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CONVOLUTIONAL CODING FOR BPSK WITH PILOT TONE OVER RAYLEIGH FADING CHANNELS

C. Tellambura and V. K. Bhargava

Indexing terms: Digital communication systems, Coding, Phase-shift keying

To improve the performance of digital communication systems over fading channels, several pilot signal techniques have been proposed in the literature. The Letter investigates the use of binary convolutional coding for such systems and derives an upper bound on the bit error probability (BEP), the optimum power split ratio between data and pilot signals, and the channel cutoff rate.

Introduction: To improve the performance of digital mobile communications over fading channels, the use of a pilot tone to provide a phase reference at the receiver has been suggested in the literature [1]. To maximise the effectiveness of the technique, both pilot and data signals must be subjected to the same fading. Also of importance is the fraction of total power allocated to the pilot. This Letter analyses interleaved convolutional coding for such schemes. An upper bound on the BEP, the optimum power split ratio between the data and pilot signals, and the channel cutoff rate are derived.

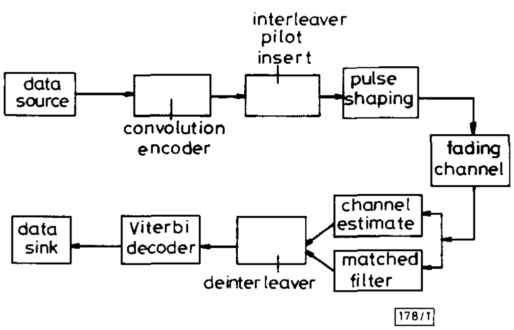

Fig. 1 *Coded BPSK with pilot tone*

Performance analysis: In the proposed system (Fig. 1), coded binary PSK symbols are interleaved, and transmitted along with the pilot tone. As proposed in Reference 2, a pilot tone extraction filter can be used to estimate the channel gain for each coded symbol. The channel gain estimates are then used to form the metric that is used in the Viterbi decoder, following deinterleaving. In the following, we assume perfect receiver timing, fading to be constant over a bit and, consequently, that there is no intersymbol interference. We also assume ideal interleaving/deinterleaving.

The transmitted signal can be represented by a complex envelope [2]

$$s(t) = A \sum_{k=-\infty}^{\infty} b_k p(t - kT_s) + B \tag{1}$$

where $p(t)$ is a unit energy pulse, A is the amplitude of the data signal, B is the amplitude of the pilot signal, T_s is the symbol duration, and b_k is the baseband equivalent of the convolutional encoder output, assuming values ± 1 (i.e. '0' to 1 and '1' to '-1'). Here we define the energy ratio between the data signal and the pilot signal as $r = B^2 T_s / A^2$.

The received data signal is detected with a matched filter whose impulse response is $p^*(-t)/\sqrt{(N_0)}$, where normalisation by the additive noise power spectral density N_0 is made for notational convenience [2]. The filter output is then given by

$$r_k = \frac{A\alpha_k}{\sqrt{(N_0)}} b_k + n_k \tag{2}$$

where n_k is the additive white Gaussian noise with zero mean and unit variance, and α_k is complex Gaussian with zero mean and variance σ^2.

As in Reference 2, the received pilot signal is detected with a matched filter with frequency response

$$H(f) = \begin{cases} \dfrac{A}{B\sqrt{(N_0)}} & -W_p/2 \le f \le W_p/2 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Once again, normalisation by the factor $A/[B\sqrt{(N_0)}]$ is made for notational convenience. To allow for undistorted measurement of the fading process, the bandwidth of the pilot extraction filter should be chosen such that $W_p = 2f_d$ where f_d is the maximum Doppler spread. The output of the pilot extraction filter is then given by

$$v_k = \frac{A\alpha'_k}{\sqrt{(N_0)}} + \eta_k \tag{4}$$

where η_k is a complex Gaussian with zero mean and a variance of $W_p A^2/B^2 = W_p T_s/r$. η_k is uncorrelated with the noise term n_k in eqn 2. α'_k is the Rayleigh fading term affecting the pilot, which is assumed to be strongly correlated with (if not equal to) α_k in eqn. 2. Let $\rho = \rho_r + i\rho_i$ be the correlation coefficient between them. The received signal energy to noise-spectral density ratio (SNR) can be defined, including both the data and pilot signals, as

$$\gamma_s = \frac{E_s}{N_0} = \frac{\sigma^2 A^2}{N_0} + \frac{\sigma^2 B^2 T_s}{N_0} \tag{5}$$

Turning now to the decoder, with the pilot-tone estimates v_k, the Viterbi decoder uses the metric $|r_k - v_k b_k|^2$, which is the maximum likelihood metric if v_k is a perfect estimate of α_k. When evaluating the bit error probability of linear convolutional codes, it is customary to assume, without loss of generality, that all-zero codeword $\mathbf{0} = (111 \ldots)$ is transmitted. Thus, an error event occurs when the decoder chooses a nonzero codeword $\boldsymbol{x}_j \neq \mathbf{0}$. Let $\{x_{j1}, x_{j2}, x_{j3}, \ldots\}$ denote the baseband representation of $\boldsymbol{x}_j$ where $\{x_{jk} = \mp 1\}$ $(k = 1, 2, 3, \ldots)$. Thus, an error event happens if $\sum |r_k - v_k|^2 \ge \sum |r_k - v_k x_{jk}|^2$. Clearly, only the components $x_{jk} \neq 1$ contribute to this decision rule. On simplification of this decision rule, the decision variable is

$$U = \operatorname{Re}\left(\sum_{w_j} r_k v_k^*\right) \tag{6}$$

where w_j is the Hamming distance between $\mathbf{0}$ and $\boldsymbol{x}_j$. In the above, r_k and v_k are a pair of correlated complex Gaussian random variables, with all w_j pairs mutually statistically independent and identically distributed. To find an upper bound on the error event probability, using the Chernoff bound (Pr

$[U < 0] \le E(e^{sU}))$ and selecting s to minimise $E(e^{sU})$ [3, 4], we have

$$P_e(0 \to x_j) \le \frac{1}{2}\left[\frac{1}{1+\theta}\right]^{w_j} \tag{7}$$

where

$$\theta = \frac{\gamma_s^2 \rho_r^2}{\gamma_s^2(1 - |\rho|^2) + (1 + r)\gamma_s + W_p T_s \dfrac{(1+r)^2}{r} + W_p T_s \gamma_s \dfrac{1+r}{r}} \tag{8}$$

Note that γ_s, as defined by eqn. 5, accounts for the power allocated to the pilot as well. To derive the above, the variances of r_k and v_k and the correlation between them are obtained from eqns. 2 and 4. The factor of $\frac{1}{2}$ appearing in eqn. 7 follows from Reference 4. To obtain an upper bound on the BEP, eqn. 7 and union bounding methods can be used.

Minimising the denominator of θ with respect to r, it follows that the optimum choice of power split ratio r between data and pilot signals is

$$r_{opt}^2 = \frac{W_p T_s(1 + \gamma_s)}{(W_p T_s + \gamma_s)} \simeq W_p T_s \tag{9}$$

Note that as γ_s increases, r_{opt} is solely dependent on $W_p T_s$. Thus, the power allocated to the pilot tone need not be dynamically changed with increasing SNR. This has also been observed in References 2 and 5.

Now the cutoff rate R_0 for a Rayleigh fading channel with pilot-tone aided detection may be defined as [4]

$$R_0 = 1 - \log_2\left[1 + \frac{1}{1+\theta}\right] \text{ bit/symbol} \tag{10}$$

R_0 can be considered as a practical upper bound on the channel code rate, and is obtained as a function of the system parameters. This allows for a quick appraisal of system performance irrespective of the specific convolutional code.

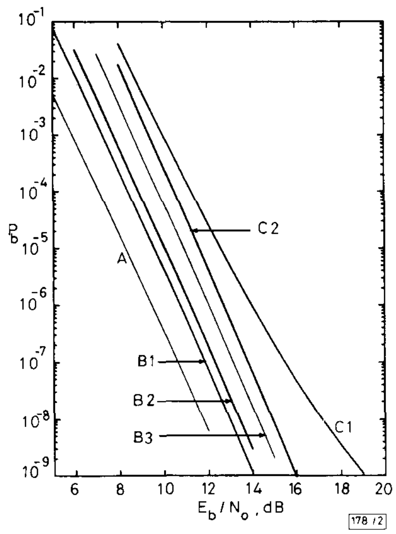

Fig. 2 *Bit error probability against E_b/N_0 ($R = 1/2$, $K = 7$)*

Rayleigh fading channel with different modulation techniques
A: ideal BPSK
B1: pilot tone 1%
B2: pilot tone 2%
B3: pilot tone 5%
C1: DPSK 5%
C2: ideal DPSK

Example: As an example, consider an optimal convolutional code with rate $R_0 = \frac{1}{2}$ and the constraint length $K = 7$. Using the weight distribution of this convolution code [4], the bit error upper bound is

$$P_b \le \tfrac{1}{2}[36D^{10} + 211D^{12} + 1404D^{14} + 11633D^{16} + \ldots]\Big|_{D=1/(1+\theta)} \tag{11}$$

The BEP curves of pilot tone technique are depicted in Fig. 2 for three normalised, maximum Doppler rates $f_d T_s$ of 1, 2 and 5%. As indicated by eqn. 9, the power split ratio between the pilot and the data signals r is 0·14, 0·2 and 0·32, respectively. By comparison, also shown are the BEP curves for ideal coherent detection and differential detection under ideal and fast fading conditions. In particular, for the pilot tone technique and ideal differential detection the correlation coefficient ρ is taken to be unity, whereas for differential detection with fast fading, $\rho = J_0(2\pi f_d T_s)$ [2]. We see that the pilot tone scheme is at most 3 dB worse than ideal, unattainable, coherent detection. It is also better than differential detection, even under very slow fading (i.e., $\rho \simeq 1$). Moreover, differential detection causes error floors, which in this case are suppressed by the powerful code being used.

Conclusions: This Letter analyses convolutional coding for binary PSK with a pilot tone for phase correction over Rayleigh fading channels. As might be anticipated, the performance of this scheme lies between that of ideal and differential detection techniques. At moderate and high SNRs, the power allocated to the pilot is essentially fixed by the maximum Doppler shift.

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SIDE-LIGHT-INJECTION MQW BISTABLE LASER USING SATURABLE ABSORPTION AND GAIN QUENCHING

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Indexing terms: Semiconductor lasers, Lasers

The characteristics of set-on and set-off operations by input light of the same wavelength have been observed in a side-light-injection MQW bistable laser. In this structure, the main bistable laser was located perpendicular to two waveguides for amplifying input light. Two intersections, the gain quenching region and the saturable absorption region, were spatially separated. Input light of 40 μW results in saturable absorption in one intersection biased at +0·65 V, and 570 μW causes gain quenching in the other intersection biased at +0·93 V.

Much attention has been given to all-optical memory devices because they are expected to be key elements in future telecommunications switching and optical signal processing