where $\epsilon_S^2(\theta, \omega_m)$ is given by eqn. 6. Note that $\overline{E_n^2(\theta,\Omega)}$ will have local minima at $\theta = \omega_1, ..., \omega_M$.

Based on the assumptions of zero-mean white noise and independent sources, we can find the estimate $\hat{\Omega}$ of Ω from the minima of $(1/N)\Sigma_{n=1}^{N} E_n^2(\theta, \omega)$ (with respect to θ) which lie below a chosen threshold. Sources whose powers cause the corresponding minima to lie above the threshold (i.e. weak sources) will not be detected.

Simulation results: Simulations have been run on the computer for $L = 24$, $N = 100$ (i.e. a ULA of 24 sensors using 100 snapshots) for both a single source and two independent equipower sources, corrupted by additive white noise of mean zero when $d/\lambda = 0.5$. The results are shown in Tables 1 and 2.

The accuracy of the estimates and the resolution is not as high as those of MUSIC and associated subspace methods [1, 2]. However, the method has two advantages over subspace methods: (i) the processing is simpler because on-line eigendecomposition of the data correlation matrix is not needed, and (ii) the number of sources present need not be known *a priori*.

21 April 1998
Electronics Letters Online No: 19980894

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Efficient computafion of MRC diversity performance in Nakagami fading channel with arbitrary parameters

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A very efficient numerical expression is described for MRC diversity performance in a Nakagami fading channel with arbitrary parameters, including all possible conditions of received signal strength and arbitrary fading figures.

Introduction: The Nakagami distribution (*m*-distribution) is a versatile statistical distribution which can accurately model a variety of fading environments. It has greater flexibility in matching some empirical data than the Rayleigh, lognormal or Rice distributions, owing to its characterisation of the received signal as the sum of vectors with random moduli and random phases. It also includes the Rayleigh and the one-sided Gaussian distributions as special cases for fading figure $m = 1$ and $m = 0.5$, respectively. Moreover, the *m*-distribution can closely approximate the Rice distribution by the relationship $m = (K + 1)^2/(2K + 1)$ [1], or by a linear map $m = 0.499853K + 0.762216$ for Rice factor $K \geq 2$.

It is well-known that the deep fades experienced in wireless channels can result in a large penalty on the power requirement to meet a prescribed error rate performance. However, in most cases, a large amount of power is not possible due to technical and economical constraints. As a consequence, diversity techniques are usually employed to alleviate this issue. Among the various known diversity linear combining techniques, the maximal ratio combiner (MRC) is considered optimum because it yields the best statistical reduction of deep fades, as well as providing the highest average output signal-to-noise ratio (SNR). In this technique, all the L independent diversity branches are first co-phased, and then weighted in proportion to their signal level before summing. Recent advances in digital signal processing have resulted in widespread use of MRC diversity in current systems (e.g. in rake receivers and antenna arrays). Moreover, efficient evaluation of MRC diversity performance in various fading environments is desirable because it represents the highest possible improvement that a diversity system can attain through a fading channel.

In [2], the authors have derived a closed-form analytical expression for MRC diversity performance over a Nakagami fading environment, but their results are limited to integer m and identical fading parameters across all the diversity branches. However, in an actual mobile link, the radio waves take different paths and may undergo differential fading before arriving at the receiver. Therefore, it is more realistic to assume that the mean signal strength and/or the fading parameter may take arbitrary values and be different for different diversity branches, due to the random nature of the propagation channel. In [3], Eng and Mustein examined the bit error rate (BER) performance of an MRC rake receiver for an arbitrary value of fading figure (but kept this value identical for all diversity branches) by exploiting the approximate probability density function for the sum of squares of Nakagami random variates presented in [1]. As a consequence, their results deviate noticeably from the actual BER performance if the unbalances in mean signal strength among the diversity branches are large (e.g. heavily decayed exponential multipath intensity profile). More recently, Efthymoglou *et al.* [4] have analysed the performance of a coherent DS-CDMA system with arbitrary fading parameters. However, their result is in the form of an integral and a Gaussian quadrature integration method is used to approximate the final result.

In contrast, in this Letter we present a very accurate and computationally efficient closed-form expression for analysing the bit error rate performance of coherent MRC diversity systems with binary antipodal (PSK) and orthogonal (FSK) signalling. Subsequently, a BER expression to evaluate a more general case of *M*-ary PSK is outlined. Our results are sufficiently general to allow for arbitrary fading parameters as well as different mean signal strengths across the diversity branches.

Theory: The SNR at the output of the maximal-ratio combiner is

$$\gamma = \sum_{l=1}^{L} \alpha_l^2 \quad (1)$$

where α_l is a Nakagami random variable and L denotes the order of diversity. We also define $\Omega_l = E[\alpha_l^2]$ as the average SNR per branch where $E[\cdot]$ is the mean function. Assuming the branch fading as well as the branch noises to be mutually statistically independent, then the moment generating function (MGF) of γ is given by

$$\varphi_\gamma(s) = E[\exp(-s\gamma)] = \prod_{l=1}^{L}\left(1 + \frac{s}{\lambda_l}\right)^{-m_l} \quad (2)$$

where $\lambda_l = m_l/\Omega_l$. For binary coherent demodulation in the presence of AWGN, the conditional error probability is

$$P_b|\gamma = \frac{1}{2}\mathrm{erfc}(\sqrt{\gamma}a) = \frac{a}{\pi}\int_0^\infty \frac{\exp[-\gamma(s^2 + a^2)]}{s^2 + a^2}ds \quad (3)$$

where the integral expression is given in [5], eqn. 7.4.11, and the coefficient a indicates the type of modulation:

$$a = \begin{cases} 1 & \text{antipodal signalling (PSK)} \\ \frac{1}{\sqrt{2}} & \text{orthogonal signalling (FSK)} \end{cases} \quad (4)$$

Therefore the average BER is given by

$$P_b = \frac{a}{\pi}\int_0^\infty \frac{\varphi_\gamma(s^2 + a^2)}{s^2 + a^2}ds = \frac{1}{2\pi}\int_{-1}^{1} \frac{\varphi_\gamma(2a^2/(1 + x))}{\sqrt{1 - x^2}}dx \quad (5)$$

where the second integral is obtained via variable transformation $s^2 + a^2 = 2a^2/(x + 1)$.

Now, using the Gauss-Chebyshev approximation [5], eqn. 25.4.38, [6, 7] and [7], leads eqn. 5 directly to a closed-form expression in a very compact form:

$$P_b = \frac{1}{2n}\sum_{j=1}^{n}\prod_{l=1}^{L}\left[1 + \frac{a^2 \sec^2(\theta_j)}{\lambda_l}\right]^{-m_l} + R_n \quad (6)$$

where n is a small positive integer, $\theta_j = (2j - 1)\pi/(4n)$ and R_n denotes the remainder term (which becomes negligible as n

increases). For the sake of comparison, we shall reproduce the BER expression of MRC diversity system obtained in [4] below:

$$P_b = \frac{1}{2\pi}\int_0^\infty \prod_{l=1}^{L}\left[1+\left(\frac{t}{\lambda_l}\right)^2\right]^{-0.5m_l}$$
$$\times\left\{\cos\left[\sum_{l=1}^{L} m_l \tan^{-1}\left(\frac{t}{\lambda_l}\right)\right]\frac{a^{0.25}\sin[0.5\tan^{-1}(\frac{t}{\sqrt{a}})]}{(t^2+a)^{0.25}}\right.$$
$$\left.+\sin\left[\sum_{l=1}^{L} m_l \tan^{-1}\left(\frac{t}{\lambda_l}\right)\right]\left[1-\frac{a^{0.25}\cos[0.5\tan^{-1}(\frac{t}{\sqrt{a}})]}{(t^2+a)^{0.25}}\right]\right\}\frac{dt}{t} \quad (7)$$

Comparison between eqns. 6 and 7 reveals that the new expression developed in this Letter is very simple and can be readily programmed without much difficulty. Moreover, this elegant formula yields very accurate results even for a truncated series with $n = 10$. Note the implications of eqn. 6: we are simply sampling the MGF at n points. So as long as the MGF exists and is computable, this method can work very effectively. Of course, its accuracy will be high if the high-order derivatives of the MGF vanish rapidly. In particular, the remainder term can be bounded using the results of Appendix A in [7]. However, this is not necessary in practice, since one simply computes eqn. 6 for several increasing values of n, and stops when the result converges (i.e., satisfying a prescribed accuracy). Also in a correlated fading scenario, this method works equally well because we have the MGF of the total received power [9], eqn. 8. For a more general case of M-ary PSK where $M > 2$, the BER in an AWGN channel can be closely approximated by [8 (eqn. 5-2-62)],

$$P_b|\gamma \simeq \frac{1}{q}\mathrm{erfc}\left(\sqrt{q\gamma}\sin\frac{\pi}{M}\right) \quad (8)$$

where $q = \log_2 M$ (number of bits per symbol) and γ is the bit energy-to-noise ratio. Thus, the average BER in a Nakagami fading channel becomes

$$P_b \simeq \frac{1}{nq}\sum_{j=1}^{n}\prod_{l=1}^{L}\left[1+q\sin^2(\pi/M)\frac{\sec^2(\theta_j)}{\lambda_l}\right]^{-m_l} + R_n \quad (9)$$

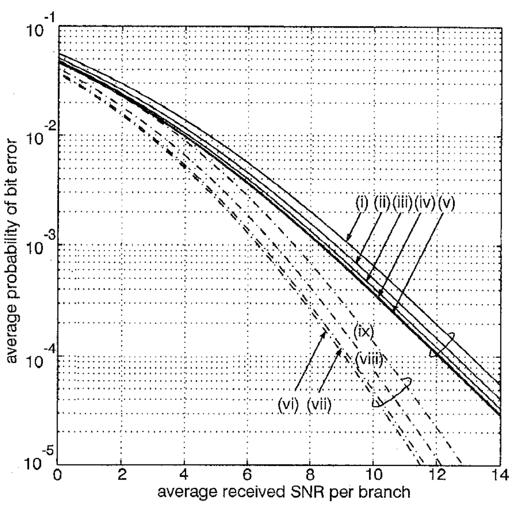

Fig. 1 *Effect of nonidentical fading parameters on average BER performance of dual-branch MRC diversity system*

(i) $m_1 = 0.5$, $m_2 = 2.5$
(ii) $m_1 = 0.75$, $m_2 = 2.25$
(iii) $m_1 = 1.0$, $m_2 = 2.0$
(iv) $m_1 = 1.25$, $m_2 = 1.75$
(v) $m_1 = 1.5$, $m_2 = 1.5$
(vi) $m_1 = 2.5$, $m_2 = 2.5$
(vii) $m_1 = 1.8$, $m_2 = 3.2$
(viii) $m_1 = 1.1$, $m_2 = 3.9$
(ix) $m_1 = 0.6$, $m_2 = 4.4$

Results: Fig. 1 illustrates the effect of noninteger and/or nonidentical fading parameters on the error performance of a dual diversity MRC system with different combinations of fading figures. It is observed that the performance curves for these cases vary considerably. This suggests that the effect of noninteger and/or nonidentical fading parameters cannot be ignored in the analysis of a diversity receiver operating in Nakagami fading environments. Also, it is apparent that the best error performance is attained when the statistically independent diversity branches undergo the same level of fading. Therefore, the assumption of identical fading figure tends to yield a rather optimistic result.

Conclusions: A simple yet extremely accurate BER expression for MRC diversity systems in Nakagami fading environments with arbitrary fading parameters has been derived. The closed-form BER formula (which is applicable for coherent MPSK and BFSK modulation schemes) can be easily programmed and evaluated efficiently (for instace, there is no need to work around Gaussian quadrature tables). Hence the generality (i.e. arbitrary signal strength and fading figure) and the computational efficiency of the new results, rendering them powerful means for both theoretical analysis and practical applications.

9 March 1998
Electronics Letters Online No: 19980863

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Frequency-domain forward/backward LMS adaptive predictors with reduced complexity

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The implementation of forward/backward least mean square linear predictors in the frequency-domain is known to require two DFT/FFT operations. Here, computationally efficient structures are derived using proper data augmentation methods. As a result, the following are achieved: (i) reduction of a DFT/FFT operation; and (ii) further reduction in complexity when the input signal is real-valued.

Introduction: The simultaneous use of forward and backward prediction errors in deriving the least mean square (LMS) adaptive line enhancer has been reported in [1]. The results indicated that the forward/backward algorithm converges to the optimal Wiener