{[d_k^I * h_k]^4\}}{E\{[d_k^I * h_k]^2\}} \end{cases} \tag{7}$$

Z_k^R and Z_k^I are the variable confidence zones of the real and imaginary parts, respectively, and $\hat{y}_k^r$ and $\hat{y}_k^I$ are the real and imaginary parts, respectively, of the decision result of the multipath diversity combiner output.

PN code tracking loop: Here, we describe the operations in the modified PN code tracking loop. The early-late structure is replaced by the very simple digital correlator with the half-integer-instant stream and the code difference stream as its inputs. The incommg delayed (one symbol interval, $T_b = MT_c$, delayed) half-integer-instant samples $\tilde{r}_{k-\frac{1}{2}-M-m}$, $m = 0, 1, ..., L-1$, are cross-correlated with the code difference stream, $c_{k-M}^{\Delta} = c_{k-(L-1)-M} - c_{k-L-M}$. After passing through the arm filter, h_k, the arm error signal z_k^m on the mth arm is generated in the following form: $z_k^m = \tilde{r}_{k-\frac{1}{2}-M-m} \times c_{k-M}^{\Delta} * h_k$. The arm error signal z_k^m on the mth arm has the same error characteristics as those of the conventional delay locked loop (DLL) operated on a single-path channel with AWGN, but here it is corrupted by the corresponding channel tap weight and the information-bearing symbol. The compound error signal, e_k, can be obtained by combining the arm error signals, z_k^m $\forall m$, with the tap weights, w_k^m $\forall m$, which are the same as those exploited in the multipath diversity combiner; then, the data modulation effect on z_k^m compensated for by the decision-directed method:

$$e_k = \mathrm{Re}\left\{\hat{y}_{[k]_M}^* \sum_{m=0}^{L-1} w_k^m z_k^m\right\} \tag{8}$$

The effects of channel multipath fading, the carrier phase error and data modulation on the arm timing error signal, z_k^m, can be simultaneously overcome by exploiting multipath diversity combining.

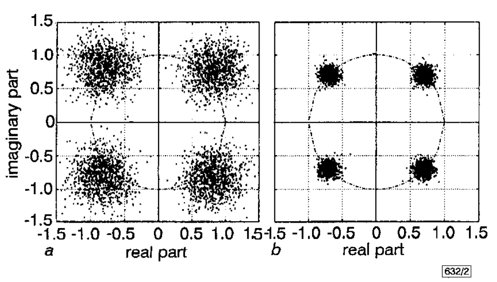

Fig. 2 *Signal constellations after multipath diversity combiner with EKF-based estimator* [2] *and proposed technique*

a EKF when SNR = –5dB
b MKMA when SNR = –5dB

Simulation results: Computer simulation results illustrating the performance of the proposed technique are presented in this Section. Fig. 2 shows the signal constellations obtained by a multipath diversity combiner with the EKF-based estimator [2] and the proposed technique, in the case of a frequency offset $\Delta f/(1/T_c) = 10^{-4}$ and SNR = –5dB. The multipath diversity combiner, aided by the EKF-based estimator [2], and with the proposed technique, can track the carrier frequency offset and cluster the output signal constellation at the right position. It is also obvious that the multipath diversity combiner with the proposed technique can cluster the output signals at the right position much better than can one aided by the EKF-based estimator [2].

Conclusion: A technique of joint blind multipath diversity combining and code tracking is proposed in this Letter. It has been shown that this technique can accomplish multipath diversity combining in the blind mode. The simulation results show that such a technique can certainly achieve simultaneous improvement of multipath diversity combining and code tracking on frequency-selective fading channels.

26 May 1998
Electronics Letters Online No: 19981040

Jia-Chin Lin (*Department of Electrical Engineering, Room 531, National Taiwan University, 106-17 Taipei, Taiwan, Republic of China*)

E-mail: jiachin@speech.ee.ntu.edu.tw

References

1 SHEEN, W.-H., and STUBER, G.L.: 'Effects of multipath fading on delay-locked loops for spread spectrum systems', *IEEE Trans. Commun.*, 1994, **42**, pp. 1947–1956
2 ILTIS, R.A.: 'An EKF-based joint estimator for interference multipath, and code delay in a DS spread-spectrum receiver', *IEEE Trans. Commun.*, 1994, **42**, pp. 1288–1299
3 LIN, J.-C., and LEE, L.-S.: 'A modified blind equalization technique based on a constant modulus algorithm'. ICC'98, 1998 (accepted)
4 OH, K.N., and CHIN, Y.O.: 'Modified constant modulus algorithm: Blind equalization and carrier phase recovery algorithm'. Proc. ICC'95, Seattle, WA, June 1995, pp. 498–502

Comment

Bit error rate performance of π/4 DQPSK for Nakagami-lognormal channels

C. Tellambura

A convergent infinite series has been presented for the bit error performance of π/4 DQPSK for Nakagami-lognormal channels if the fading figure m is an integer. The authors show that this infinite series can be replaced by a finite series. A convergent series for the BER is also derived when the fading figure, m, is real.

Introduction: In a recent Letter [1], Tjhung and Chai have derived an infinite series for the bit error rate (BER) of π/4 DQPSK for Nakagami-lognormal (NLN) channels provided the Nakagami fading figure is constrained to be an integer. This modulation scheme, differential quadrature phase shift keying, has been adopted for several practical mobile systems. Therefore, evaluation of its performance under general fading and shadowing conditions is useful. The purpose of this Letter is to suggest some improvements to the infinite-series solution.

Theory: The conditional BER, $P_b(e|s)$, derived in [1] utilises a BER expression from [2] which involves the generalised Marcum's Q function and an infinite series of modified Bessel functions. Consequently, eqn. 8 in [1] is an infinite series. However, it possible to replace this by a finite series. From [3], the conditional BER can be written as

$$P_b(e|s) = \frac{1}{2\pi}\int_0^{\pi} \frac{1}{(\sqrt{2}-\cos\theta)(\alpha-\beta\cos\theta)^m} d\theta \qquad (1)$$

where

$$\alpha = 1 + \frac{2s^2\gamma_b}{m}$$

$$\beta = \frac{\sqrt{2}s^2\gamma_b}{m}$$

Here, s, γ_b and m are as defined in [1]. Eqn. 1 holds for arbitrary $m \geq 0.5$ and is equivalent to eqn. 8 in [1].

Integral m: The integrand in eqn. 1 can be expanded as

$$\frac{1}{(\sqrt{2}-\cos\theta)(\alpha-\beta\cos\theta)^m} = \frac{1}{(\sqrt{2}-\cos\theta)} - \beta\sum_{k=1}^{m}\frac{1}{(\alpha-\beta\cos\theta)^k} \qquad (2)$$

Substituting this into eqn. 1 and using a tabulated integral formula [4], we obtain

$$P_b(e|s) = \frac{1}{2}\left[1 - \sum_{k=1}^{m}\frac{\beta}{(\alpha^2-\beta^2)^{k/2}}P_{k-1}\left(\frac{\alpha}{\sqrt{\alpha^2-\beta^2}}\right)\right] \qquad (3)$$

where $P_k(x)$ is the kth order Legendre polynomial. This is a finite-sum replacement for eqn. 13 of [1]. Moreover, Legendre polynomials with consecutive indices satisfy the recurrence relation [4]:

$$(n+1)P_{n+1}(x) - (2n+1)xP_n(x) + nP_{n-1}(x) = 0 \qquad n = 1, 2, \ldots \qquad (4)$$

with $P_0(x) = 1$ and $P_1(x) = x$. Therefore, combining two equations above obviates the need for computing the Gauss hypergeometric function (cf. [1]). So, eqn. 3 is readily programmable using common numerical computing environments such as MATLAB.

Arbitrary m: We define:

$$I_n(\alpha) = \frac{1}{\pi}\int_0^{\pi}\frac{\cos^n\theta}{(\alpha-\cos\theta)}d\theta \qquad (5)$$

where $|\alpha| > 1$ and $n \geq 0$ is an integer. By converting this integral into a contour integral defined on the unit circle, we can readily prove that

$$I_n(\alpha) = \frac{\alpha^n}{\sqrt{\alpha^2-1}} + \frac{1}{2^n}\sum_{r_0, r_0+2, \ldots, n}\binom{n}{\frac{n+r}{2}}\frac{(\alpha-\sqrt{\alpha^2-1})^r - (\alpha+\sqrt{\alpha^2-1})^r}{\sqrt{\alpha^2-1}} \qquad (6)$$

where $r_0 = 1(2)$ if n is odd(even). For any $|z| < 1$, the following holds:

$$\frac{1}{(1-z)^m} = \sum_{n=0}^{\infty}\frac{(m)_n}{n!}z^n \qquad (7)$$

where $(m)_n = m(m+1) \ldots (m+n-1)$. We use this to expand $(\alpha - \beta\cos\theta)^{-m}$ in eqn. 1 and obtain:

$$P_b(e|s) = \frac{1}{2}\sum_{n=0}^{\infty}\frac{\beta^n(m)_n}{\alpha^{n+m}n!}I_n(\sqrt{2}) \qquad (8)$$

This series is rapidly convergent.

Non-shadowing case: By setting $s = 1$ in eqn. 1 or eqn. 8, the BER for Nakagami fading can be obtained. Also, note that as $\gamma_b \to \infty$, $\alpha/\beta \to \sqrt{2}$. This means:

$$P_b(e) \simeq \frac{P_m(\sqrt{2})}{2}\left(\frac{m}{\sqrt{2}\gamma_b}\right)^m \qquad (9)$$

exhibiting an mth order diversity-like effect.

Results and conclusion: Using the substitution:

$$\frac{\ln s - h\mu_S}{\sqrt{2}h\sigma_S} = x$$

in eqn. 1 of [1], we obtain:

$$P_b(e) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}P_b[e|\exp(\sqrt{2}h\sigma_S x + h\mu_S)]\exp(-x^2)\,dx \qquad (10)$$

This can be evaluated using a Gauss-Hermite quadrature formula. The BER of $\pi/4$-DQPSK in NLN fading is shown in Fig. 1. The fading figure varies from 0.5 to 6.5.

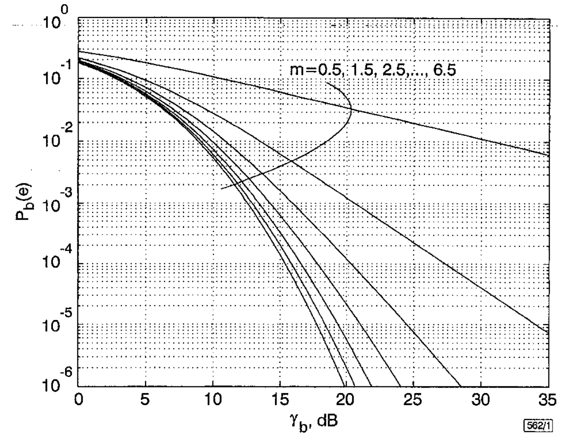

Fig. 1 *BER of DQPSK in NLN fading for $\mu_s = 0$ and $\sigma_s = 2dB$*

A finite series has been derived for the bit error performance of $\pi/4$ DQPSK over Nakagami-lognormal channels if the fading figure m is an integer. A convergent series for the BER has also been derived for real values of the fading figure.

19 May 1998
Electronics Letters Online No: 19981125

C. Tellambura (*Communications and Networking Research Group, School of Computer Science and Software Engineering, Monash University, Wellington Road, Clayton, Victoria 3168, Australia*)

E-mail: chintha@dgs.monash.edu.au

References

1 TJHUNG, T.T., and CHAI, C.C.: 'Bit error rate performance of $\pi/4$ DQPSK for Nakagami-lognormal channels', *Electron. Lett.*, 1998, **34**, pp. 625–626
2 PROAKIS, J.G.: 'Digital communications' (McGraw Hill, New York, 1995), 3rd edn.
3 TELLAMBURA, C., and BHARGAVA, V.K.: 'Unified error analysis of DQPSK in fading channels', *Electron. Lett.*, 1994, **30**, pp. 2110–2111
4 RYZHIK, I.M., and GRADSHTEYN, I.S.: 'Table of integrals, series, and products' (Academic Press, Inc., 1994), 5th edn.

Efficient unfolding procedure for DSP applications

J. Kim

An FSFG can be used to obtain a rate-optimal schedule. There are some criteria to measure the optimality. Normally, an iteration period bound (IPB) is used for the optimal implementation. If an iteration period (IP) is the same as the IPB, the schedule is called rate-optimal. Unfolding can reduce the IP and guarantee rate-optimal schedules with an optimum unfolding factor. A new unfolding procedure, called JK's unfolding, is introduced which has low complexity and can be described using graphical methods.

Introduction: Algorithms for DSP applications can be described as a fully specified flow graph (FSFG). From the FSFG, we obtain the optimality criterion. An iteration period (IP) and an iteration period bound (IPB) are normally used to measure the optimality [3, 4]. The IP and the IPB are described as follows:

$$IP = \frac{throughput\ delay\ in\ loop\ l}{number\ of\ delays\ in\ loop\ l}$$

$$IPB = \max\frac{throughput\ delay\ in\ loop\ l}{number\ of\ delays\ in\ loop\ l}$$