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Unified error analysis of DQPSK in fading channels

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Indexing terms: Error statistics, Fading, Phase shift keying

A general result is derived for the bit error rate (BER) of differential quadrature phase shift keying (DQPSK) for reception in slow fading and additive white Gaussian noise (AWGN). Fading models include Rayleigh, Rician, Nakagami and shadowed Rician, which describe a wide range of fading conditions encountered in mobile communications. Easily computable, the result is potentially useful in evaluating the BER of such systems.

Introduction: Because the US and Japanese digital cellular system standards recommend using DQPSK modulation, recently there has been a flurry of research activity regarding the performance of such systems in fading channels. This Letter focuses on the slow fading case, where the data rate is significantly higher than the rate at which the channel gain varies. Numerous models have been used in the literature to describe the fading of the amplitude of the received signal. Typically, the Rician fading model is used for microcellular and mobile satellite channels, Rayleigh for macrocellular channels, Nakagami [1] for urban multipath channels, and the shadowed Rician (SR) for the Canadian mobile satellite channel [2]. The BER of DQPSK for Rician fading is derived in [3] and for Nakagami fading in [4], where both results are given as an infinite series of integrals which must be evaluated numerically. This Letter, however, derives a general expression for the BER given by a single finite integral.

Analysis: The probability density function (PDF) of the signal amplitude ρ for each fading model is expressed as

$$\begin{aligned} P(\rho) &= \frac{\rho}{\sigma^2}\exp\left(-\frac{(\rho^2+A^2)}{2\sigma^2}\right) I_0\left(\frac{\rho A}{\sigma^2}\right) && \text{Rician} \\ &= \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m \rho^{2m-1}\exp\left(-\frac{m\rho^2}{\Omega}\right) && \text{Nakagami} \\ &= \frac{\rho}{b_0\sqrt{2\pi d_0}}\int_{-\infty}^{\infty}\exp\left(-\left(\frac{\rho^2+e^{2(\mu_0+t)}}{2b_0}+\frac{t^2}{2d_0}\right)\right) \\ &\qquad \times I_0\left(\frac{\rho e^{(\mu_0+t)}}{b_0}\right)dt && \text{SR} \end{aligned} \tag{1}$$

where, for Rician fading, A is the direct signal component, $2\sigma^2$ is the power of the multipath component and $I_0(x)$ is the zero-order Bessel function of the first kind; for Nakagami fading, $\Gamma(m)$ is the gamma function, Ω is the mean square value of ρ and $m \geq 0.5$ is the fading severity parameter; for SR fading, $2b_0$ is the power of the multipath signal, and the logarithm of the direct signal is Gaussian with mean μ_0 and variance d_0. For the Rician model, the ratio of direct and multipath signal powers is usually referred to as the K factor, given by $K = A^2/(2\sigma^2)$. The Rayleigh PDF can be obtained as a special case of either Rician or Nakagami fading, i.e. when there is no direct signal component, i.e. $A = 0$ or $m = 1$.

The instantaneous signal/noise ratio is defined as $\gamma = (\rho^2 T)/(2N_0)$ where T is the symbol duration and N_0 is the one sided power spectral density of the AWGN in W/Hz units. According to this definition, the mean value of γ depends on the mean value of ρ^2, which can be evaluated from eqn. 1. Therefore the average SNR $\bar{\gamma}$, which determines the average BER, is

$$\bar{\gamma} = \begin{cases} (1+K)\sigma^2 T/N_0 & \text{Rician} \\ \Omega T/(2N_0) & \text{Nakagami} \\ (1+\exp\{2(\mu_0+d_0)\}/(2b_0))b_0 T/N_0 & \text{SR} \end{cases} \tag{2}$$

Because the BER of DQPSK with Gray coding in AWGN can be expressed in terms of the generalised Q-function and a Bessel function ([5], eqn. 4.2.118), previous derivations [3, 4] of the effect of fading on the BER have used this formula. However, here we use an alternative formula, and the reason for this choice will be apparent shortly. By using a result from eqn. 7 in [6], the BER for this case can be alternatively expressed as

$$P_b(\rho) = \frac{1}{2\pi}\int_0^{\pi}\frac{\exp\{-(2-\sqrt{2}\cos(\phi))\gamma\}}{\sqrt{2}-\cos(\phi)}d\phi \tag{3}$$

The average BER can be expressed as follows:

$$P_b = \int_0^{\infty} P_b(\rho)P(\rho)d\rho \tag{4}$$

By substituting eqns. 1 and 3 in eqn. 4, we have a double integral of (ρ, ϕ), which is integrated first over ρ from 0 to ∞ to obtain P_b as an integral of ϕ. To evaluate this we first define the function

$$\psi(\alpha,\beta,\delta,n) \equiv \frac{\alpha}{2\pi}\int_0^{\pi}\frac{\exp\left(\frac{\beta}{1-\delta\cos(\phi)}\right)}{(\sqrt{2}-\cos(\phi))(1-\delta\cos(\phi))^n}d\phi \qquad |\delta|<1 \tag{5}$$

As will been seen shortly, the results for all the fading models can be expressed in terms of this function. We should also add that because this integral can be evaluated with a simple numerical method, there is really no need to obtain a series expansion for it, which is possible for certain specific cases.

For Rician fading, with $P(\rho)$ given in eqn. 1, by integrating eqn. 4 over ρ, with the help of the identity that the integral of $xe^{-p^2x^2}I_0(ax)$ from $x = 0$ to ∞ is equal to $\exp\{a^2/(4p^2)\}/(2p^2)$, we obtain the average BER as

$$P_b = \psi\left(\left(\frac{(1+K)\exp(-K)}{1+K+2\bar{\gamma}}\right),\frac{K(1+K)}{1+K+2\bar{\gamma}},\frac{\sqrt{2}\bar{\gamma}}{1+K+2\bar{\gamma}},1\right) \tag{6}$$

For Nakagami fading, with $P(\rho)$ given in eqn. 1, by integrating eqn. 4 over ρ, with the help of the indentity that for $n > 0$, $p > 0$ the integral of $e^{-px}x^{n-1}$ from $x = 0$ to ∞ is equal to $\Gamma(n)/p\nu$, we obtain the average BER as

$$P_b = \psi\left(\left(\frac{m}{m+2\bar{\gamma}}\right)^m,0,\left(\frac{\sqrt{2}\bar{\gamma}}{m+2\bar{\gamma}}\right),m\right) \tag{7}$$

This also implies that, for large SNRs ($\bar{\gamma} \to \infty$, $\delta \to 1/\sqrt{2}$ and $\alpha \to (m/(2\bar{\gamma}))^m$), P_b varies asymptotically as the inverse mth power of $\bar{\gamma}$. This suggests that a Nakagami-m channel resembles a Rayleigh fading channel with m orders of diversity.

For the nonfading case, i.e. $K \to \infty$ or $m \to \infty$, we can readily show that eqns. 6 and 7 reduce to eqn. 3. Note that for Rayleigh fading, by substituting either $K = 0$ in eqn. 6 or $m = 1$ in eqn. 7 and evaluating the resulting integral in eqn. 5, we have

$$P_b = \frac{1}{2}\left(1-\frac{\sqrt{2}\bar{\gamma}}{\sqrt{1+4\bar{\gamma}+2\bar{\gamma}^2}}\right) \tag{8}$$

This expression is identical to eqn. 18 in [3].

For the SR model, BER evaluation is more complicated, since the PDF itself is represented as an integral. This implies that direct evaluation of eqn. 4 now needs a triple integral. We can show that the SR PDF in eqn. 1 can be expressed as a Rician PDF with $A = \exp(t)$ where t is Gaussian with mean μ_0 and variance d_0. Thus, $K = A^2/(2\sigma^2)$ in eqn. 6 is now a random variable, so we define $K_s = \exp(2t)/(2b_0)$. The BER is then

$$P_b = E\left\{\psi\left(\left(\frac{(1+K_s)\exp(-K_s)}{1+K_s+2\bar{\gamma}}\right), \frac{K_s(1+K_s)}{1+K_s+2\bar{\gamma}}, \frac{\sqrt{2}\bar{\gamma}}{1+K_s+2\bar{\gamma}}, 1\right)\right\} \quad (9)$$

where $E(x)$ is the average over t.

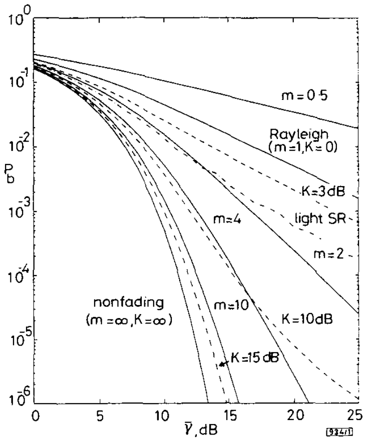

Fig. 1 *BER performance of DQPSK in Rician, Nakagami and SR fading and AWGN*

- - - - Rician
——— Nakagami
- · - · SR

Results and conclusions: Fig. 1 shows the BER of DQPSK in the aforementioned fading channels. The parameters for the SR model are obtained from [2]. For Nakagami fading, we find that the asymptotic slope is m.

We have presented a new, general result for the BER of DQPSK for reception in slow fading and AWGN. The result is computationally simple and is potentially useful in evaluating the performance of DQPSK systems that operate on a wide variety of fading channels.

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Correction of the PO current density close to perfectly conducting wedges by the UTD

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Indexing terms: Geometrical theory of diffraction, Physical optics

A correction to the conventional physical optics (PO) current density based on the uniform geometrical theory of diffraction (UTD) is proposed in the Letter to account for the effects of the edges of perfectly conducting wedges. This improved high frequency current approximation is well suited for combination with the method of moments, resulting in a current-based hybrid formulation.

Introduction: Current-based hybrid formulations represent a suitable method for the analysis of electromagnetic scattering problems over a wide frequency range. An overview is given in [1]. Usually PO currents, currents based on the physical theory of diffraction (PTD) or Fock-currents [2], are employed in the asymptotic region. Here we propose PO currents with superimposed correction terms to account for the influence of edges of perfectly conducting wedges of the scattering body. In contrastto an application of the PTD, only surface current densities and no additional electric and magnetic line currents along the edges are involved. A similar modification for the edges of flat polygonal plates based on Sommerfeld's exact solution for the half-plane scattering problem has already been presented by the authors in [3].

Correction of PO current: The conventional PO current density $\mathbf{J}^{PO} = 2\hat{n} \times \mathbf{H}_i$ in the lit, and $\mathbf{J}^{PO} = \mathbf{0}$ in the shadowed, part of the scattering body, respectively, is heuristically corrected by terms $\mathbf{J}_m^{UTD}$ corresponding with the $m = 1 \ldots N_w$ wedges of finite length, representing the boundary of the surface under consideration:

$$\mathbf{J}^{CPO} = \mathbf{J}^{PO} + \sum_{m=1}^{N_w} \mathbf{J}_m^{UTD} \quad (1)$$

Here an upper index CPO has been introduced to indicate the corrected PO current density. Application of the UTD yields the correction terms

$$\mathbf{J}_{n,o}^{UTD} = \pm A(s) e^{-jks} \mathbf{H}_i(Q_E) \left[\hat{\beta}_0' \hat{s} D_h - \hat{\phi}' \hat{\beta}_0 \frac{1}{jks} \frac{\partial D_s}{\partial \phi}\right] \quad (2)$$

where the index m has been omitted. The notation of [4] has been used in eqn. 2. $A(s)$ describes how the amplitude varies, $\mathbf{H}_i(Q_E)$ denotes the incident magnetic field at the diffraction point Q_E on the edge, which may be caused by the excitation or, in context with a hybrid method, by the currents radiating in the MM-region [3]. $D_{s,h}$ represent the scalar diffraction coefficients for soft and hard polarisation, respectively. The positive sign in eqn. 2 refers to the n-face at $\phi = n\pi$, and the negative sign to the o-face of the wedge at $\phi = 0$. See [4] for the definition of ϕ and n.

Note that approximate expressions for $\mathbf{J}^{UTD}$ have already been published by Schretter and Bolle [5]. These are in good agreement with our results even very close to the edge of the wedge.

Example: Fig. 1 shows a perfectly conducting cube with sidelength 2λ. A z-polarised plane electromagnetic wave propagating in the positive y-direction acts as the excitation. However, to distinguish uniquely between lit and shadowed surfaces, we assume the direction of incidence as specified by $\vartheta_i = \pi^-/2$ and $\varphi_i = 3\pi^-/2$, where the upper index '–' indicates that, for example, ϑ_i approaches $\pi/2$ but is slightly smaller than $\pi/2$ in the limiting case. Consequently, the three surfaces of the cube at $x = -\lambda$, $y = -\lambda$, and $z = \lambda$ are illuminated by the plane wave, whereas the remaining three surfaces are shadowed. The PO approximation yields a surface current density $\mathbf{J}^{PO} = 2H_i\hat{z}$ on the surface at $y = -\lambda$, a value of $\mathbf{J}^{PO} = 2He^{-jky}\hat{y}$ on the surface at $z = \lambda$, and $\mathbf{J}^{PO} = \mathbf{0}$ on the remaining four sides. The magnitude of this PO current density is represented by the grey scale shading in Fig. 1. A reference solution based on the method of moments is depicted in Fig. 2. A comparison clearly demonstrates the deficiencies of the conventional PO approximation.