Outage probability analysis for cellular mobile radio with multiple, arbitrary Nakagami interferers [1]

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Abstract - In this study, we derive new expressions for the probability of outage where a mobile unit receives a Nakagami desired signal and multiple, cochannel, independent Nakagami interferers. Unlike some previous results, the expressions are usable when the Nakagami fading parameter, m, is not necessarily an integer. An exact formula is also derived for the outage probability when the desired signal power must exceed a minimum threshold for satisfactory reception. Typically, in mobile radio, the received signal also experience log-normal shadowing. The outage probability is derived when the received signals suffer both Nakagami-fading and shadowing simultaneously.

I. INTRODUCTION

The design of cellular mobile radio systems is fundamentally based upon the concept of frequency reuse. This gives rise to cochannel interference (CCI). Thus, it is the goal of system design to ensure that for any mobile unit in given cell, the desired-signal-to-interference ratio be higher than a specified minimum. When this condition fails, a signal outage occurs, degrading the quality of service.

In a previous paper, Wojnar [1] derives the outage probability for a single Nakagami interferer. Abu-Dayya and Beaulieu [2] obtain the outage probability for cellular mobile systems in the presence Nakagami interferers. However, some of the results therein apply only when $0.5 \le m < \infty$ is constrained to take integer values. For nonintegral m, the use of linear extrapolation is suggested to get the exact result for m. To overcome this difficulty, Zhang [3] proposes a method based on direct numerical inversion of the characteristic function.

To circumvent such difficulties, we derive a new series solution for the probability of outage when the signals experience Nakagami fading with arbitrary parameters, with or without specifying a minimum signal power level for satisfactory reception. This method of evaluating the probability of outage avoids numerical integration and residue calculation. We also analyze the case where the signals experience both Nakagami fading and log-normal shadowing simultaneously.

A. Description of the System Model

The total received signal can be expressed as

$$v(t) = \sum_{j=0}^{L} \rho_j(t) \cos[\omega_c t + \theta_j(t) + \psi_j(t)] \tag{1}$$

where ω_c is the carrier frequency, $\rho_j(t)$ ($j = 0,1,\ldots L$) are the random fading amplitudes, $\theta_j(t)$ are the random phase, and $\psi_j(t)$ represent the data-bearing signal phase of each carrier. The desired signal is indexed with $j = 0$ and the interfering signals with $j = (1,2,\ldots,L)$. In typical cases of interest, the variation of $\rho_j(t)$ and $\theta_j(t)$ over the data symbol duration T is negligible. Thus, the short-term power of each carrier signal is expressed as $p_j = \rho_j^2(t)$ ($j = 0, 1, \ldots, L$).

Assume that the signal amplitudes $\rho_j(t)$ are mutually independent Nakagami random variables (RVs). Then, the short-term power p_j is a Gamma RV with probability distribution (PDF) given by [1]

$$f_{p_j}(p_j|\bar{p}_j) = \left(\frac{m_j}{\bar{p}_j}\right)^{m_j} \frac{p_j^{m_j-1}}{\Gamma(m_j)} \exp\left(-\frac{m_j}{\bar{p}_j} p_j\right) \tag{2}$$

where $m_j \ge 0.5$ and $\bar{p}_j$ denotes the local-mean power of the j th signals. $\Gamma(x)$ is the gamma function, and m_j is referred to as the fading parameter.

Assuming incoherent power addition, the total interference power, denoted by I, is $I = p_1 + \ldots + p_L$. Since the characteristic function of I is readily available (14), for integral m_j, the PDF of I can be obtained via residue calculation, which, however, fails for nonintegral m_j. For the subsequent derivations, it is convenient to define the usual average-signal to average-interference power ratio, concerning each interferer, as $\Lambda_j = \bar{p}_0 / \bar{p}_j$.

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II. Analysis

The signal outage probability is the probability that the signal-to-interference ratio drops below a minimum threshold q. The outage probability due to L interferers is then expressed as

$$O_L \equiv \Pr(S/I < q) = \int_0^{\infty} f_I(y) \int_0^{qy} f_{p_0}(s)\, ds\, dy \qquad (3)$$

where q is a power protection ratio and S and I are the short-term powers of the desired signal and the composite interfering signal, respectively. $f_I(y)$ is the PDF of the total interference variable I. The power protection ratio, q, usually specified in decibels, is fixed by the type of modulation and transmission technique employed and the quality of service desired. Typical values for q are 18 dB for analog FM systems [2] and on the order of 20 dB for linear SSB modulation.

Suppose that $K(x) = P\{0 \le S \le x\}$. Then (6) becomes

$$O_L = \int_0^{\infty} K(qy) f_I(y)\, dy \qquad (4)$$

which can be viewed as an integral transform of the PDF of the joint interference power. The kernel $K(qy)$ is the cumulative distribution of the short-term power p_0, and since p_0 is a Gamma variate (2), the kernel is expressed by the incomplete gamma function [4]: $K(x) = \gamma(m_0, m_0 x/\bar{p}_0)/\Gamma(m_0)$.

A. Identical Interferers

When all the interfering signal sources are roughly equidistant from the receiving mobile, and interfering signals experience the same degree of fading, we have, for $j = 1,2,\ldots,L$, $m_1 = m_j$ and $\bar{p}_1 = \bar{p}_j$. Consequently, the total interference I has the PDF $f_I(y)$ given by Eq. (3) with the parameters $m_1 L$ and $\bar{p}_1 L$. Substituting this in (4) and integrating term-by-term, we can readily prove that

$$O_L = k_1 z^{m_0} (1-z)^{m_1 L} \Phi(m_0 + m_1 L, m_0 + 1, z) \qquad (5)$$

where $\Phi(a, b, z)$ is the confluent hypergeometric series [4], $k_1 = \Gamma(m_0 + m_1 L)/(\Gamma(m_1 L)\Gamma(m_0 + 1))$ and $z = m_0 q/(m_0 q + m_1 \Lambda_1)$. This series is always convergent and in particular for large SIRs (signal-to-interference ratios), only a few terms need to be calculated to get very high accuracy. This result, in a slightly different format, has also been derived in references [1] and [2].

FIGURE 1. shows the outage probability, computed using (5), for several cases. Note that $m = 0.5$ is the most severe fading, and the results show that for $L = 1$, an Λ_1/q of almost 40 dB is essential to achieve an outage probability of 10^{-2}, while this figure drops to about 5 dB when $m = 6.5$. This clearly shows that increasing m leads to a nonfading channel.

B. Arbitrary Parameters

In the general case, we let $m_j \neq m_k$ and $\bar{p}_j \neq \bar{p}_k$ for $j \neq k$ $(j, k = 1, 2, \ldots, L)$. Note that if any two or more interferers have the same parameters, then they can be combined to a single interferer, as was the case in the previous section. Using the PDF of I given in (16) and the series expression for the kernel and , by integrating term-by-term in the series expansion we obtain the probability of outage as

$$O_L = \phi_1 z^{m_0} \sum_{r_2, \ldots, r_L}^{\infty} D(M;\Delta;R)\,\Gamma(m_T + r_t) \times \Phi(m_T + r_t, m_0 + 1, z) \qquad (6)$$

where, for convenience, $m_T = m_0 + \ldots + m_L$, $r_t = r_2 + \ldots + r_L$, $\phi_1 = \left(z_1^{m_0} \ldots z_L^{m_L}\right)/\Gamma(m_0 + 1)$,

The only requirement, one which is not at all restrictive, for the convergence of (6) is that $m_1\Lambda_1 > m_j\Lambda_j$ for $j = 2, 3, \ldots, L$. The convergence rate depends on $m_1\Lambda_1$ and the differences $m_1\Lambda_1 - m_j\Lambda_j$, and with favorable conditions, i.e., $m_1\Lambda_1/q \gg 1$ and $\delta_j \ll 1$, the convergence of the multiple series is extremely rapid.

Using (6), we have calculated the probability of outage for the following case. Consider six Nakagami interferers with the parameters given by $m = \{4.7, 1.3, 2.1, 1.8, 0.9, 2.25\}$ and $\bar{p} = \{3.1, 2.2, 1.6, 1.2, 1.5, 1.9\}$. For calculations, $\mathrm{SIR} = \bar{p}_0/(\bar{p}_1 + \ldots + \bar{p}_6)$. In the following, outage probabilities are computed as a function of the quantity SIR/q. FIGURE 2. shows the probability of outage for a Nakagami fading desired signal with a varying fading parameter. The effect of interferers reduces significantly as the desired signal is subjected to less fading.

C. Rayleigh-fading Desired Signal

Our general expression (6) simplifies considerably if the desired signal is Rayleigh fading (i.e., $m_0 = 1$). Thus, the kernel $K(qy) = 1 - \exp(qy/\bar{p}_0)$. By substituting this in (4), the probability of outage becomes

$$O_L = 1 - \prod_{j=1}^{L} \left[\frac{m_j\Lambda_j/q}{1 + m_j\Lambda_j/q}\right]^{m_j}. \qquad (7)$$

For multiple uncorrelated interferers in a Rayleigh fading environment, $m_j = 1$ for all $j = 1, 2, \ldots, L$, this reduces to a previously derived result [6].

FIGURE 3. shows, given an outage due a single interferer, the incremental outage caused by an addition of a second interferers as a function of the relative strength of the second interferer. Three cases where both the interferers are heavily faded $m_1 = m_2 = 0.5$, first one is heavily faded $m_1 = 0.5$ and the second one less faded $m_2 = 7.5$, and vice versa. It can be seen, for instance, if the first interferer causes an outage probability of 20%, the additional interferer must be at least 2.0 dB, 3.0 dB and 3.5 dB, respectively, weaker than the first one in order that the total outage not exceed 30%.

D. Desired Signal with an Integral m

Suppose the Nakagami fading parameter of the desired signal is an integer (i.e., m_0 is an integer). In this case the kernel can be expressed as a finite polynomial of y [4]. By substituting this in (4) and using the fact that differentiation of the Laplace transform of a function corresponds to the multiplication of the function by $-y$, we obtain

$$O_L = 1 - \sum_{k=0}^{m_0-1} \frac{(-1)^k}{k!} s^k \phi_I^k(s) \Big|_{s = m_0 q/\bar{p}_0} . \tag{8}$$

where $\phi_I^k(s)$ is the k th derivative. Since $\phi_I(s)$ is given in (14), this can be alternatively used to compute the probability of outage.

III. Nakagami Fading and Log-Normal Shadowing

Shadowing refers to a large-area distribution of the signal power. Many experimental studies show that the local-mean signal power fluctuates with a log-normal distribution about the area-mean signal power. This means, using the notation in (2), $\bar{p}_j$ is a log-normal RV with the mean denoted by $\tilde{p}_j$ for $j = 0, 1, \ldots, L$. Mathematically, this is expressed as [5]

$$f_{\bar{p}_j}(\bar{p}_j|\tilde{p}_j) = \frac{1}{\sqrt{2\pi}\sigma_s \bar{p}_j} \exp\left(-\frac{\ln^2(\bar{p}_j/\tilde{p}_j)}{2\sigma_s^2} \right) \tag{9}$$

where σ_s is the logarithmic standard deviation (expressed in natural units) of shadowing and $\ln(x)$ is the natural logarithm of x. When signal powers are specified in decibels, the standard deviation s_s is given by $s_s = 4.34\sigma_s$ [5]. Typical values for the standard deviation range from 5 to 12 dB for most channels. In the following we make simplifying assumption that the shadowings of the signals are uncorrelated.

For a given area-mean power $\tilde{p}_j$, the composite PDF of the local-mean signal power p_j is then given by averaging (2) over (9). When $m_j = 1$, this reduces to the Suzuki distribution [5]. There appears to be no analytical solution for this integral.

It is quite useful find the characteristic function on the composite PDF above. To express this more compactly, if we let

$$\phi_{\sigma_s}(s, m) \equiv \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-x^2)dx}{\left(1 + s\exp(\sqrt{2}x\sigma_s)\right)^m}, \tag{10}$$

then, we can show that [7] $\phi_{p_j}(s) = \phi_{\sigma_s}(s\tilde{p}_j/m_j, m_j)$.

In order to compute the integral in (10) efficiently, following the method of Linnartz [5], we can use an n-point Hermite-Gauss integration formula. According to this, where the abscissas x_i (ith root of an nth order Hermite polynomial) and weights w_i are tabulated in Table 25.10 of reference [4]. In all our calculations we have used $n = 20$.

For simplicity we only consider the case where the desired signal is Rayleigh fading. Then following the reasoning in (7), we have

$$O_L|\bar{p}_0 = 1 - \prod_{j=1}^{L} \phi_{\sigma_s}\left(\frac{\tilde{p}_j q}{\bar{p}_0 m_j}, m_j \right). \tag{11}$$

By averaging this over the shadowing distribution of the desired signal (9), the probability of outage is obtained.

For mobile radio communications, the area-mean power received by the mobile can be expressed as [5] $\tilde{p}_j = \kappa r_j^{-\beta}, 0 \le j \le L$, where κ is a constant common to all the signals, β is a propagation loss exponent, and r_0 and r_j ($j = 1, 2, \ldots, L$) are the radio-path distances from the mobile unit to the desired and interfering base-station transmitters. For calculations, we assume an inverse fourth power propagation low (i.e., $\beta = 4$).

Given an idealized cellular network with a hexagonal cell layout, let the cell radius be r. Worst-case cochannel interference occurs when the mobile unit is at the cell boundary, i.e., $r_0 = r$. Assume that there are L active cochannel interferers at the reuse distance. This means a normalized reuse distance can be defined as [2] $U = (r_0 + r_j)/r = 1 + r_j/r_0$.

Based on the Hermite integration formula, we have shown that [7]

$$O_L = 1 - \frac{1}{\sqrt{\pi}} \sum_{i=1}^{n} w_i \left(\prod_{j=1}^{L} \phi_{\sigma_s}\left(\frac{q e^{-\sqrt{2}\sigma_s x_i}}{m_j (U-1)^{\beta}}, m_j \right) \right). \tag{12}$$

Using (12), the probability of outage is plotted in FIGURE 4. as a function of the reuse distance with q and σ_s as parameters for a macrocellular system. The results show that the effect of shadowing is to require increasing reuse distances to ensure the quality of service desired.

IV. MINIMUM SIGNAL CONSTRAINT

For adequate reception of the desired signal, the short-term SIR must be greater than q, and the desired signal power level exceeds, simultaneously, a minimum power level denoted by γ_0 [6]. Therefore, the outage probability is expressed as the probability union: $\tilde{O}_L \equiv \Pr\{[S/I<q] \cup [S<\gamma_0]\}$.

Then we obtain [7]

$$\tilde{O}_L = K(\gamma_0) + \int_{\gamma_0/q}^{\infty} f_I(y)\,(K(qy) - K(\gamma_0))\,dy. \qquad (13)$$

Observe that this equation implies a floor on the outage probability because the first term on the right hand side is a constant regardless of cochannel interference. The second term vanishes as $\gamma_0 \to \infty$. Of course, for $\gamma_0 = 0$, (13) reduces to (4).

V. CONCLUSIONS

Analytical expressions have been derived for calculating the probability of outage in a cellular mobile radio system where all the received radio signals experience arbitrary Nakagami fading. We have found that the series expressions can be easily computed numerically. This method of computing the outage probability thus supplements the previous reported methods [2]-[3]. When shadowing is present, analysis becomes much more difficult, but we have derived a computationally efficient solution for this case.

Numerical results for the probability of outage have been presented for various fading and shadowing conditions. These results should lend insight into the behavior of the communication reliability of a cellular system vis-a-vis many system parameters such as SIRs, the power protection ratio, the degree of shadowing and so on.

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Appendix

The characteristic function of the total interference variable is [2]

$$\phi_I(s) = \prod_{j=1}^{L}\left[\frac{m_j/\bar{p}_j}{s + m_j/\bar{p}_j}\right]^{m_j}. \qquad (14)$$

Define $\mu_j = m_j/\bar{p}_j$, for $j = 1,2,\ldots,L$, $m_t = \sum_{j=1}^{L} m_j$, and

$$D(M;\Delta;R) = \left(\prod_{j=2}^{L} \frac{(m_j)_{r_j}\delta_j^{r_j}}{r_j!}\right) / (\Gamma(m_t + r_t)) \qquad (15)$$

where $M = (m_2, \ldots, m_L)$, $\Delta = (\delta_2, \ldots, \delta_L)$ with $\delta_j = (\mu_1 - \mu_j)$ for $j = 2,\ldots,L$, and $R = (r_2, \ldots, r_L)$.

By taking the inverse Laplace transform of (14), we obtain [7]

$$f_I(y) = \phi_0 \sum_{r_2,\ldots,r_L}^{\infty} D(M;\Delta;R)\, y^{r_t + m_t - 1} e^{-\mu_1 y} \qquad (16)$$

for $0 < y < \infty$.

VI. REFERENCES

[1] A. Wojnar, "Unknown bounds on performance in Nakagami fading channels," *IEEE Trans. Commun.,* vol. COM-33, pp. 22-24, Jan. 1986.

[2] A. Abu-Dayya and Norman C. Beaulieu, "Outage probabilities of cellular mobile radio systems with multiple Nakagami interferers," *IEEE Trans. Vehic. Techn.,* vol. VT-40, no. 6, pp. 757-767, Nov. 1991.

[3] Q. T. Zhang, "Exact formulas for outage probability in the presence of multiple Nakagami interferers with arbitrary parameters," *Proc. Wireless 94*, Canada, pp. 689-699, July 1994.

[4] M. Abramowitz and I. Stegun, "*Handbook of Mathematical Functions.* Washington: National Bureau of Standards, 1972.

[5] Jean-Paul Linnartz, *Narrowband land-mobile radio networks.* Norwood, MA: Artech, 1993

[6] K. Sowerby and A. Williamson, "Outage probability calculations for multiple cochannel interferers in cellular mobile radio systems," *Inst. Elec. Eng. Proc.,* pt. F, vol. 135, no. 3, pp. 208-215, June 1988.

[7] C. Tellambura and V. K. Bhargava, "Outage probability analysis for cellular mobile radio systems subject to Nakagami fading and shadowing," submitted to *ICECE Trans.*

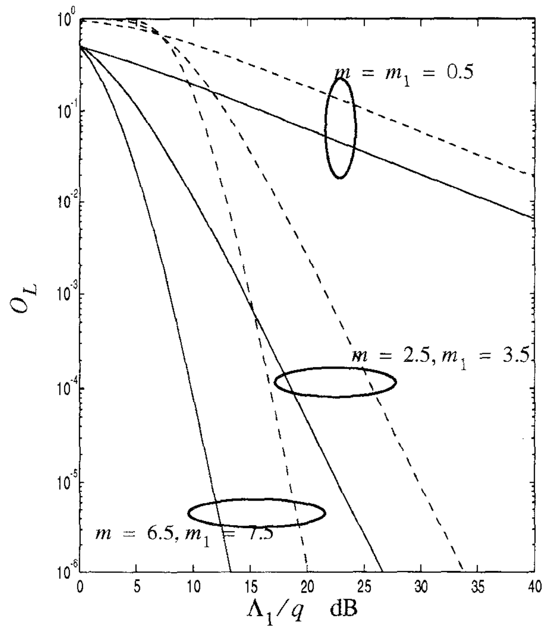

FIGURE 1. The outage probability of a Nakagami fading signal and L interferers as a function of Λ_1/q. Dashed lines correspond to $L = 6$ and solid lines to $L = 1$.

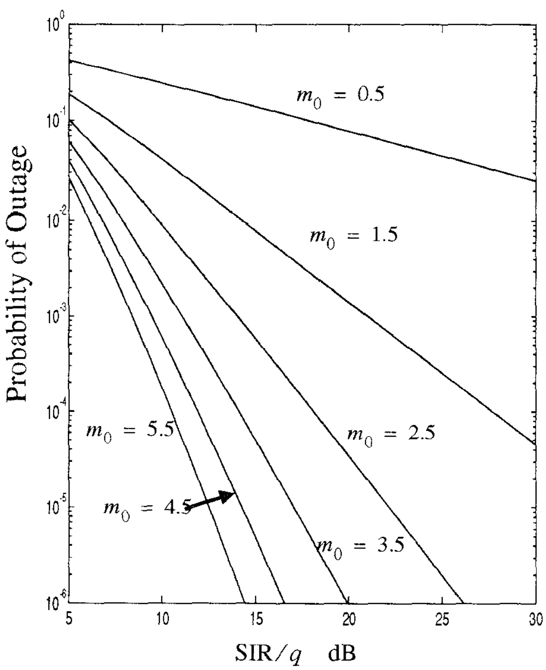

FIGURE 2. The outage probability as a function of SIR/q for a Nakagami fading signal and six interferers.

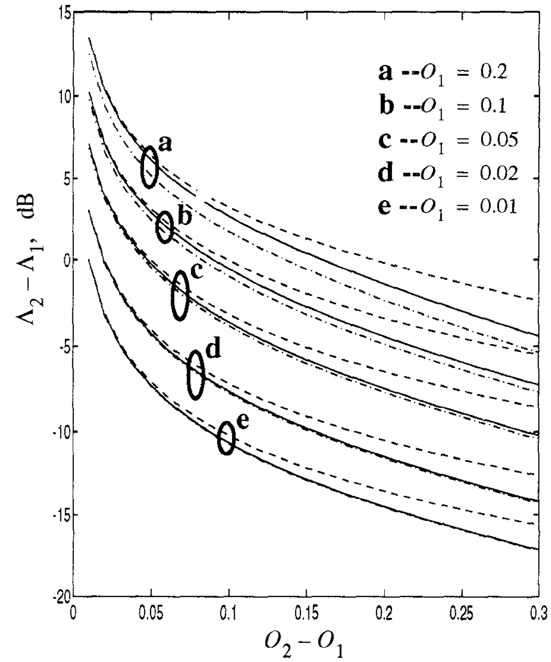

FIGURE 3. The difference in dB between the SIRs of two interferers versus the increase in the outage probability. The solid lines correspond to $m_1 = m_2 = 0.5$, the dashed lines to $m_1 = 0.5, m_2 = 7.5$, and the dashdot lines to $m_1 = 7.5, m_2 = 0.5$.

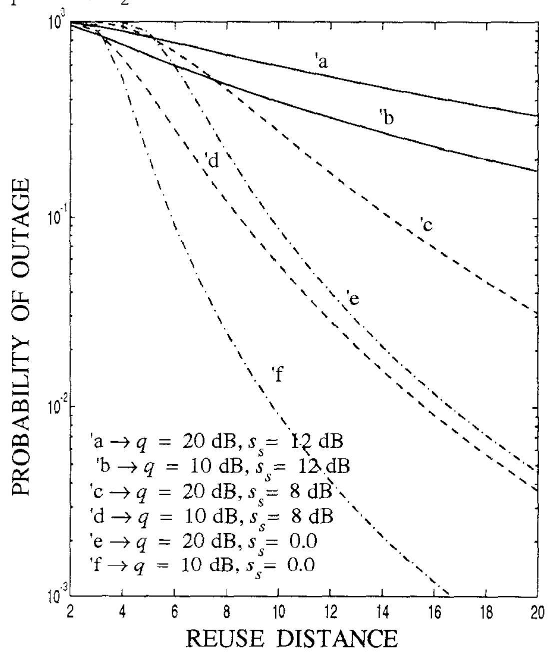

FIGURE 4. The outage probability versus normalized reuse distance for six interferers in macrocellular. All signals suffer fading and log-normal shadowing.