Unified error analysis of DQPSK in fading channels

C. Tellambura and V.K. Bhargava

Indexing terms: Error statistics, Fading, Phase shift keying

A general result is derived for the bit error rate (BER) of differential quadrature phase shift keying (DQPSK) for reception in slow fading and additive white Gaussian noise (AWGN). Fading models include Rayleigh, Rician, Nakagami and shadowed Rician, which describe a wide range of fading conditions encountered in mobile communications. Easily computable, the result is potentially useful in evaluating the BER of such systems.

Introduction: Because the US and Japanese digital cellular system standards recommend using QPSK modulation, recently there has been a flurry of research activity regarding the performance of such systems in fading channels. This Letter focuses on the slow fading case, where the data rate is significantly higher than the symbol rate, and the channel properties change slowly, over a time period of several symbol intervals. This Letter, however, derives a general expression for the BER given by

\[ P_b = \frac{1}{2\pi} \int_0^{2\pi} \exp\left(-\frac{1}{2}(\sin(\theta))^2\right) d\theta \]

The average BER can be expressed as follows:

\[ P_b = \frac{1}{2\pi} \int_0^{2\pi} P_e(p) d\theta \]

For Nakagami fading, with \( P_e(p) \) given in eqn. 1, by integrating eqn. 4 over \( p \), with the help of the identity that the integral of \( \exp(-ax) \) from \( x = 0 \) to \( \infty \) is equal to \( \frac{1}{a} \), we obtain the average BER as

\[ P_b = \frac{1}{2\pi} \int_0^{2\pi} P_e(p) d\theta \]

For Nakagami fading with \( P_e(p) \) given in eqn. 1, by integrating eqn. 4 over \( p \), with the help of the identity that for \( n > 0, p > 0 \) the integral of \( e^{-ax} \) from \( x = 0 \) to \( \infty \) is equal to \( \gamma \), we obtain the average BER as

\[ P_b = \frac{1}{2\pi} \int_0^{2\pi} P_e(p) d\theta \]

This also implies that, for large SNRs \( \gamma \rightarrow \infty, n \rightarrow \infty \), and \( m \rightarrow (m(2^n))^{1/2} \), \( P_b \) varies asymptotically as the inverse n-th power of \( \gamma \). This suggests that a Nakagami-m channel resembles a Rayleigh fading channel with m orders of diversity.

For the nonfading case, i.e. \( K \rightarrow 0 \) or \( m = 1 \), we can readily show that eqns. 6 and 7 reduce to eqn. 3. Note that for Rayleigh fading, by substituting either \( K = 0 \) in eqn. 6 or \( m = 1 \) in eqn. 7 and evaluating the resulting integral in eqn. 5, we have

\[ P_b = \frac{1}{2\pi} \int_0^{2\pi} \exp(-\frac{1}{2}(\sin(\theta))^2) d\theta \]

This expression is identical to eqn. 18 in [3].

For the SR model, BER evaluation is more complicated, since the PDF itself is represented as an integral. This implies that direct integration is impossible. For large SNRs, this can be alternatively expressed as

\[ P_b = \frac{1}{2\pi} \int_0^{2\pi} P_e(p) d\theta \]

This expression is identical to eqn. 18 in [3].

For the SR model, BER evaluation is more complicated, since the PDF itself is represented as an integral. This implies that direct integration is impossible. For large SNRs, this can be alternatively expressed as

\[ P_b = \frac{1}{2\pi} \int_0^{2\pi} P_e(p) d\theta \]

This expression is identical to eqn. 18 in [3].

For the SR model, BER evaluation is more complicated, since the PDF itself is represented as an integral. This implies that direct integration is impossible. For large SNRs, this can be alternatively expressed as

\[ P_b = \frac{1}{2\pi} \int_0^{2\pi} P_e(p) d\theta \]

This expression is identical to eqn. 18 in [3].

For the SR model, BER evaluation is more complicated, since the PDF itself is represented as an integral. This implies that direct integration is impossible. For large SNRs, this can be alternatively expressed as

\[ P_b = \frac{1}{2\pi} \int_0^{2\pi} P_e(p) d\theta \]

This expression is identical to eqn. 18 in [3].
Correction of the PO current density close to perfectly conducting wedges by the UTD

U. Jakobs and F.M. Landstorfer

Introduction: Current-based hybrid formulations represent a suitable method for the analysis of electromagnetic scattering problems over a wide frequency range. An overview is given in [1]. Usually, PO currents, based on the physical theory of diffraction (PTD) or Fock-currents [2], are employed in the asymptotic region. Here we propose PO currents with superimposed correction terms to account for the influence of edges of perfectly conducting wedges of the scattering body. In contrast to an application of the PTD, only surface current densities and no additional electric and magnetic line currents along the edges are involved. A similar modification for the edges of flat polygonal plates based on Sommerfeld’s exact solution for the half-plane scattering problem has already been presented by the authors in [3].

Correction of PO current: The conventional PO current density \( \mathbf{J}^{\text{PO}} = 2 \pi \mathbf{H}_0 \), in the lit, and \( \mathbf{J}^{\text{PO}} = 0 \) in the shadowed part of the scattering body, respectively, is heuristically corrected by terms \( \mathbf{J}^{\text{UTD}} \) corresponding with the m = 1 ... Nw wedges of finite length, representing the boundary of the surface under consideration:

\[
\mathbf{J}^{\text{CPO}} = \mathbf{J}^{\text{PO}} + \sum_{m=0}^{\infty} \mathbf{J}^{\text{UTD}}
\]

Here an upper index CPO has been introduced to indicate the corrected PO current density. Application of the UTD yields the correction terms

\[
\mathbf{J}^{\text{UTD}} = \pm A(\phi) e^{-i\pi m} \mathbf{H}_0 \left( \frac{\partial}{\partial \phi} - \frac{1}{2} \frac{\partial}{\partial \phi} \right) \mathbf{D}_w \]

where the index \( m \) has been omitted. The notation of [4] has been used in eqn. 2. \( A(\phi) \) describes how the amplitude varies, \( \mathbf{H}_0 \) denotes the incident magnetic field at the diffraction point \( Q_0 \) on the edge, which may be caused by the excitation or, in context with a hybrid method, by the currents radiating in the MM-region [3]. \( D_w \) represent the scalar diffraction coefficients for soft and hard polarisation, respectively. The positive sign in eqn. 2 refers to the n-face at \( \phi = \pi \), and the negative sign to the o-face of the wedge at \( \phi = 0 \). See [4] for the definition of \( \phi \) and \( m \).

Note that approximate expressions for \( \mathbf{J}^{\text{UTD}} \) have already been published by Schiesser and Bolle [5]. These are in good agreement with our results even very close to the edge of the wedge.

Example: Fig. 1 shows a perfectly conducting cube with side-length 2A. A z-polarised plane electromagnetic wave propagating in the positive y-direction acts as the excitation. However, to distinguish uniquely between lit and shadowed surfaces, we assume the direction of incidence as specified by \( \theta = \pi/2 \) and \( \varphi = \pi/2 \), where the upper index ‘c’ indicates that, for example, \( \theta \) approaches \( \pi/2 \) but is slightly smaller than \( \pi/2 \) in the limiting case. Consequently, the three surfaces of the cube at \( x = \lambda, y = \lambda \), and \( z = \lambda \) are illuminated by the plane wave, whereas the remaining three surfaces are shadowed. The PO approximation yields a surface current density \( \mathbf{J}^{\text{PO}} = 2 \pi \mathbf{H}_0 \) on the surface at \( y = \lambda \), and \( \mathbf{J}^{\text{PO}} = 0 \) on the remaining four sides. The magnitude of this PO current density is represented by the grey scale shading in Fig. 1. A reference solution based on the method of moments is depicted in Fig. 2. A comparison clearly demonstrates the deficiencies of the conventional PO approximation.