photon-induced start pulse, the time interval was terminated by the first following pulse derived from the electrical drive to the laser. Fig. 2 shows a typical distribution of start pulse to stop pulse time intervals plotted as a function of time. The three temporal windows in which the photo-counts occur are separated by 11 ns as expected, and only the amplitude of the central peak varied when the phase shift in the interferometer was adjusted. Consequently, the system had sufficient time resolution to discriminate between interfering and noninterfering events. The amplitude of the single-photon interference fringes was investigated by scanning the PZT position point by point and integrating the number of photo-counts occurring in the central peak. In addition, at each position the dark count background was estimated from data lying outside the photo-count time window and subtracted from the total count.

Fig. 2 Time-interval distribution between single-photon detection start pulses and (first following) laser drive pulses

A typical series of experiments was preceded by the adjustment of two fibre-polarisation controllers in the interferometer (not shown in Fig. 1 for clarity), in order to match the polarisation states of the interfering pulses. Single photon data were then acquired as a function of PZT drive voltage. For each PZT voltage, the number of counts in the central peak was normalised to the number of counts obtained from the noninterfering pulses, and the dark count background was subtracted. Fig. 3 shows a typical single photon fringe pattern obtained in this way. A fringe visibility of 91% was calculated from the observed maxima and minima, and the solid line shows a cosinusoidal fit to the data using this value. There are indications of a variation in periodicity in the data, and this probably arises from small amounts of drift in the relative path lengths in the interferometer due to environmental fluctuations which occur during the data acquisition time. The fringe visibility obtained without background subtraction was still relatively high at 78%, indicating that the dark count fraction was relatively small. At the fringe maxima, the number of photo-counts occurring in the central peak corresponded to an average count rate of 1.1 kHz. The Letter derives a Chernoff based error bound on the performance of trellis coded modulation (TCM) schemes operating on this channel.

Conclusion: In summary, we have demonstrated a prototype single photon channel that has the potential to form the basis of a future quantum cryptographic system. Relatively high single-photon count rates of ~20 kHz were obtained in the experiment, suggesting that data rates of this order are potentially achievable in a system based on the current design. We note that because the objective of a quantum cryptographic channel is to establish a relatively short secret key (perhaps a few hundred bits), rather than the transmission of the encrypted data itself, a data rate of order 20 kHz may be more than adequate.

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P. D. Townsend (BT Laboratories, Martlesham Heath, Ipswich, Suffolk IP5 7RE, United Kingdom)
J. G. Rarity and P. R. Tapster (DRA Malvern, St Andrews Road, Malvern, Worcestershire WR14 3PS, United Kingdom)

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TRELLIS CODED MODULATION SCHEMES FOR SHADOWED RICIAN FADING CHANNELS

C. Tellambura and V. K. Bhargava

Indexing terms: Trellis codes, Modulation, Satellite links

The Canadian mobile satellite (MSAT) channel has been modelled as the sum of lognormal and Rayleigh components to represent foliage attenuation and multipath fading, respectively. The Letter derives a Chernoff based error bound on the performance of trellis coded modulation (TCM) schemes operating on this channel.

Introduction: The Canadian mobile satellite (MSAT) channel has been modelled as the sum of lognormal and Rayleigh components, by which foliage attenuation and multipath fading are represented [1, 2]. As such, it is closely related to a Rician channel. Typically, mobile satellite channels have been modelled as Rician fading channels. The shadowed Rician model appears to be a better representation for the MSAT...
Hence, to approximate this integral, by substituting $\log(R\text{ayleigh distributed})$, $J(\phi)$ for light shadowing, for example, for variance the degree of shadowing three cases have been identified, and as [Reference 3] where binary input data are convolutedly encoded at rate $n/(n+1)$ the encoded $n+1$ bit words are block interleaved and mapped into a sequence $x = (x_1, x_2, \ldots, x_n)$ of $M$-ary PSK symbols, which constitute a normalized constellation, i.e., $|x|^2 = 1$ for all symbols. For simplicity, we assume ideal interleaving/deinterleaving. The corresponding channel output sequence is [3]

$$y_k = p_k x_k + n_k$$

where $p_k$ denotes the random fading amplitude, and $n_k$ is a complex Gaussian noise sample with zero mean and variance $(2E_b/N_0)^{-1}$. According to the shadowed Rician fading model, each $p_k$ is described by the probability density function (PDF) [2]

$$p(p_k) = \frac{\mu}{b_0/(2\pi d_k)} \int_{0}^{z} \exp \left[ (-\mu z^2 + 2\mu x + z^2) \right] \frac{z^2}{b_0} dz$$

for $0 \leq \mu < \infty$. Here, $z = e^t; x$ is Gaussian with mean $\mu_0$ and variance $d_0$, $b_0$ is the variance of the multipath component (Rayleigh distributed), $\log(\cdot)$ is the natural logarithm, and $I_0(\cdot)$ is the zero order modified Bessel function. For light shadowing, for example, $b_0 = 0.158, \mu_0 = 0.115$, and $\sqrt{d_0} = 0.115$ [2].

The PDF in eqn. 2 is difficult to handle mathematically. Hence, to approximate this integral, by substituting $z = \mu_0 = t$, it can be converted to Laplace type:

$$p(p_k) = \frac{\mu}{b_0/(2\pi d_k)} \int_{0}^{\infty} \exp \left[ (-\mu z^2 + 2\mu x + z^2) \right] \frac{z^2}{b_0} dz$$

$$= \int_{-\infty}^{\infty} g(t) \exp \left[ -(y^2) \right] dt$$

where $\gamma = 1/(2d_0)$ and $g(t)$ denotes the rest of the integrand in eqn. 3. Because $\gamma$ is quite large for both light and average cases, this integral can be approximated in terms of $g(0), g'/(0)$ and $\gamma$. Hence, using the second order Laplace approximation (Reference 4, eqn. 53) for this integral, we have

$$p(p_k) = \frac{\mu}{b_0} \left[ c_0 g(p_k) - \epsilon c_1 g'(p_k) \right] + \epsilon c_2 p_k^2 g(p_k) + \epsilon c_3 p_k^4 g''(p_k)$$

$$\epsilon = -1/(2\gamma^2)$$

where $c_0 = 1 + 0.5d_0 p_k^2 - 2\mu_0, c_1 = 0.5d_0 p_k^2, c_2 = 0.25d_0 p_k^4, \epsilon = (p_k)^2, \epsilon = c_1, \epsilon = c_2, \epsilon = c_3, \epsilon = c_4$. The pairwise error probability is bounded as follows:

$$P(x \rightarrow \hat{x}) \leq \prod_{k=1}^{n} \exp \left[ -\frac{E_b}{4N_0} (x_k - \hat{x}_k)^2 \right]$$

$$\times \exp \left[ -\frac{\epsilon (x_k - \hat{x}_k)^2}{1 + b_0 E_b/2N_0} \right]$$

$$\leq \exp \left[ -\frac{\epsilon (x_k - \hat{x}_k)^2}{1 + b_0 E_b/2N_0} \right]$$

In deriving eqn. 7, we have made use of certain identities involving Bessel functions, and these can be found in Reference 4.
Example: In this study, we use a rate 2/3, eight-state binary convolutional encoder (see Reference 6, Fig. 7) to confirm the accuracy of eqn. 7. The transfer function for this code is given in Reference 6, eqn. 19, and the exact Chernoff bound, computed by integration of eqn. 46, Reference 6 using quadrature techniques, or eqn. 7 can be used with the transfer function bound.

Fig. 1 presents $P_b$ against $E_b/N_0$ performance for this code, and we observe that eqn. 7 is virtually indistinguishable from the exact result. As for computational speed, computing eqn. 7 consumes negligible time in comparison to quadrature integration necessary for the exact result. The transfer function bound is within 1 dB of simulation points, confirming the validity of simulation points. Also shown is the performance of equivalent uncoded 4PAMSK. It is apparent that a significant coding gain is realisable in this case.

Conclusions: A new approximation for the Chernoff bound on the pairwise error probability of TCM schemes operating on the shadowed Rician fading channel has been derived. The approximation gives excellent accuracy and allows for easier bounding of the bit error probability, and so is useful in analysing the coded system performance of channels that are subjected to shadowing.

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C. Tellambura and V. K. Bhargava (Department of Electrical & Computer Engineering, University of Victoria, PO Box 3055, Victoria BC, V8W 3P6, Canada)

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GUIDED QUASISTATIC FIELDS IN ELECTROMAGNETIC MEASUREMENT-WHILE-DRILLING
M. Y. Xia and Z. Y. Chen

Indexing terms: Electromagnetic theory, Guided waves

The quasistatic fields of extremely low frequencies (ELF) employed to communicate from underground to the ground surface in the electromagnetic measurement-while-drilling (EM-MWD) system are investigated. Three types of well, the vertical well, directional well and horizontal well, are considered. Computed results are obtained for various parameters, including the operating frequencies and the Earth's conductivities. It is demonstrated that long drill strings have a guiding effect which is advantageous to the data transmission.

Introduction: In spite of the fact that the electromagnetic measurement-while-drilling (EM-MWD) system has been studied for several decades, it is fair to say that this method is still only in its infancy. It is a real time data transmission system consisting of two parts, the underground part and the surface part. The sophisticated device of the underground part, the drill string, acts as a transmitter and is fixed inside the narrow drill string or drill collar. The surface part, acting as a receiver, detects the signals and decodes the quantities that are being measured.

Compared with another system, the so-called mud-pulse method (MPM), the EM-MWD system has three main advantages:

1. it is a real time system, and does not need drilling to be stopped
2. it has higher data rates and can measure more varieties of quantities
3. particularly because of the much greater commercial value of directional and horizontal wells, which are increasingly being drilled in practice, the EM-MWD system is becoming more desirable, as MPM is effective only for vertical wells and cannot be used for these types of well. The models for the three types of well under consideration are shown in Fig. 1a-c (the mud is not shown in the latter two types). The measured quantities, which may include the moment of torsion of the drill string, the underground temperature and pressure, are first transformed into electric signals by different sensors, then the signals are applied to an 'isolation' gap between the drill string and the drill collar and transmitted to the surface by electromagnetic means. They are the E-field excitations. It has been proven that E-field excitations are more efficient than H-field excitations (a ring of magnetic current). The measurement is accomplished by determining the voltage between the upper drill-string and an electrode placed beneath the surface at a distance.

Fig. 1 Three well models
a Vertical well
b Directional well
c Horizontal well

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