Calculation of impulse response function: From the previous estimates we can study the parameters $\tau_{\text {sms }}$ and $\tau_{m}$ against known contour conditions of the transmission. The following stage compares the measured $h(t)$ with some known distributions. The two main options, the Rayleigh and Gamma functions, were tested. In the fitting comparison, Cramer-Von Mises and the minimum square error criteria (Table 1) were used. We have tested six positions, equally spaced on the main diagonal of the room, from the centre to the corner, at a height of 1.5 m . The emitter and receiver are considered to be at the same position, pointing to the centre of the ceiling. Fig. 4 shows some examples of a transfer function comparing estimated values (using the Rayleigh function) with those calculated using classical iterative methods. The estimated parameters $\tau_{m m s}$ and $\tau_{m}$, were calculated using eqns. 2 and 3 .


Fig. 4 Comparison of measured and estimated normalised impulse transfer functions
Relative positions of emitter and receiver, both pointing to centre of ceiling, are also given

Conclusion: A statistical method to calculate the impulse response of a wireless-indoor-IR is presented. The results are similar to those obtained with iterative methods. This new approach offers increased flexibility and reduced computer complexity for calculating the IR-indoor-diffuse- $h(t)$ function.

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R. Pérez-Jiménez and J. Berges (Departamento Seriales y Comunicaciones, ULPGC, Campus Universitario de Tafira, 35017 Las Palmas de Gran Canaria, Spain)
M.J. Betancor (DETA, ETSI Telecomunicación, ULPGC, Campus Universitario de Tafira, 35017 Las Palmas de Gran Canaria, Spain)
E-mail: rperez@cibeles.teleco.ulpgc.es

## References

1 BARRY, J.R.: 'Wireless infrared communications' (Kluwer Academic Publishers 1994)
2 LOPEZ-HERNANDEZ, F.J: "Estimation of the IR channel on indoor diffuse channels'. Internal report, DTF-UPM. Madrid, Spain, 1996
3 KHAN, J.M., KRAUSE, W.J., and CARRUTHERS, J.B.: 'Experimental characterisation of non-directed indoor infrared channels', IEEE Trans. Commun., 1995, 43, pp. 1613-1623
4 hashemi, h., yun, G., Kaverhrad, m., and behbahani, f.: 'Indoor propagation measurements at infrared frequencies for wireless local area networks applications', IEEE Trans. Veh. Technol., 1994, 43, (43) pp. 562-575

5 santamaria, a., and lópez-hernandez, f.J.: 'Wireless LAN systems' (Artech-House, 1993)
6 PEREZ-IIMENEZ, R., MELIÁN, V.M., and BETANCOR, M.J.: 'Analysis of multipath impulse response of diffuse and quasi-diffuse optical links for IR-WLAN'. Proc. INFOCOM'95, Boston, MA, USA, 1995, pp. 7d.3.1-7d.3.7

## Use of $m$-sequences for OFDM peak-toaverage power ratio reduction

## C. Tellambura


#### Abstract

Indexing terms: Frequency division multiplexing, Block codes A block coding scheme has previously been proposed using maximal-length sequences to reduce the peak-to-average power ratio (PAP) for orthogonal frequency division multiplexing (OFDM). It has also been claimed that PAP can be limited to the range $0.08-0.6 \mathrm{~dB}$ for some $m$-sequences. These numbers are incorrect and $m$-sequences can only yield PAP values in the range $5-7 \mathrm{~dB}$.


Introduction: Multicarrier modulation techniques have been proposed for digital TV broadcasting and high speed wireless networks over multipath channels [1]. A main drawback of multicarrier modulation is that the peak transmitted power may be up to $N$ times the average, where $N$ is the number of carriers. Linear amplifiers that can withstand the peak power are less efficient. Hard limiting of the transmitted signal to reduce the peak power causes performance degradation and spectral sidelobe growth. Thus, several solutions for overcoming this drawback have recently appeared in literature [2-5].

Maximum-length shift register (MLSR) codes are a class of ( $2^{m}$ $-1, m)$ cyclic codes where $m$ is a positive integer. All cyclic shifts of an $m$-sequence form an MLSR code. In [5], the use of $m$ sequences for peak-to-average power (PAP) reduction is proposed. This is done by mapping a block of $m$ input bits to an $m$-sequence, $\left[c_{0}, \ldots, C_{x_{-1}}\right]$, of length $N=2^{m}-1$. This results in a code rate of $m /$ $\left(2^{m}-1\right)$. It is then claimed that the PAP can be limited to 0.58 , $0.28,0.14,0.07$, and 0.03 dB for $m=3,4,5,6$ and 7 , respectively, by this block coding technique. This Letter shows that these PAP estimates are incorrect and $m$-sequences for $3 \leq m \leq 10$ can only yield PAP values of 5.4 to 7.3 dB .

The complex envelope of the transmitted signal is given by [4]

$$
\begin{equation*}
e(t)=\sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} c_{n}^{m} e^{i 2 \pi(n / T) t} u(t-m T) \tag{1}
\end{equation*}
$$

where $i=\sqrt{ }-1, c_{n}^{m}= \pm 1$ is the binary phase shift-keying modulation symbol for the $n$th carrier of $m$ th OFDM (orthogonal frequency division multiplexing) symbol, $T$ is the duration of an OFDM symbol and $u(t)$ is a rectangular pulse of duration $T$. In practice, eqn. 1 is generated by means of an inverse fast Fourier transform (IFFT). For notational simplicity, we omit the index $m$ of $c_{n}{ }^{m}$ and define

$$
\begin{equation*}
P(t)=\sum_{n=0}^{N-1} c_{n} e^{i 2 \pi(n / T) t} \tag{2}
\end{equation*}
$$



Fig. 1 Instantaneous power profile for 15-channel OFDM signal

Note that $P(t)$ is periodic with period $T$. The PAP in this case is defined as

$$
\begin{equation*}
\gamma=\max _{0 \leq t \leq T}|P(t)|^{2} / N \tag{3}
\end{equation*}
$$

The PAP, $\gamma$, is useful for specifying the power capability of transmitters. It can easily be shown that $1<\gamma \leq N$. Also, by eliminating binary sequences for which $\gamma$ exceeds a given threshold, PAP reduction coding schemes can be constructed [2].

An $N$-point IFFT on $\left[c_{0}, \ldots, c_{N-1}\right]$ only gives samples of $P(t)$ at time instants $t=k T / N$ for $k=0,1, \ldots, N-1$. Thus, the peak amplitude of the IFFT of $\left[c_{0}, \ldots, c_{N-1}\right]$ does not necessarily yield the PAP. That is:

$$
\begin{equation*}
\gamma \geq \max \left\{|P(k T / N)|^{2} / N \mid k=0,1, \ldots, N-1\right\} \tag{4}
\end{equation*}
$$

Therefore, PAP cannot be established solely on the basis of $N$ samples of $P(t)$, which only yields a lower bound to the PAP.

Table 1: Worst-case PAP

| $m$ | Octal $g(x)$ | PAP |
| :---: | :---: | :---: |
| 3 | 13 | 5.5 |
| 4 | 23 | 5.4 |
| 5 | 45 | 6.0 |
| 6 | 103 | 6.3 |
| 7 | 211 | 7.1 |
| 8 | 435 | 7.3 |
| 9 | 1021 | 7.6 |
| 10 | 2011 | 7.2 |

Fig. 1 shows $|P(t)|^{2 / N}$ for an $m$-sequence of length 15 . The peak of this curve is 2.44 , therefore the PAP is $10 \log _{10}(2.44)=3.87 \mathrm{~dB}$, whereas [5] claims this to be 0.28 dB . Circled points on the curve correspond to the 15 -point IFFT on the $m$-sequence. Clearly, the peaks of the waveform do not occur on these points. Also, if the $m$-sequence is shifted cyclically, then the actual peaks are not necessarily the same, although the 15 IFFT points would be the same. Thus, in general, to find the worst-case PAP, all $2^{m}-1$ non-zero code words must be tested. Table 1 shows the worst-case PAP for $m=3, \ldots, 10$, which is obtained by calculating eqn. 3 for all the code words in each MLSR code and selecting the largest $\%$. The table also contains the octal representation of the generator polynomial of each MLSR code tested (if $g(x)=g_{0}+g_{1} x+\ldots+g_{n} x^{n}$, then $g_{0}$ on right to $g_{n}$ on left, [6]).

Conclusions: The use of $m$-sequences for PAP reduction has recently been investigated. However, the resulting PAP ranges from 5.4 to 7.6 dB for $m$ ranging from 3 to 10 . While these are large PAP reductions, they fall short by several dBs of the estimates given in [5]. A further problem with this approach is the extremely low code rates for large values of $m$.

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C. Tellambura (Department of Digital Systems, Monash University, Wellington Road, Clayton, Victoria 3168, Australia)
E-mail: chintha@dgs.monash.edu.au

## References

1 BINGHAM, J.A.C.: 'Multicarrier modulation for data transmission: an idea whose time has come', IEEE Commun. Mag., 1990, pp. 5-14
2 JONES, A.E., WILKINSON, T.A., and BARTON, S.K.: 'Block coding scheme for reduction of peak to mean envelope power ratio of multicarrier transmission schemes', Electron. Lett., 1994, 30, pp. 2098-2099
3 FRIESE, M.: 'Multicarrier modulation with low peak-to-average power ratio', Electron. Lett., 1996, 11, pp. 713-714
4 mestdagh, d.s.g., and spruyt, p.m.P.: 'A method to reduce the probability of clipping in DMT based transceivers', IEEE Trans. Commun., 1996, 44, pp. 1234-1238
5 LI, X., and Rficey, J.A.: 'M-sequences for OFDM peak-to-average power ratio reduction and error correction', Electron. Lett., 1997, 33, pp. 554-555
6 ZIEMER, R.E., and PETERSON, R.L.: 'Digital communications and spread spectrum systems' (Macmillan, 1985)

# Scheduling algorithm for real-time imprecise computations to minimise maximum weighted errors using the linear programming method 

Choi Kyunghee, Yun Sungjae, Jung Gihyun and Kim Naeheon


#### Abstract

Indexing terms: Linear progranming, Computer aided analysis An optimal scheduling algorithm for imprecise systems is presented. The proposed algorithm aims at minimising the maximum weighted errors. A novel property of the algorithm is that the errors are evenly distributed among scheduled tasks. The complexity of the proposed algorithm is $O\left(N^{3}\right)$ in the worst case, where $N$ is the number of tasks.


Introduction: Shih and Liu proposed a pre-emptive algorithm for scheduling imprecise real-time tasks [1]. In their algorithm they tried to minimise the maximum normalised error that is usually introduced in real-time systems during processing imprecise tasks. The algorithm is based on the algorithm $F$ that focuses on minimising the sum of errors produced in all tasks. According to the algorithm in [1], the average qualities of processed tasks may be improved. However, when tasks have different weights, minimising only the maximum normalised error is not sufficient.

To resolve the above problem while trying to both minimise the total error and distribute weighted errors evenly among scheduled tasks, we propose a new scheduling approach. In the proposed algorithm, the maximum value of the weighted errors produced in all scheduled tasks is minimised under some restrictions. To do this, we formulate the problem of scheduling imprecise real-time systems as a linear programming problem. The formulated linear program is solved efficiently by considering its unique characteristics. While the complexity of the simplex method is exponential and the complexity of the algorithm by Karmarkar is $O\left(N^{1 / 2}\right)$, the complexity of the algorithm proposed here is $O\left(N^{3}\right)$.

Formulation and scheduling algorithm: We assume that an imprecise system is composed of $N$ imprecise pre-emptive tasks running on a single processor. A task $t_{i}$ is composed of the mandatory part $M_{i}$ and the optional part $O_{\text {, }}$, and characterised by the ready time $r_{i}$, the deadline $d_{i}$, the weight $\alpha_{i}$ and the computation requirements $m_{i}$ and $o_{i}$ for $M_{i}$ and $O_{i}$, respectively. Let $p_{i}$ be the sum of $m_{i}$ and $o_{i}$. We also assume that all tasks are sorted according to their deadlines, i.e. $d_{i} \leq d_{i+1}$ for $i=1, \ldots, N-1$.

Let $a_{i}$ be one of either $r_{i}$ or $d_{j}$, such that $a_{i}<a_{i+1}$ and $I_{i}$ is an interval $\left(a_{j}, a_{j+1}\right]$. Then, the maximum $2 N$ mutually disjoint intervals are constructed, even though the actual number is usually smaller than that (i.e. $M$ ). Let $w_{i, j}$ be the amount of processing time assigned to task $j$ in interval $I_{i}=\left(a_{i}, a_{i j}\right)$, and let $e_{j}$ be the error. Note that, to satisfy the minimum service requirement, the total processing time given to task $j, w_{j}$ (defined as the sum of $w_{i, j}$ for task $j$ ), should be equal to or greater than $m$.

To minimise $z$, the minimum value of the maximum weighted error can be formulated with the following constraints. Constraint (i) is obvious; constraint (ii) ensures that mandatory parts of all tasks are scheduled. Constraint (iii) requires that the error of each task is equal to the difference between the total computational requirement and the service time actually given. Constraint (iv) means that the sum of all service times assigned in an interval cannot exceed the length of the interval. Constraint (v) represents non-negativity of all variables. Considering that a bounded linear program with a feasible solution possesses an optimal solution, the following linear problem must have an optimal solution if all mandatory parts are schedulable:

Min $z$ subject to:
(i) $z \geq \alpha_{j} e_{j}$ for $j=1, \ldots, N$
(ii) $e_{j} \leq p_{j}-m_{j}$ for $j=1, \ldots, N$
(iii) $e_{i}+\Sigma_{i} w_{i j}=p_{j}$ for $j=1, \ldots, N$
(iv) $\Sigma_{j} w_{i, j} \leq c_{i}$ for $i=1, \ldots, M$, where $c_{i}$ is the length of interval $I_{i}$
(v) $w_{i, j} \geq 0, e_{j} \geq 0, z \geq 0$ for $j=1, \ldots, N, i=1, \ldots, M$

