CONCATENATED CODING AND ARQ SYSTEMS USING TCM FOR RAYLEIGH FADING

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ABSTRACT

Trellis coded modulation (TCM) schemes can be employed in conjunction with concatenated coding or automatic-repeat-request (ARQ) systems. The performance of such hybrid schemes operating in Rayleigh fading channels is addressed in this paper. The fading is assumed to be slow with respect to the symbol rate. New approximate error expressions are developed when a pilot tone is used to estimate the channel gain, and when the Viterbi decoding algorithm is modified such that side information can be generated to erase symbols or request retransmission.

Key Words: Rayleigh fading, trellis coding, data communications.

1. INTRODUCTION

TCM schemes can be employed in conjunction with concatenated coding or ARQ systems. The performance of such hybrid schemes operating in Rayleigh fading channels is addressed in this paper. The fading is assumed to be slow (e.g., ≤ 5 % of the symbol rate), nonfrequency selective. Since in a fading environment perfect coherent detection (e.g., by using phase-locked loops) is impossible, a phase reference can be transmitted to assist demodulation. Common reference systems include differential detection, pilot-tone aided detection [1], and pilot-symbol aided detection [2]. Applying a pilot-tone model to study such systems, this paper derives general error-rate expressions for analyzing concatenated or ARQ systems. Moreover, the analysis considers a modified Viterbi algorithm.

The idea of modifying the conventional Viterbi algorithm (VA) so as to generate reliability information has received attention recently. When decoding a received sequence, the conventional VA selects the path (codeword) that maximizes the log-likelihood function. The modified VA's given in [3-5] essentially amount to observing the metric difference between the best and the second best path in the trellis and erasing symbols if the difference is less than a certain threshold.

In the literature, several papers which use modified VA's have dealt with the problem of TCM in concatenated coding and ARQ systems. In [5], concatenated coding systems consisting of trellis inner codes and Reed-Solomon (RS) outer codes have been considered for AWGN channels; coding gains of 4-6 dB have been obtained for specific concatenations of TCM and RS codes. TCM-RS concatenated coding for fading channels has been investigated in [4], where the Chernoff bound approach is used to determine the performance of the inner code. In [6], the performance of TCM in conjunction with retransmission strategies for both AWGN and fading channels is addressed, but only the case of ideal channel state information is treated. As a result, here the pilot-tone model is used to derive the PEP (pairwise error probability) expressions and erasure (or retransmission) probabilities when the modified VA is used for decoding, and when the inner channel is ideally interleaved.

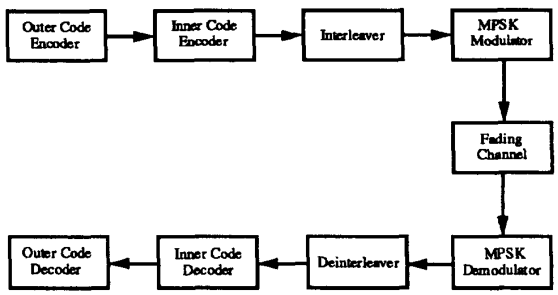

Figure 1: A Concatenated Coding System.

2. SYSTEM AND CHANNEL DESCRIPTION

The system under consideration is shown in Figure 1. In this system, a trellis code with Viterbi decoding is used as the inner code along, and an (n_2, k_2) Reed-Solomon code with symbols from the Galois Field GF(2^l), where $n_2 = 2^l - 1$, is used as the outer code. The trellis code is defined by k_1 encoder input bits per unit time and 2^{k_1+1}-ary PSK modulation. Binary input data is encoded by the outer encoder with $k_2 l$ information bits each being converted to an n_2-byte codeword. This codeword is symbol-interleaved and then further encoded by the trellis encoder. To estimate the true channel gain, a pilot tone is added to the transmitted signal; thus, this is referred to as the pilot-tone model. The received M-ary symbols are rearranged by the deinterleaver and decoded by the Viterbi decoder, and then passed onto the RS decoder. For the purposes of analysis, we assume that both the outer channel and the inner channel are ideally interleaved (note that the outer channel interleaver is not shown in Fig. 1).

Neglecting intersymbol interference, which is justified because of slow fading, the k-th output of the channel can be

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expressed as [7]

$$y_k = \alpha_k x_k + n_k \tag{1}$$

where x_k denotes the transmitted multilevel PSK symbol at time k, α_k is a zero-mean, complex Gaussian variate – an approximation of the fading process $\alpha(t)$ for $kT \le t \le (k+1)T$– and n_k is an additive Gaussian sample. The normalized autocorrelation function of $\alpha(t)$ for a land mobile radio system is given by [8]

$$R(\tau) = J_0(2\pi f_d \tau) \tag{2}$$

where $J_0(\bullet)$ is the zero-order Bessel function and f_d denotes the maximum Doppler frequency.

An estimate of α_k, $\hat{\alpha}_k$ is derived at the receiver by filtering the pilot tone. Using this estimate, for a codeword $\mathbf{x}$, the maximum-likelihood estimator derived in [7] maximizes the metric[1]

$$M_{\mathbf{x}} = -\sum_k |y_k - \beta x_k \hat{\alpha}_k|^2 \tag{3}$$

where $\beta = \mu\frac{\sigma_1}{\sigma_2}$, and where σ_1^2 is the variance of α_k, σ_2^2 is the variance of $\hat{\alpha}_k$, and μ is the normalized correlation coefficient between them.

3. MODIFIED VA

In this section, we consider a modified VA, which generates side information used by the outer decoder. As stated, we investigate only the Rayleigh channel. In the following, the pilot-tone model is used to derive expressions for the asymptotic PEP and erasure probability as a function of the threshold.

In order to derive the side (erasure) information the following algorithm can be utilized [3-5]. Consider the codeword $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ and another codeword $\hat{\mathbf{x}} = (\hat{x}_k, \hat{x}_2, \ldots, \hat{x}_N)$. Let $\mathbf{y} = (y_1, y_2, \ldots, y_N)$ be the corresponding received vector. As defined in Eq. (3), the decoder computes the path metrics $M_{\mathbf{x}}$ and $M_{\hat{\mathbf{x}}}$ for these two codewords. At the beginning of the decoding ($k = 0$), all paths in the trellis are labelled as C. At each next step k, ($k = 1, 2, \ldots$), the decoder selects the paths $\mathbf{x}$ and $\hat{\mathbf{x}}$ that have the largest and second largest metrics, $M_{\mathbf{x}}$ and $M_{\hat{\mathbf{x}}}$, respectively. If

$$M_{\mathbf{x}} - M_{\hat{\mathbf{x}}} > T \tag{4}$$

where T is an erasure threshold, then $\mathbf{x}$ survives with as correct, reliable codeword. Otherwise, path $\mathbf{x}$ survives as unreliable. If all survivors are unreliable at one point, the decoder, when used in a concatenated system, attaches erasure flags to symbols back to time unit $k - D$ where D denotes the code decision length, so that the RS outer decoder can use this information.

Let $\mathbf{x}$ be the transmitted code. Then, the PEP, the probability $\hat{\mathbf{x}}$ survives as a reliable path, is

$$P(\mathbf{x} \to \hat{\mathbf{x}}) = \Pr\{\Xi \ge T\} \tag{5}$$

where

$$\Xi = \sum_{k\in\eta} |y_k - \beta\hat{\alpha}_k \hat{x}_k|^2 - |y_k - \beta\hat{\alpha}_k x_k|^2 \tag{6}$$

[1] Several symbol definitions differ from those of [7]

and where $\eta = \{k | x_k \ne \hat{x}_k, k = 1, \ldots, N\}$, and $y_k, \beta, \hat{\alpha}_k$ have been defined previously. When the inner channel is ideally interleaved, the above sum consists of independent random variables, and the residue method given in [7] can be used to compute Eq. (5).

Because of ideal interleaving, the characteristic function of Ξ follows from [7, Eq. (18)], where it is shown that

$$\phi_{\Xi}(s) = \left(\prod_{k\in\eta} \frac{1 + (1 - |\mu|^2)\gamma_s}{|\mu|^2 |x_k - \hat{x}_k|^2 \gamma_s}\right) \prod_{k\in\eta} \frac{-1}{(s - v_{1k})(s - v_{2k})} \tag{7}$$

where μ is the correlation coefficient between α_k and $\hat{\alpha}_k$, γ_s denotes the signal-to-noise ratio, $w_k = 1/2$, and

$$\begin{bmatrix} v_{1k} \\ v_{2k} \end{bmatrix} = w_k \pm \sqrt{w_k^2 + \frac{1 + (1 - |\mu|^2)\gamma_s}{|\mu|^2 |x_k - \hat{x}_k|^2 \gamma_s}}. \tag{8}$$

In Eq. (7), the convergence region in the complex plane is given by $v_{2k} < \text{Real}(s) < v_{1k}$.

The exact PEP in Eq. (5) can be readily computed by the method given in [7], which involves summing up all residues at the right-plane poles of the Laplace transform of cumulative PDF of the random variable Ξ. That is,

$$P(\mathbf{x} \to \hat{\mathbf{x}}) = -\sum_{k\in\eta} \underset{s=v_{1k}}{\text{Residue}} \left[\frac{e^{-sT}\phi_{\Xi}(s)}{s}\right]. \tag{9}$$

Calculating this for each error event is time consuming. Alternatively, we derive an easy-to-compute, asymptotic expression for $P(\mathbf{x} \to \hat{\mathbf{x}})$. When $\mu \simeq 1$ and $\gamma_s \to \infty$, from Eq. (8) we have $v_{1k} \simeq 1$ and $v_{2k} \simeq 0$. Substituting these values in Eqs. (7) and (9) results in

$$P(\mathbf{x} \to \hat{\mathbf{x}}) \simeq - \underset{s=1}{\text{Residue}} \left[\frac{(-1)^L e^{-sT}}{4^L s^{L+1}(s-1)^L} \cdot \left(\prod_{k\in\eta} \frac{1 + (1 - |\mu|^2)\gamma_s}{|\mu|^2 |x_k - \hat{x}_k|^2 \gamma_s / 4}\right)\right] \tag{10}$$

where L is the Hamming distance associated with this error event. By evaluating the L-th order residue at $s = 1$, we get

$$P(\mathbf{x} \to \hat{\mathbf{x}}) \simeq \psi(L, T) \left(\prod_{k\in\eta} \frac{1 + (1 - |\mu|^2)\gamma_s}{|\mu|^2 |x_k - \hat{x}_k|^2 \gamma_s / 4}\right) \tag{11}$$

where

$$\psi(L, T) \triangleq 4^{-L} e^{-T} \sum_{r=0}^{L-1} \frac{(L+r)!}{r! L! (L-1-r)!} T^{L-1-r}. \tag{12}$$

Using the union bound technique and the above expression for the PEP, the bit error probability at the output of the inner decoder could be bounded by

$$P_b \le \frac{1}{k_1} \sum_{j=1}^{\infty} m_j \psi(L_j, T) \left(\prod_{k\in\eta_j} \frac{1 + (1 - |\mu|^2)\gamma_s}{|\mu|^2 |x_{jk} - \hat{x}_{jk}|^2 \gamma_s / 4}\right) \tag{13}$$

where m_j is the number of information bit errors associated with the j-th error event, L_j is the Hamming distance of the j-th error event, and η_j is defined for the j-th event as in (6). This bound can be readily specialized to the following cases.

3.1. Ideal Coherent Detection

By definition, we now have an ideal phase reference; that is, $\hat{\alpha}_k = \alpha_k$. Thus the correlation coefficient, μ, is unity, which leads to the upper bound

$$P_b \leq \frac{1}{k_1}\sum_{j=1}^{\infty} m_j \psi(L_j,T)\left(\prod_{k\in\eta_j}\frac{1}{|x_{jk}-\hat{x}_{jk}|^2\gamma_s/4}\right). \quad (14)$$

3.2. Differential Detection

Here, the previous signal element act as the phase reference. Consequently, the correlation coefficient is given by [7]

$$|\mu|^2 = \frac{J_0^2(2\pi f_d T_s)}{1+\gamma_s^{-1}} \quad (15)$$

which when substituted in (13) gives a bound on P_b.

3.3. Pilot-Tone Aided Detection

As a phase reference, a pilot tone can be transmitted along side the data bearing signal, and, at the receiver, it is extracted by a filter of bandwidth $2f_d$. It can be readily shown that [7, 9]

$$|\mu|^2 = \frac{1}{1+2f_dT_s(\frac{1+r}{r})\gamma_s^{-1}} \quad (16)$$

where r is the ratio of pilot power to data signal power and γ_s now accounts for the energy spent on both the pilot and the data. Substituting this μ in (13), we get a bound on P_b.

3.4. Erasure Probability

As before, denote by $\mathbf{x}$ the transmitted codeword. Let the decoder select $\hat{\mathbf{x}}$ as the correct one. An erasure event can be divided into two parts. First, the decoder makes the correct choice (i.e., $\hat{\mathbf{x}} = \mathbf{x}$) and, yet, sets up an erasure flag. Second, the decoder makes an error (i.e., $\hat{\mathbf{x}} \neq \mathbf{x}$) and sets up an erasure flag. Thus, the erasure event probability can be obtained as [6]

$$\begin{aligned} P(\text{erasure}) &= P(\text{erasure}|\hat{\mathbf{x}}=\mathbf{x})P(\hat{\mathbf{x}}=\mathbf{x}) + \\ &\quad P(\text{erasure}|\hat{\mathbf{x}}\neq\mathbf{x})P(\hat{\mathbf{x}}\neq\mathbf{x}). \end{aligned} \quad (17)$$

Based on the erasure generating rule in (4), the event $(\text{erasure}|\hat{\mathbf{x}}=\mathbf{x})$ occurs when there exists another codeword $\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_N)$ which, when compared with $\hat{\mathbf{x}}$, causes the erasure.

Following the method used in deriving Eq. (10), it can be shown that

$$\begin{aligned} P(\text{erasure}|\mathbf{x}=\hat{\mathbf{x}}) &\simeq 4^{-L}\sum_{r=0}^{L}\frac{(L+r-1)!}{r!(L-1)!(L-r)!}T^{L-r} \\ &\quad \cdot\left(\prod_{k\in\eta}\frac{1+(1-|\mu|^2)\gamma_s}{|\mu|^2|x_k-\hat{x}_k|^2\gamma_s/4}\right). \end{aligned} \quad (18)$$

The remaining terms in Eq. (17) can be bounded as follows. The correct decoding probability $P(\hat{\mathbf{x}}=\mathbf{x}) \leq 1$, and $P(\text{erasure}|\hat{\mathbf{x}}\neq\mathbf{x}) \leq 1$. The pairwise error probability $P(\hat{\mathbf{x}}\neq\mathbf{x})$ is obtained by setting $T=0$ in (11). Using union bound arguments and Eq. (17), the erasure probability at the output of the inner decoder is

$$P_e \leq \sum_{j=1}^{\infty}\psi_1(L_j,T)\left(\prod_{k\in\eta_j}\frac{1+(1-|\mu|^2)\gamma_s}{|\mu|^2|x_{jk}-\hat{x}_{jk}|^2\gamma_s/4}\right). \quad (19)$$

where

$$\begin{aligned} \psi_1(L,T) &\triangleq 4^{-L}\left[\frac{(2L-1)!}{(L-1)!(L-1)!} + \right. \\ &\quad \left.\sum_{r=0}^{L}\frac{(L+r-1)!}{r!(L-1)!(L-r)!}T^{L-r}\right]. \end{aligned} \quad (20)$$

This bound can be specialized to the three detection methods following Sections (3.1.)-(3.3.), though the details will not be shown here.

4. CONVENTIONAL VA

For this case, the error expressions developed earlier can be directly used with the setting $T=0$. Since no erasure information is generated at the output of the Viterbi decoder, the RS decoder performs errors-only decoding. Let t_2 the maximum number of symbol errors that can be corrected (the error correcting capability) of the (n_2,k_2) RS code. If the number of symbol errors in a codeword is greater that t_2, decoding fails. Assuming decoding symbol errors occur independently in RS decoding, the decoded block error probability is given by

$$P_{bl} \simeq \sum_{i=t_2+1}^{n_2}\binom{n_2}{i}P_s^i(1-P_s)^{n_2-i}, \quad (21)$$

which is used to find the decoded bit error probability [4].

4.1. Erasures and Errors

In this case, the inner decoder output symbols can contain both erasures and errors. Let P_s and P_e be the symbol error probability and symbol erasure probability at the input of the outer decoder. The maximum number of erasures that can be allowed within a codeword is given by

$$T_{es} \leq d_2 - 1. \quad (22)$$

Let i and j designate, respectively, the number of symbol erasures and the number of symbol errors in a codeword of the (n_2,k_2) RS code. If $i > T_{es}$, the outer decoder will erase the entire codeword. Otherwise, the decoder is capable of correcting the received codeword if $i \leq T_{es}$ and $j \leq t(i)$ where

$$t(i) = \left\lfloor\frac{d_2-i-1}{2}\right\rfloor. \quad (23)$$

The block-error probability is upper bounded by using [5, (22)], which is related to the decoded bit error probability.

5. ARQ SYSTEMS

Trellis-coded hybrid automatic-repeat-request (TCM-HARQ) protocols, as defined in [6], work as follows. At the transmitter, the TCM encoder output is transmitted as N-symbol packets. Each received packet is decoded with a Viterbi decoder; the test given in (4) is used to generate retransmission requests. That is, if the received packet satisfies (4), it is accepted and delivered to the data sink. Otherwise, the packet is rejected and a retransmission is requested. This process continues until a reliable packet is decoded.

Theoretically, this protocol might need an infinite number of retransmissions before a packet is accepted. Therefore, the probability $P(E)$ that an accepted packet contains errors is given by [10]

$$P(E) = \frac{P_{err}}{1 - P_r} \quad (24)$$

where P_r is the probability of retransmission of any packet and P_{err} is the probability that an accepted packet on any transmission contains errors. Clearly, P_{err} is given by (11) and by informationally weighting each error event, we obtain (13), which is the bit error probability of any given packet. Therefore, the average bit error probability, taking into account the possibility of retransmissions, is

$$P_b \leq \frac{1}{k_1}\left(\frac{1}{1-P_r}\right)\sum_{j=1}^{\infty} m_j \psi(L_j, T) \cdot \left(\prod_{k\in\eta_j} \frac{1+(1-|\mu|^2)\gamma_s}{|\mu|^2|x_{jk}-\hat{x}_{jk}|^2\gamma_s/4}\right). \quad (25)$$

The retransmission requests are generated in accordance with (4). Consequently, the retransmission probability is

$$P_r \leq \sum_{j=1}^{\infty} \psi_1(L_j, T)\left(\prod_{k\in\eta_j} \frac{1+(1-|\mu|^2)\gamma_s}{|\mu|^2|x_{jk}-\hat{x}_{jk}|^2\gamma_s/4}\right). \quad (26)$$

Then the throughput is

$$\eta = \left(\frac{k_1}{k_1+1}\right)(1 - P_r). \quad (27)$$

6. RESULTS

To illustrate the results derived in this chapter, the eight-state TCM in [11] combined with 8-PSK modulation is considered.

First, to establish the accuracy of the approximations developed thus far, consider the error event of length two between the two codewords $\mathbf{x} = (1, 1, \ldots)$ and $\hat{\mathbf{x}} = (e^{j\pi/2}, e^{j\pi}, 1, \ldots)$. For the Rayleigh fading channel, Fig. 2 depicts the exact PEP and the approximation (11) as functions of the signal-to-noise ratio and the threshold T. The exact PEP is computed by evaluating Eq. (9). As might be expected, the accuracy of the approximations increases as the signal-to-noise ratio ratio increases and T decreases.

Figure 3 shows the byte error probability for error and erasure decoding as a function of T and $\overline{E}_b/N_0$. Paradoxically, increasing $\overline{E}_b/N_0$ *increases* the bit error rate until it peaks and then declines. The initial behaviour is due to the fact that the increasing erasure probability can increase the block error probability.

Figure 4 depicts the throughput of the TCM-HARQ scheme in a Rayleigh fading channel where ideal coherent detection is employed. The individual curves are labelled with the threshold T. Increased T causes a reduction in throughput due to the increased retransmissions.

For the same case, Figure 5 shows the average bit error probability as a function of the threshold. Increased reliability is achieved by increasing T, at the cost of reduced throughput.

7. CONCLUSIONS

Easy-to-compute, asymptotic expressions for the probabilities of pairwise error event and erasure were derived. The analysis considered the generation of side information by modifying the conventional Viterbi algorithm and the correction of channel phase errors by using a phase reference. These expressions were used for evaluating the performance of concatenated coding systems and ARQ systems.

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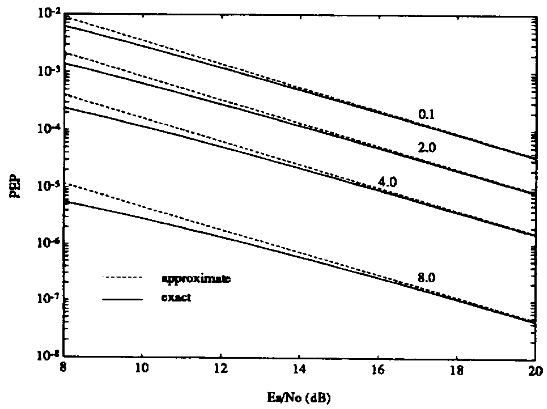

Figure 2: Comparison of exact and approximate PEP (coherent detection).

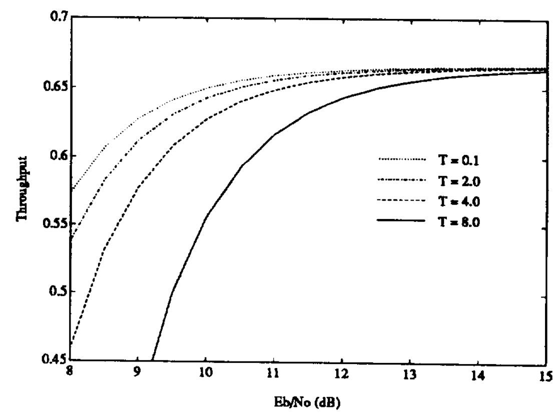

Figure 4: Throughput performance (coherent detection).

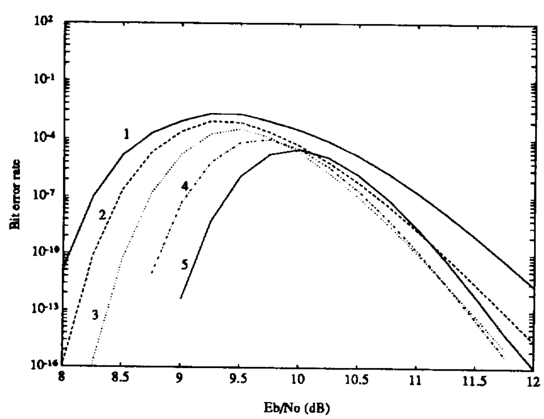

Figure 3: Bit error performance for errors and erasures decoding (coherent detection). 1: $T = 1.0$, 2: $T = 2.0$, 3: $T = 3.0$, 4: $T = 4.0$, 5: $T = 5.0$.

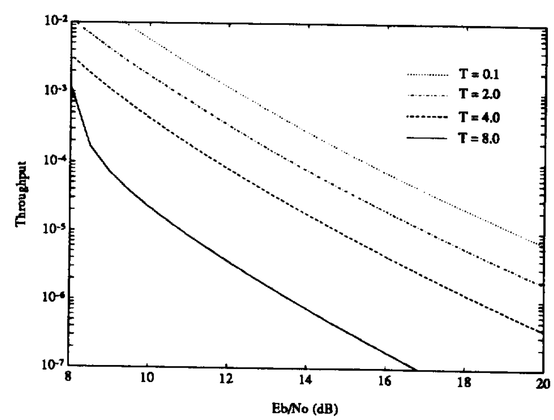

Figure 5: Bit error performance (coherent detection).