Efficient computation of MRC diversity performance in Nakagami fading channel with arbitrary parameters

A. Annamalai, C. Tellambura and V.K. Bhargava

A very efficient numerical expression is described for MRC diversity performance in a Nakagami fading channel with arbitrary parameters, including all possible conditions of received signal strength and arbitrary fading figures.

Introduction: The Nakagami distribution (m-distribution) is a very flexible statistical distribution which can accurately model a variety of fading environments. It has greater flexibility in matching some empirical data than the Rayleigh, lognormal or Rice distributions, and the Nakagami distribution is a versatile statistical distribution which can accurately model the Nakagami ratio variable. The Nakagami distribution is a Nakagami random variable and denotes the order of diversity. We also define \( \Omega = \frac{\lambda}{\sigma^2} \) as the Nakagami fading parameter for arbitrary fading parameters as well as different mean signal strengths across the diversity branches.

Theory: The SNR at the output of the maximal-ratio combiner is

\[
\gamma = \sum_{l=1}^{L} \alpha_l^2
\]

where \( \alpha_l \) is a Nakagami random variable and \( L \) denotes the order of diversity. The Nakagami fading parameter is defined as, \( \Omega = \frac{\lambda}{\sigma^2} \) as the Nakagami fading parameter for arbitrary fading parameters as well as different mean signal strengths across the diversity branches.

\[
\varphi_{\gamma}(s) = E[\exp(-s\gamma)] = \frac{1}{\Omega} \int_{0}^{\infty} \frac{\exp(-x)}{x^{\frac{m}{2}+1}} dx
\]

where \( \varphi_{\gamma}(s) \) is the integrand expression in [5], eqn. 7.1, and the coefficient \( s \) indicates the type of modulation:

\[
a = \begin{cases} 
1 & \text{antipodal signalling (PSK)} \\
\frac{1}{\sqrt{2}} & \text{orthogonal signalling (FSDK)}
\end{cases}
\]

Therefore, the average BER is given by

\[
P_b = \frac{1}{2\pi} \int_{0}^{\infty} \varphi_{\gamma}(s^2 + \sigma^2) ds = 2 \pi \left( \frac{1}{2\pi} \right) \int_{0}^{\infty} \varphi_{\gamma}(s^2 + \sigma^2) ds
\]

where the second integral is obtained via variable transformation \( s^2 + \sigma^2 = 2\pi^2 x - 1 \). Now, using the Gauss-Chebyshev approximation [5], eqn. 7.4.38, [6, 7] and [7], leads eqn. 5 directly to a closed-form expression in a very compact form:

\[
P_b = \frac{2\pi}{2\pi} \sum_{j=1}^{L} \left[ 1 + a^2 \sec^2(\theta_j) \right]^{-\frac{m}{2}} + R_n
\]

where \( n \) is a small positive integer, \( \theta_j = (2j - 1)\pi/(2\pi) \) and \( R_n \) denotes the remainder term.
Performance of dual-branch MRC diversity systems with nonidentical fading parameters cannot be ignored in the analysis of a diversity receiver operating in Nakagami fading environments. Also, it is apparent that the best error performance is attained when the statistically independent diversity branches undergo the same level of fading. Therefore, the assumption of identical fading tends to yield a rather optimistic result.

Conclusions: A simple yet extremely accurate BER expression for MRC diversity systems in Nakagami fading environments with arbitrary fading parameters has been derived. The closed-form BER formula (which is applicable for coherent MPSK and BFSK modulation schemes) can be easily programmed and evaluated efficiently (for instance, there is no need to work around Gaussian quadrature tables). Hence the generality (i.e. arbitrary signal strength and fading figure) and the computational efficiency of the new results, rendering them powerful means for both theoretical analysis and practical applications.

References


Frequency-domain forward/backward LMS adaptive predictors with reduced complexity

Jae-Chon Lee

The implementation of forward/backward least mean square linear predictors in the frequency-domain is known to require two DFT/FFT operations. Here, computationally efficient structures are derived using proper data augmentation methods. As a result, the following are achieved: (i) reduction of a DFT/FFT operation; and (ii) further reduction in complexity when the input signal is real-valued.

Introduction: The simultaneous use of forward and backward prediction errors in deriving the least mean square (LMS) adaptive line enhancer has been reported in [1]. The results indicated that the forward/backward algorithm converges to the optimal Wiener