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Maximum Likelihood Detection in Communication using Metaheuristic Search Methods

by

Mehtaz Sharmin

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of **Master of Science**.

Department of Electrical and Computer Engineering

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled **Maximum Likelihood Detection in Communication using Metaheuristic Search Methods** submitted by Mehtaz Sharmin in partial fulfillment of the requirements for the degree of **Master of Science**.

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Dr. Chintha Tellambura (Supervisor)

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Dr. Ehab Elmallah (External)

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Dr. Masoud Ardakani

Date: _______________

To my parents.

Abstract

Maximum-Likelihood (ML) detection problem in communication is known to be NP(Non-deterministic Polynomial time)-hard. The computational complexity of solving ML detection is exponential in the size of the problem with exhaustive search that provides optimal solution. Several suboptimum algorithms have been proposed in the literature that provide reliable performance with reduced complexity. But still there is a large gap between the performance of the sub-optimal detectors and that of the optimal detector. Motivating by this, the main objective of this research is to achieve near-optimal performance of detector while maintaining computational efficiency.

In this thesis, we look at several metaheuristic optimization methods to get approximate optimal solution. We improve the performance of $(1 + \lambda)$ Evolutionary Strategy (ES) based multiuser detector for synchronous Direct Sequence Code Division Multiple Access (DS-CDMA) system by applying hybrid $(1 + \lambda)$ ES algorithm. We also applied this hybrid method for ML detection in Multicarrier CDMA (MC-CDMA) and Multiple Input Multiple Output (MIMO) systems. We proposed Simulated Annealing (SA) algorithm for ML detection and applied to these systems. Based on a new type Evolutionary Computation (EC) algorithms named Estimation of Distribution Algorithms (EDAs), we developed a new detection scheme. We applied an EDA approach named Population-based Incremental Learning (PBIL) algorithm and also modified. Simulation results are presented to demonstrate the efficacy of the proposed algorithms over the existing detectors.

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Acronyms

Acronyms	**Definition**
1G	first generation
2G	second generation
3G	third generation
4G	forth generation
ACO	ant colony optimization
AMPS	advance mobile phone system
AWGN	additive white Gaussian noise
BER	bit error rate
BPSK	binary phase shift keying
CD	conventional detector
CO	combinatorial optimization
DF	decision feedback
DS-CDMA	direct sequence code division multiple access
EC	evolutionary computation
EDA	estimation of distribution algorithm
EM	expectation-maximization
EP	evolutionary programming
ES	evolutionary strategy
FDD	frequency division duplex
FDMA	frequency division multiple access

GA	genetic algorithm
GLS	guided local search
GP	genetic programming
GRASP	greedy randomized adaptive search procedure
GSM	global system for mobile
ILS	iterated local search
IMT-2000	international mobile telecommunications-2000
IP	internet protocol
IS-95	interim standard 95
ICI	inter-carrier interference
ITU	international telecommunication union
LAN	local area network
LR	lattice-reduction
MAI	multiple access interference
MAP	maximum *a posteriori*
MC-CDMA	multi-carrier code division multiple access
MF	matched filter
MIMO	multiple-input multiple-output
ML	maximum-likelihood
MMSE	minimum mean square error
MUD	multiuser detection
NP	non-deterministic polynomial-time
OFDM	orthogonal frequency division multiplexing
PBIL	population-based incremental learning
PSO	particle swarm optimization
QAM	quadrature amplitude modulation
QoS	quality-of-service
SA	simulated annealing
SAGE	space-alternating expectation-maximization

SD	sphere decoder
SDR	semidefinite relaxation
SER	symbol error rate
SIC	successive interference cancellation
SNR	signal-to-noise ratio
SS	scatter search
TACS	total access communication system
TDMA	time division multiple access
TD-SCDMA	time division synchronous CDMA
TS	tabu search
V-BLAST	vertical bell laboratories space time
VNS	variable neighborhood search
WCDMA	wideband CDMA
ZF	zero-forcing
ZMCSCG	zero mean circularly symmetric complex gaussian

Chapter 1

Introduction

1.1 Background

1.1.1 Wireless Systems

Wireless technology enables high-speed, high-quality communication between mobile devices. Potential wireless applications include cell phones, 802.11-based wireless Local Area Networks (LANs), Bluetooth, smart homes and appliances, voice and data communication over the Internet, and video conferencing. The wireless communications industry has advanced drastically in the past decade and emerged as one of the fastest growing sectors in telecommunications. Although the enormous demand for mobile phones has driven the early developments, the latest generations of wireless systems are also designed to provide broadband multimedia applications.

The first generation (1G) of mobile telephony systems was introduced in the early 1980s for voice-only services using analog transmission techniques. 1G cellular systems include the Advance Mobile Phone System (AMPS) in North America and the Total Access Communication System (TACS) in Europe. All 1G standards were based on Frequency Division Multiple Access (FDMA). 1G systems frequently suffered from busy signals and dropped calls because of the low system capacity.

The second generation (2G) wireless systems introduced in the early 1990s were based on digital signalling to cope with increased traffic using limited bandwidth. These 2G systems were designed mainly for voice transmission. The most popular 2G wireless technology, known as GSM (Global System for Mobile), was first implemented in 1991. The 2G cellular systems enjoyed incredible success and were quickly adopted worldwide. The 2G standards include Time Division Multiple Access (TDMA), FDMA, Frequency Division Duplex (FDD) and Code Division Multiple Access (CDMA). The first 2G CDMA cellular standard is Interim Standard 95 (IS-95). 2G technology has limited data transmission capabilities such as fax and short message service at data rates up to 14.4 kb/s [1]. With the virtual explosion of internet usage, users demand data delivery on mobile devices, and hence the 2G standards have evolved to 2G+ packet-based technology with data rates up to 384 kb/s.

The Third Generation (3G) of telephone systems provides for both voice and data applications. The advantages of 3G systems are universal global roaming, increased data rates (up to 2Mb/s), and improved spectral efficiency. International Mobile Telecommunications-2000 (IMT-2000) is the global standard for 3G wireless systems set by the International Telecommunication Union (ITU). Almost all IMT-2000 radio standards are based on CDMA: CDMA2000, wideband CDMA (WCDMA) and time division synchronous CDMA (TD-SCDMA) [1]. The first 3G wireless system was deployed in Japan in 2001. The global evolution of wireless communication standards from 1G to the 3G is summarized in Fig. 1.1 [1].

Research has continued on the improvement of 3G networks while they are being deployed. Fourth generation (4G) systems will be super-enhanced versions of 3G, using the Internet Protocol (IP) technology [2]. 4G systems will support all broadband wireless services such as interactive multimedia; high Quality-of-Service (QoS); high data rates (up to 100Mb/s); significantly increased spectral efficiency; and low deployment, maintenance, and operation costs. Moreover, the new systems will assure universal roaming by using a single handheld device. These features impose demanding technical challenges on the system design. Currently, several possible technologies are under consideration to meet these demands for the 4G air interface: Multiple In-

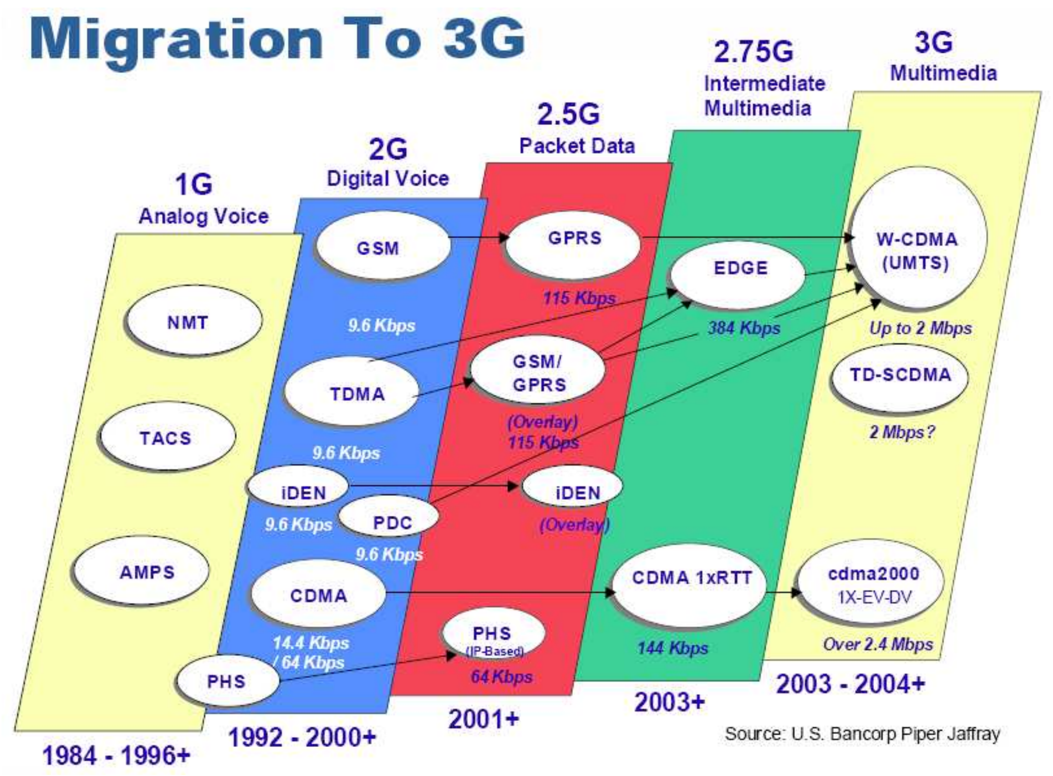

Fig. 1.1. Evolution of wireless communication standards [1].

put Multiple Output (MIMO), Multicarrier CDMA (MC-CDMA), and Orthogonal Frequency Division Multiplexing (OFDM).

1.1.2 Code Division Multiple Access

Capacity-enhancing techniques for current digital cellular systems have received much interest. The major resource constraints are the spectrum and transmission power. Multiple-access communication allows multiple users to share the common spectrum efficiently. Efficient spectrum use enables high throughput, integration of services and flexibility. Of several multiple access schemes, CDMA has taken a major role in cellular and personal communication systems as it can meet these requirements. The bandwidth sharing and inter-symbol interference rejection capabilities of CDMA are desirable in cellular systems and wireless LANs, making CDMA the basis for both second- and third-generation cellular systems as well as second-generation wireless

LANs [3]. In 1990, Qualcomm introduced IS-95 (Interim Standard-95) based on CDMA air-interface technology. The CDMA capacity can be 4-6 times that of TDMA and is nearly 20 times that of analog networks [4].

In Direct Sequence CDMA (DS-CDMA), each user signal is spread by using a unique pseudorandom signature sequence. The transmitted signal is spread over the whole frequency band, which is much lower than the minimum bandwidth required for a single user. Low cross-correlation among the signature sequences allows different users to share the same frequency band with low interference. Thus, CDMA is now a driving force behind the rapidly developing wireless telecommunication industry.

MC-CDMA, a multicarrier version of CDMA, is being considered for the 4G wireless physical layer [4]. MC-CDMA offers several advantages over single carrier systems [5] and combines the advantages of OFDM and DS-CDMA [6]. The available spectrum is partitioned into several narrow subchannels (subcarriers), which experience flat fading. OFDM sub-channels are overlapping and orthogonal, thus eliminateing mutual interference. The MC-CDMA transmitter spreads the each user's data over several subcarriers by using a frequency-domain signature sequence. Thus, MC-CDMA uses frequency domain spreading while DS-CDMA uses time domain spreading. It is shown in [7] that MC-CDMA outperforms DS-CDMA in multipath channels while in an Additive White Gaussian Noise (AWGN) channel, their performances are identical.

1.1.3 Multiple Input Multiple Output System

MIMO systems with multiple antennas at both the transmitter and receiver offer high-data-rate wireless transmissions without increasing the bandwidth or transmit signal power [8]. By using an appropriate MIMO space-time processing technology, enormous capacities can be extracted from the rich-scattering multipath channel [9]. The MIMO technology is also a leading physical layer candidate for emerging 4G wireless networks and has already been implemented in current standards such as the

IEEE 802.16e (WiMAX) [10]. MIMO systems can achieve diversity gains and multiplexing gains. Diversity gains are realized by sending space-time coded information signals over multiple antennas and thus improving the performance (bit error rate) at the receiver. Multiplexing gains are realized by sending different information signals over multiple antennas, resulting in an increase in the information transmission rate.

1.1.4 Multiuser Detection

The conventional CDMA detection uses the single user detection approach. A user's data is detected by correlating the composite received signal with that user's unique signature sequence. Other users' signals are, in this case, treated as additional Gaussian noise [11]. This conventional detector (CD) consists of a bank of matched filters. Since the signature sequences are not perfectly orthogonal, this detector suffers from multiple access interference (MAI) and also from the near-far problem [12]. That is, if an interferer is significantly stronger than the desired user, then the low-power user may be swamped out, and the high-power user can potentially dominate performance as even a small amount of correlation will lead to significant interference. To avoid this problem, precise power control, which is very difficult to maintain, is required in system design. In a mobile environment, multipath fading further aggravates the near-far problem.

The effects of MAI and the near-far problem can be eliminated by multiuser detection (MUD). MUD techniques have been investigated since the mid-1980's and provide different tradeoffs between the bit error rate (BER) performance and computational cost [13]. Reference [14] provides an overview of the most common MUD techniques. Verdú proposed the optimal maximum likelihood (ML) detector for MUD to mitigate the MAI problem of the conventional single-user detection approach [15]. Verdú introduced a K-user ML sequence detector, which consists of a bank of K single user matched filters followed by a Viterbi algorithm with 2^{K-1} states. The complexity of Verdú's detector per binary decision is $O(2^K)$. Consequently, this detector is unfeasible for a large number of users.

Although ML-MUD achieves the minimum probability of error, the ML cost function must be evaluated over the set of all feasible user data sequences, resulting in a non-deterministic polynomial time hard (NP-hard) optimization problem. Since computational complexity grows as $O(2^K)$, several suboptimum detectors have been developed with reduced (polynomial) complexity. Decorrelating or minimum mean square error (MMSE) detectors are introduced in [16] and [17]. These linear suboptimal receivers have linear computational complexity in the number of users, K. Suboptimal multistage nonlinear detectors are proposed in [18]. A successive interference cancelation algorithm is presented in [19]. A nonlinear technique using a variant of the decorrelating detector in conjunction with feedback is introduced [20]. All these detectors require knowledge of signature sequences and timing information about the desired user and interferer, the received amplitudes and other parameters. Blind MUD schemes, proposed in [21, 22], require no more knowledge than that required by a conventional detector, i.e., only the desired user's signature waveform and its timing are required.

In sphere decoding (SD), the closest lattice point to the received signal within a hyper-sphere is searched. The sphere decoder has been shown to offer ML performance at polynomial complexity in the high signal-to-noise ratio (SNR) region and for moderately sized problems [23]. However, the performances of the sphere decoder are dependent mainly on the search radius, and the complexity is large when the SNR is low or when the problem dimension is high [24]. The semidefinite relaxation (SDR) detector uses the cone of semidefinite matrices to obtain a good approximation to the ML-MUD problem [25]. Cutting planes are introduced in [25] to strengthen the approximation, and the semidefinite programming arising from the relaxation is solved by using the interior point method. Clearly, the tighter the relaxation in the solution, the more precise result can be obtained. However, the semidefinite relaxations encounter difficulty in practice because the cost of solving semidefinite programming goes up quickly as the problem dimension increases. Exact methods such as the branch and bound algorithm have also been proposed for MUD [26]. To approximately solve the ML-MUD problem, metaheuristic methods can also be applied [27]-[29].

1.1.5 Metaheuristics

For NP-hard problems, complete methods may have exponential time complexity in the worst-case. In the last 20 years, such problems have been attacked by using *metaheuristics*. The fundamental idea is to combine basic heuristic methods with higher level frameworks aimed at exploring the search space by using the concepts derived from artificial intelligence and the biological, neural and physical sciences [30]. A heuristic is a method that finds near-optimal solutions in a short time without being able to guarantee either feasibility or optimality [31]. Heuristic methods have been widely investigated for combinatorial problems [31].

Metaheuristic algorithms include Evolutionary Computation (EC) algorithms comprising Genetic Algorithm (GA), Evolutionary Programming (EP) and Evolutionary Strategy (ES); Tabu search (TS); and Ant Colony Optimization (ACO) have previously been employed for solving the ML-MUD problem [27]-[33]. Neighborhood search methods like 1-*opt*, *k-opt* local search have already been introduced to solve the ML-MUD problem [34]-[36]. A GA-based detection strategy was used in [27] for a multiuser receiver, highlighting the feasibility of using GA as a powerful solution for MUD. EP converges to an optimal solution with a small number of generations and has lower complexity than GA [28]. Wang, Zhu and Kang proposed a new MUD algorithm based on $(1 + \lambda)$ ES for an asynchronous DS-CDMA system in [29]. They showed that their algorithm performed better when the number of users was large, while other evolutionary MUD algorithms performed poorly. In [37], a heuristic algorithm based on a nonlinear nonconvex programming relaxation for the CDMA ML problem is presented. The BER performance of this heuristic detector is similar to that of the SDR detector, but has lower average CPU time. Reference [32] compares relaxations, exact and heuristic search methods and shows that when the number of users increases, the heuristic search methods such as TS and Iterated Local Search (ILS) are more effective than the SDR approach.

1.2 Contributions

The full potential of DS-CDMA, MC-CDMA and MIMO can be realized only with increased hardware cost due to the significantly more complex signal processing algorithm in the receiver (and also in the transmitter side in the case of MIMO). Signal detection algorithms for these systems have thus been heavily researched. Much of the recent research has focused on the appropriate trade-offs between complexity and performance. In this thesis, we focus on computationally efficient detection algorithms that can achieve near-optimum performance with low computational complexity. Several metaheuristic detection methods are developed for DS-CDMA, MC-CDMA and MIMO systems. The key contributions of this thesis can be summarized as follows:

1. In [29], it is shown that the $(1 + \lambda)$ ES-based detector outperformed other EC-based detectors. Inspired by this result, we develop a $(1 + \lambda)$ ES-based detector for MC-CDMA and MIMO systems. We also propose a hybrid $(1+\lambda)$ ES algorithm for synchronous DS-CDMA, MC-CDMA, MIMO systems. This hybrid algorithm employs a $(1+\lambda)$ ES for the basic search, directing the search process towards an elitist solution space. A simple 1-*opt* local search is applied for a thorough search in the elitist region. The simulation results demonstrate the efficiency of the proposed algorithm as for a large number of users, it offers a near optimal bit-error rate (BER) performance with lower computational complexity compared to the maximum-likelihood (ML) detector.

2. The SA algorithm is proposed for ML detection in DS-CDMA, MC-CDMA and MIMO systems. The average computational cost is significantly reduced in comparison to that of the optimal ML detector but degrades the BER performance.

3. A new type of EC algorithms named Estimation of Distribution Algorithms (EDAs) [38] eliminate the drawbacks of typical EC algorithms. We develop a novel ML detection scheme based on the PBIL algorithm (one of the first EDAS) for DS-CDMA, MC-CDMA and MIMO systems. We also propose a modified PBIL algorithm which employs simple PBIL with 1-*opt* local search to provide a better performance. Computer simulation reveals that these detection methods achieve near-optimal performance with low computational complexity.

1.3 Outline of the Thesis

Chapter 2 introduces the DS-CDMA, MC-CDMA and MIMO systems. ML detection for these systems and the existing MUD techniques are reviewed.

Chapter 3 discusses the metaheuristic optimization methods used in this thesis for ML detection.

Chapter 4 applies these metaheuristic methods to the ML-MUD problem for the CDMA system, and the ML problem for the MIMO system. This chapter focuses on the efficient detection algorithms based on these metaheuristic methods for different systems. Numerical results are presented to show the BER and complexity of these proposed detectors.

Chapter 5 concludes the thesis and outlines future works.

Chapter 2

System Models and Maximum Likelihood Detection

This chapter describes the DS-CDMA, MC-CDMA and MIMO systems, reviews MUD for CDMA and mathematically formulates the ML detection problem. Section 2.1 describes the CDMA mathematical model and several multiuser detectors. MC-CDMA systems and ML detection are introduced in Section 2.2. Section 2.3 discusses MIMO systems and ML detection.

2.1 Synchronous DS-CDMA Systems

We consider a synchronous single carrier DS-CDMA system in which K users simultaneously transmit BPSK signals (Fig. 2.1). Each user signal is multiplied by a distinct signature sequence, $s_k(t)$ (which is also called a spreading code, spreading sequence, or chip sequence). The k-th signature sequence can be expressed as

$$s_k(t) = \sum_{j=0}^{L-1} c_k(j)\, p(t - jT_c), \quad (0 \le t \le T_s), \tag{2.1}$$

where $\{c_k(j) : (0 \le j \le L-1)\}$ is a code sequence of L chips that take value $\{\pm 1\}$, and $p(t)$ is a pulse of duration T_c, where T_c is the chip interval, and T_s is the symbol interval. The spreading gain is $L = T_s/T_c$. Without loss of generality, all K signatures are normalized to the unit energy [11].

The data sequence of the k-th user is denoted by $\{b_k(m)\}$, where $b_k(m) \in \{\pm 1\}$. All data sequences are equally probable. We consider a block of M transmitted data symbols. The received signal for synchronous transmission may be expressed as

$$r\left(t\right) = \sum_{k=1}^{K} \sqrt{\varepsilon_k} \sum_{m=1}^{M} b_k(m) s_k(t) + n(t), \tag{2.2}$$

where ε_k is the k-th user signal energy, $n(t)$ is an AWGN process with mean zero and power spectral density $N_0/2$ [11].

The received signal $r\left(t\right)$ (2.2) consists of the sum of all user signals and is demodulated by using a bank of matched filters. The sampled output of the matched filter of the k-th user is given by

$$y_k = \int_0^{T_s} r(t) s_k(t)\, dt, \qquad (1 \leq k \leq K). \tag{2.3}$$

By using (2.2) in (2.3), y_k can be expressed as

$$y_k = \sqrt{\varepsilon_k} b_k(1) + \sum_{k \neq l} \sqrt{\varepsilon_l} b_l(1) \rho_{lk}(0) + n_k, \tag{2.4}$$

where the cross-correlation term $\rho_{lk}(0) = \int_0^{T_s} s_l(t) s_k(t)\, dt$, and the noise term $n_k = \int_0^{T_s} n(t) s_k(t)\, dt$. Arranged in vector format for all K users, the matched filter outputs can be expressed as

$$\mathbf{y} = \mathbf{R} \boldsymbol{\Sigma} \mathbf{b} + \mathbf{n}, \tag{2.5}$$

where the matched filter outputs are $\mathbf{y} = [y_1, y_2, ...y_K]^T$, $\mathbf{R}$ is the symmetric positive definite correlation matrix with elements $\rho_{lk}(0)$, $\boldsymbol{\Sigma} = \mathrm{diag}(\sqrt{\varepsilon_1}, \sqrt{\varepsilon_2}, ..., \sqrt{\varepsilon_K})$, the information vector $\mathbf{b} = [b_1, b_2, ...b_K]^T$, and $\mathbf{n} = [n_1, n_2, ..., n_K]^T$ is a noise vector of zero mean Gaussian random variables whose covariance matrix is $E(\mathbf{n}\mathbf{n}^T) = \frac{N_0}{2}\mathbf{R}$.

2.1.1 Multiuser Detectors

MUD performs joint detection to achieve a higher system capacity than is currently achieved with the conventional single-user matched filter detector. Fig. 2.2 illustrates

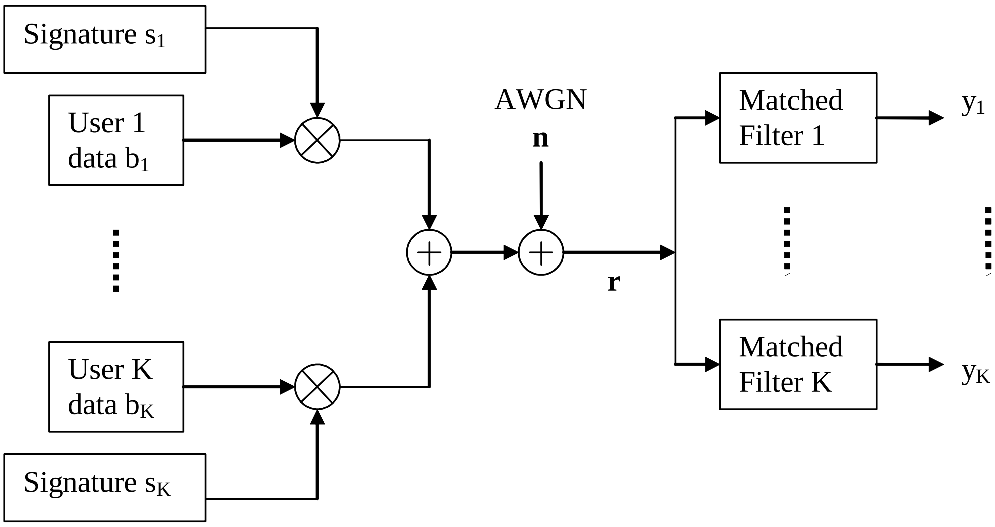

Fig. 2.1. Synchronous DS-CDMA system.

a generic multiuser detector. K matched filters at the receiver front-end match to the users' signature sequences. The multiuser detection algorithm will process the matched filter outputs y_k and provide K data decisions. A wide variety of multiuser detection algorithms has been investigated in the literature.

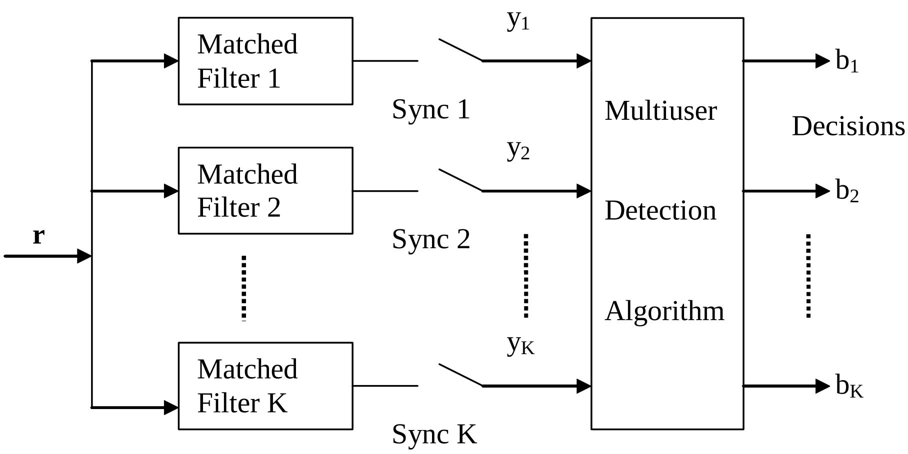

Fig. 2.2. A generic multiuser detector.

Conventional detector

The conventional single-user detector is a special case of the generic multiuser detector in Fig. 2.2. The data decisions are simply the signs of the matched filter outputs:

$$\hat{\mathbf{b}}_{CD} = \mathrm{sign}(\mathbf{y}), \tag{2.6}$$

where $\mathrm{sign}(x)$ returns 1 if x is greater than zero, 0 if it equals zero, and -1 if it is less than zero. The CD is optimal only if the spreading sequences of all users are orthogonal to each other. However, non-zero cross-correlations among spreading sequences lead to MAI, which, in general, increases with the number of users.

Optimum detector

The optimum multiuser detector proposed by Verdú in [15] minimizes the probability of error. The transmitted data vector $\mathbf{b}$ is estimated by minimizing the log-likelihood function. The decision criterion is then given by

$$\hat{\mathbf{b}} = \operatorname*{argmax}_{\mathbf{b}\in\{-1,+1\}^K} p(\mathbf{y} \mid \mathbf{b}), \tag{2.7}$$

where $\hat{\mathbf{b}}$ is the maximum-a-posteriori (MAP) estimate of $\mathbf{b}$ given the received vector $\mathbf{y}$. When all possible vectors $\mathbf{b} \in \{-1, +1\}^K$ are equally probable, MAP and ML are equivalent. The received signal $\mathbf{y}$ has a K dimensional Gaussian PDF with mean $\mathbf{R\Sigma b}$ and covariance $\mathbf{R}$:

$$p(\mathbf{y} \mid \mathbf{b}) = \frac{1}{\sqrt{(N_0\pi)^K \det \mathbf{R}}} \exp[-\frac{1}{N_0}(\mathbf{y} - \mathbf{R\Sigma b})^T \mathbf{R}^{-1}(\mathbf{y} - \mathbf{R\Sigma b})]. \tag{2.8}$$

Taking the logarithm and ignoring the constant terms, we obtain the ML detector of $\mathbf{b}$:

$$\hat{\mathbf{b}}_{opt} = \operatorname*{argmin}_{\mathbf{b}\in\{-1,+1\}^K} \mathbf{b}^T\mathbf{\Sigma}^T\mathbf{R\Sigma b} - 2\mathbf{y}^T\mathbf{\Sigma b}. \tag{2.9}$$

Since (2.9) is a discrete CO problem, an exhaustive search involves all 2^K possible combinations. Thus, the computational complexity grows exponentially with the number of users, K. Because of this increased complexity, it is critical to develop suboptimum but computationally efficient multiuser detectors.

Decorrelator

One of the most popular classes of suboptimal multiuser detectors is the linear detector, which has significantly lower computational complexity than that of the ML detector. Two common types of linear detectors are the decorrelating detector (decorrelator) and the minimum mean square error (MMSE) detector.

The decorrelator operates in two steps [16]. First, (2.9) is "relaxed" by elimination of the discrete constraint, and the relaxed solution $\tilde{\mathbf{b}} = \mathbf{R}^{-1}\mathbf{y}$ is computed. Second, the relaxed solution is mapped onto the binary constraint set via

$$\hat{\mathbf{b}}_{decor} = \mathrm{sign}(\tilde{\mathbf{b}}) = \mathrm{sign}(\mathbf{R}^{-1}\mathbf{y}). \tag{2.10}$$

The decorrelator completely eliminates the MAI but causes noise enhancement. Despite this drawback, the decorrelator provides substantial performance gains over the CD [39].

MMSE detector

The MMSE detector [17] minimizes the mean square error between the true symbol value and the linear estimate $\tilde{\mathbf{b}} = \mathbf{M}\mathbf{y}$. The linear operator $\mathbf{M}$ is found via the following optimization criterion:

$$\mathbf{M} = \underset{\mathbf{M} \in R^{K \times K}}{\operatorname{argmin}} \mathrm{E}\left\{ \left\| \mathbf{b} - \tilde{\mathbf{b}} \right\|^2 \right\}. \tag{2.11}$$

The solution to (2.11) is given by

$$\mathbf{M} = \left(\mathbf{R} + \sigma^2 \mathbf{\Sigma}^{-2}\right)^{-1}, \tag{2.12}$$

where σ^2 is the Gaussian noise power [13]. The MMSE detector output is then

$$\hat{\mathbf{b}}_{MMSE} = \mathrm{sign}(\tilde{\mathbf{b}}) = \mathrm{sign}(\left(\mathbf{R} + \sigma^2 \mathbf{\Sigma}^{-2}\right)^{-1} \mathbf{y}). \tag{2.13}$$

Since the MMSE detector takes into account both the background noise and the received signal powers, it performs better than the decorrelator. As the background noise goes to zero, the MMSE detector converges to the decorrelator. Although linear multiuser detectors outperform single-user detectors, their performance is significantly worse than the ML performance.

Multistage detector

Another popular class of suboptimal detectors is the nonlinear detector formed by decision-driven multiuser detectors including the multistage detector [18], successive interference cancelation (SIC) detector [19], and decision feedback (DF) detector [20]. These detectors use decisions on the bits of interfering users in the detection of the bit of interest.

The multistage detector employs multiple stages in detecting the user bits and canceling out the interference. In the first stage, the conventional bank of MFs is used to detect all data bits in parallel:

$$\hat{\mathbf{b}}^{(0)} = \hat{\mathbf{b}}_{CD} = \operatorname{sign}(\mathbf{y}). \tag{2.14}$$

In the next stage(s), SIC is used. The kth decision of the mth stage is based on the decisions of the $(m-1)$th stage,

$$\hat{b}_k^{(m)} = \operatorname{sign}\left(y_k - \sum_{j \neq k} \sqrt{\varepsilon_j}\rho_{jk}(0)\hat{b}_j^{(m-1)} \right). \tag{2.15}$$

Alternatively, the decorrelating detector can serve as the first stage, and each stage improves upon the previous stages' estimation [13]. In the multistage detector, different detectors can be applied each stage, but noise and data become more and more correlated with the increasing number of stages.

SIC detector

The SIC detector is based on the idea of canceling interfering signals from the received signal, one at a time as they are detected. Thus, user data bits are detected successively, one after another. The order in which the data bits are detected affects the performance of SIC detector as the reliability of all the successive decisions depends on the previous decisions. The data bits can be detected in order of decreasing received powers; i.e., the data bit of the user with the strongest received signal is detected first, and this detected data bit is used in SIC (2.16). In K-user SIC, the kth user data bit is detected on the assumption that the decisions of users $k+1, ..., K$ are correct and the presence of users $1, ..., k-1$ are neglected. Therefore, the kth user decision is

$$\hat{b}_k = \text{sign}\left(y_k - \sum_{j=k+1}^{K} \sqrt{\varepsilon_j} \rho_{jk}(0) \hat{b}_j \right), \tag{2.16}$$

where y_k is the kth user's MF output [11]. SIC suffers from error propagation: subtracting an incorrect detected symbol will double the interference, and the delay in detecting the weakest user increases linearly with the number of users.

DF detector

The DF detector detects user data bits sequentially, one at a time. This detector is analogous to the decision-feedback equalizers used for inter-symbol interference suppression in the single-user case [40]. Here, the feed-forward filter is the Cholesky factorization of the correlation matrix $\mathbf{R}$, which yields a lower-triangular matrix $\mathbf{L}$ as $\mathbf{L}^T\mathbf{L} = \mathbf{R}$. As $\mathbf{R}$ is nonsingular, so is $\mathbf{L}$. Thus MF outputs (2.6) can be written equivalently to get whitened MF outputs as

$$\tilde{\mathbf{y}} = \mathbf{L}^{-T}\mathbf{y} = \mathbf{L}\boldsymbol{\Sigma}\mathbf{d} + \tilde{\mathbf{n}}, \tag{2.17}$$

where $\mathbf{L}^{-T}$ denotes the upper triangular matrix; i.e., the inverse of $\mathbf{L}^T$ and $\tilde{\mathbf{n}} = \mathbf{L}^{-T}\mathbf{n}$ is a zero mean white Gaussian noise with the covariance matrix $\sigma^2\mathbf{I}$ [13]. The DF detector outputs are:

$$\hat{b}_k = \text{sign}\left(\tilde{y}_k - \sum_{j=1}^{k-1} \sqrt{\varepsilon_j} l_{kj} \hat{b}_j \right), \quad k = 1, 2, ..., K. \tag{2.18}$$

In the detector (2.18), a linear combination of previous decisions is subtracted from the whitened MF outputs. The performance of this DF detector depends solely on the detection order. The optimal user ordering is given in Theorem 1 of [41].

SAGE detector

In [42], a new iterative multiuser receiver is proposed by using the expectation-maximization (EM) algorithm. The EM algorithm provides an iterative approach to estimate likelihood-based parameters when direct estimation may not be feasible. Since the EM algorithm updates all estimated parameters simultaneously, it has slow convergence. The space-alternating expectation-maximization (SAGE) algorithm updates the parameters sequentially. The convergence of the likelihood function is significantly faster, and the maximization step is often simplified [43]. Each iteration of this algorithm involves the following steps:

$$Defn - step : Let \quad k = 1 : (i \mod K). \tag{2.19}$$

$$M(maximization) - step : b_k^{i+1} = \text{sign}(y_k - \sum_{j \neq k} \rho_{kj} \sqrt{\varepsilon_j} b_j^i) \tag{2.20}$$

$$b_j^{i+1} = b_j^i, \quad \forall j \neq k, \tag{2.21}$$

where i is the number of iterations. This method is similar to that of the multistage detector, except that bits are updated sequentially rather than in parallel. The interfering user data bits are treated as probabilistic missing data while updating the estimate for a given user's bit. Reference [42] showed that the SAGE detector with an M-step hard decision yielded a good performance with the following soft-decision decorrelator initialization:

$$b_k^0 = \tanh\left(\frac{\sqrt{\varepsilon_k}}{[\mathbf{R}^{-1}]_{kk}\sigma^2}[\mathbf{R}^{-1}\mathbf{y}]_k\right), \qquad k = 1, ..., K. \tag{2.22}$$

Fig. 2.3 demonstrates the BER performance of different multiuser detectors. The SAGE detector outperforms other suboptimal detectors for a DS-CDMA system with 10 users. The SIC detector performs better than conventional detector but worse than other detectors. The SAGE and DF detectors achieve better BER performance than the linear detectors.

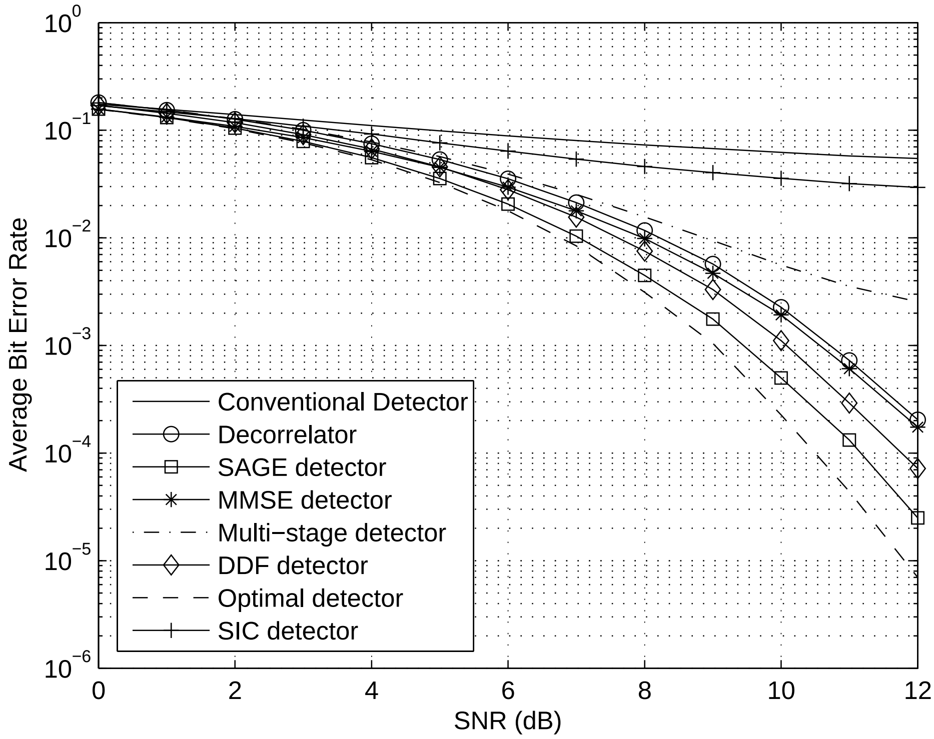

Fig. 2.3. Average BER of some existing multiuser detectors in synchronous DS-CDMA for 10 users, 10^5 monte carlo runs.

2.2 MC-CDMA Systems

MC-CDMA was first proposed in [6] as a multicarrier multiple access/digital modulation technique, which is a variation of OFDM. OFDM has recently gained much interest because it can provide higher data rates and is currently used in wireless LAN and metropolitan area network (MAN) applications, including IEEE 802.11a/g and WiMAX. MC-CDMA is essentially an OFDM technique where each data symbol is spread over multiple narrowband subcarriers with a user-specific signature code. The MC-CDMA spreads the data stream by using a spreading code and then modulates different subcarriers with each chip, i.e., by spreading the chips in the frequency domain. The spreading code associated with MC-CDMA helps in interference suppression in addition to providing high data rates.

We consider a synchronous K-user MC-CDMA system of N narrowband subcarriers. The block diagram is shown in Fig. 2.4. At time t, the data symbol $b_k(t)$ of the user k, from the signal constellation $\mathcal{B}$ is spread by the signature sequence $\mathbf{c}_k = (c_{k1}, ..., c_{kN})$. These signature sequences must have low cross-correlations. In this thesis, an orthogonal Walsh-Hadamard set of size N is used as signature sequences. In this case, the maximum number of active users K_{max} that can be supported in the MC-CDMA system is equal to the signature sequence length N; i.e., $K_{max} = N$. The resultant N chips after spreading the symbol $b_k(t)$ are modulated on the N different subcarriers using the IFFT operator and then transmitted through the channel. The propagation channel is described by the complex coefficients $h_{kn}(t)$, $k = 1, ..., K$ and $n = 1, ..., N$. The combination of spreading and channel coefficients for all users can be expressed by the $N \times K$ matrix as

$$\mathbf{D}(t) = \begin{pmatrix} c_{11}h_{11}(t) & . \quad . & c_{K1}h_{K1}(t) \\ . & & . \\ . & & . \\ c_{1N}h_{1N}(t) & . \quad . & c_{KN}h_{KN}(t) \end{pmatrix}. \tag{2.23}$$

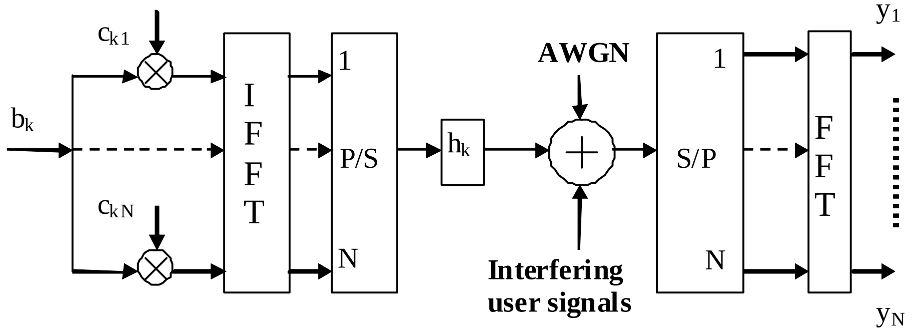

Fig. 2.4. Schamatic diagram of synchronous MC-CDMA system.

At the receiver side, an FFT operation is performed on the received samples. At time t, the received vector can be written as

$$\mathbf{y}(t) = \mathbf{D}(t)\mathbf{b}(t) + \boldsymbol{\eta}(t), \tag{2.24}$$

where the received signal vector, $\mathbf{y}(t) = [y_1(t), ..., y_N(t)]^T$; the data vector containing K transmitted data symbols, $\mathbf{b}(t) = [b_1(t), ..., b_K(t)]^T$; and $\boldsymbol{\eta}(t) = [\eta_1(t), ..., \eta_N(t)]^T$ is an AWGN vector with zero mean and variance of σ^2 per dimension. In the downlink, all users share the same channel defined by $\mathbf{H}(t) = \text{diag}[h_1(t), ..., h_N(t)]$ [**?**]. Thus, $\mathbf{D}(t) = \mathbf{H}(t)\mathbf{D}_D$, where all users' signature sequences are placed in the $N \times K$ matrix $\mathbf{D}_D = [\mathbf{c}_1, \mathbf{c}_2, ..., \mathbf{c}_K]$ where $\mathbf{c}_k = [c_{11}, c_{12}, ..., c_{1N}]^T$ for $k = 1, 2, ..., K$.

The complex system model described in (2.24) can be transformed into the following equivalent real-valued system:

$$\mathbf{y}_r = \mathbf{D}_r\mathbf{b}_r + \boldsymbol{\eta}_r, \tag{2.25}$$

where

$$\mathbf{y}_r = \begin{bmatrix} \Re\{\mathbf{y}\} \\ \Im\{\mathbf{y}\} \end{bmatrix}, \mathbf{b}_r = \begin{bmatrix} \Re\{\mathbf{b}\} \\ \Im\{\mathbf{b}\} \end{bmatrix}, \boldsymbol{\eta}_r = \begin{bmatrix} \Re\{\boldsymbol{\eta}\} \\ \Im\{\boldsymbol{\eta}\} \end{bmatrix}$$

and

$$\mathbf{D}_r = \begin{bmatrix} \Re\{\mathbf{D}\} & -\Im\{\mathbf{D}\} \\ \Im\{\mathbf{D}\} & \Re\{\mathbf{D}\} \end{bmatrix}.$$

The rank of the matrix $\mathbf{D}_r$ is generally $2 \times \min(N, K)$ so the columns of $\mathbf{D}_r$ formed a basis vector of a lattice lying in a $2K$-dimensional space.

2.2.1 MC-CDMA Detection Algorithms

The MC-CDMA system enables the realization of powerful detectors due to the avoidance of ISI and inter-carrier interference (ICI) in the detection process. These detectors can also be classified as either single-user detection or multiuser detection. With MC-CDMA in a multiuser environment, different users share the same subcarriers at the same time. Hence, MC-CDMA also suffers from MAI. With MAI, the single-user detector is suboptimal. To overcome this suboptimality, optimal MUD has been proposed where the *a priori* knowledge about the signature sequences of the interfering users is exploited in the detection process [44]. Linear MUD strategies such as decorrelation and MMSE and nonlinear MUD strategies such as SIC, multistage, and decision feedback and other strategies proposed for DS-CDMA can also be adopted to MC-CDMA.

In optimal ML-MUD, all user data symbols are detected jointly to minimize the effects of MAI. The optimal solution of (2.25) is given by the ML detector as

$$\hat{\mathbf{b}}_{opt} = \underset{\mathbf{b}_r \in \mathcal{B}_r^{2K}}{\operatorname{argmin}} \|\mathbf{y}_r - \mathbf{D}_r \mathbf{b}_r\|^2 , \tag{2.26}$$

where $\|.\|$ denotes the Euclidean norm. For instance, if the data vector $\mathbf{b}$ is taken from the 4-quadrature amplitude modulation (QAM) signal constellation, then $\mathcal{B}_r$ belongs to $\{-1, +1\}$. Thus, optimal detection via exhaustive search implies a complexity growth of $\mathcal{O}(2^{2K})$, which is exponential in K.

2.3 MIMO Systems

Multiple antenna systems promise to play a key role in future high-speed wireless applications such as wireless cellular systems and wireless LAN. MIMO systems provide high data rates by using limited bandwidth when used for spatial multiplexing.

Consider a spatial multiplexing MIMO system equipped with N_t transmit antennas and N_r receive antennas (subject to $N_t \leq N_r$) as in Fig. 2.4. At the transmitter, data bits are mapped into complex symbols from a finite constellation $\mathcal{S}$. The complex symbol stream is demultiplexed into N_t substreams, and each substream is sent through a different transmit antenna. We assume a rich scattering flat-fading channel [45]. Each receive antenna receives signals from all N_t transmit antennas. The discrete baseband received signal vector can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{2.27}$$

where $\mathbf{y} = [y_1, y_2, ...y_{N_r}]^T$ is an $N_r \times 1$ received signal vector of complex numbers, $\mathbf{s} = [s_1, s_2, ...s_{N_t}]^T$ is an $N_t \times 1$ transmitted signal vector where $s_i \in \mathcal{S}$, $i = 1, 2, ..., N_t$,

$\mathbf{H} = [h_{k,l}]$ ($k = 1, 2, ...N_r$ and $l = 1, 2, ...N_t$) is a complex $N_r \times N_t$ channel matrix, and $\mathbf{n} = [n_1, n_2, ..., n_{N_r}]^T$ is an $N_r \times 1$ additive zero mean circularly symmetric complex Gaussian (ZMCSCG) noise vector with variance of σ^2, $\mathrm{E}[\mathbf{n}\mathbf{n}^H] = \sigma^2 \mathbf{I}_{N_r}$. The elements of $\mathbf{H}$ are independent and identically distributed (i. i. d.) complex Gaussian, denoted $h_{k,l} \sim \mathcal{CN}(0, 1)$, as for the classical i. i. d. frequency-flat Rayleigh fading MIMO channel. The channel, $\mathbf{H}$, is perfectly known to the receiver. The components of the input data vector are statistically independent and are selected from the finite constellation $\mathcal{S}$ with an average unity power, so that $\mathrm{E}[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_{N_t}$, and each symbol in $\mathcal{S}$ has equal a priori probability. For simplicity, throughout this thesis, we choose $N_t = N_r = M$.

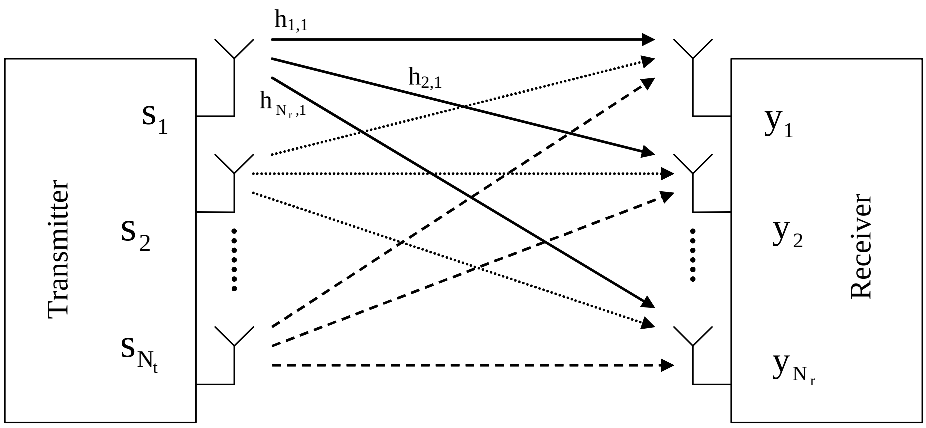

Fig. 2.5. $N_t \times N_r$ MIMO spatial multiplexing system.

With complex constellations for data transmission such as QAM, the complex system model described in (2.27) can be transformed into an equivalent real-valued system by considering the real and imaginary parts separately:

$$\mathbf{y}_r = \mathbf{H}_r \mathbf{s}_r + \mathbf{n}_r, \tag{2.28}$$

where

$$\mathbf{y}_r = \begin{bmatrix} \Re\{\mathbf{y}\} \\ \Im\{\mathbf{y}\} \end{bmatrix}, \mathbf{s}_r = \begin{bmatrix} \Re\{\mathbf{s}\} \\ \Im\{\mathbf{s}\} \end{bmatrix}, \mathbf{n}_r = \begin{bmatrix} \Re\{\mathbf{n}\} \\ \Im\{\mathbf{n}\} \end{bmatrix}$$

and

$$\mathbf{H}_r = \begin{bmatrix} \Re\{\mathbf{H}\} & -\Im\{\mathbf{H}\} \\ \Im\{\mathbf{H}\} & \Re\{\mathbf{H}\} \end{bmatrix}.$$

Note that $\mathbf{H}_r \in \mathbb{R}^{2M\times 2M}$, and $\mathbf{y}_r, \mathbf{s}_r, \mathbf{n}_r \in \mathbb{R}^{2M}$.

The equivalent real-valued data vector $\mathbf{s}_r$ is an i. i. d. symbol vector. This case is typical if the complex symbols are uniformly drawn from a decoupable constellation, $\mathcal{S}$ such as a squared QAM. Thus, a complex-valued system with statistically independent and uniformly drawn 4-QAM symbols can be rearranged as a real-valued system of double the size with statistically independent and uniformly drawn BPSK symbols; i.e., then $\mathbf{s}_r \in \{-1, +1\}^{2M}$.

2.3.1 Detection Strategies for MIMO Systems

MIMO receivers have been studied both for single-user and multiuser systems. This thesis focuses on MIMO single-user systems. Despite its optimality, the ML detector has large computational complexity (depending on the number of antennas and the size of the signal constellation), which limits its applications. Several suboptimum detectors have thus been proposed to achieve close-to-optimum performance with moderate complexity. A popular reduced-complexity MIMO detector is the V-BLAST (Vertical Bell Laboratories Space Time) [46]. Linear receivers such as zero-forcing (ZF) and MMSE have also been designed for BLAST [47]. Although they have very low complexity, their performance is significantly inferior to that of the ML detector and may even lead to numerical instability as they need to compute the pseudo-inverse of the channel gain matrix. When a low-complexity receiver such as ZF or MMSE is applied, the high diversity gain available from the MIMO channel is not realized. The difference in complexity and performance between the linear detector and the optimum ML detector has motivated the development of suboptimum alternatives that exhibit better performance/complexity tradeoffs. The BLAST-ordered decision-feedback (DF) detector, which uses a nonlinear detection strategy, can significantly outperform the linear detector by using optimal ordering [48]. Several modifications have been done to obtain optimal ordering in the DF detector with low complexity [49], [50]. Lattice-reduction (LR) is introduced in [51] for a small (2×2) system. In [52], the LR algorithm is replaced by the basis-reduction algorithm given by A. K. Lenstra, H. W. Lenstra, L. Lovász ("LLL algorithm", [53]) using an equivalent real-valued MIMO channel model. This technique can be applied to MIMO systems with an arbitrary number of antennas. Reference [54] developes a complex LLL algorithm for MIMO detection, which nearly halves the complexity compared to the real LLL algorithm without sacrificing performance.

The optimum ML detector for (2.28) decides on $\mathbf{s}_r$ by using the ML criterion that minimizes the average error probability:

$$\hat{\mathbf{s}}_{opt} = \underset{\mathbf{s}_r \in \mathcal{S}_r^{2M}}{\operatorname{argmin}} \left\| \mathbf{y}_r - \mathbf{H}_r \mathbf{s}_r \right\|^2 . \tag{2.29}$$

For the signal constellation 4-QAM, the size of the search space is $\left|\mathcal{S}_r^{2M}\right| = 2^{2M}$, so a search is required over all 2^{2M} possible combinations. Thus, the computational complexity of the optimum detector grows exponentially with the number of antennas.

ML detection can be efficiently implemented by using sphere decoding. In wireless communication, sphere decoding was first introduced in [55] for lattice code decoding. Sphere decoding achieves ML performance (or a close to approximation to it) with an average complexity in polynomial time at high SNR. Sphere decoding is proposed for multi-antenna systems and space-time codes in [56], for the CDMA system in [57], and for the MIMO system over dispersive channels in [58]. Several improvements also have been made on sphere decoding [59].

The main idea of the sphere decoding algorithm is to reduce the number of candidate symbols to be considered for (2.29) without eliminating the ML solution inadvertently. To achieve this goal, the search is constrained to only those noiseless received lattice points $\mathbf{H}_r \mathbf{s}_r$ that lie inside a hypersphere $\mathcal{D}$ of radius d around the received signal $\mathbf{y}_r$. This inequality is referred to as the sphere constraint:

$$d^2 \geq \left\| \mathbf{y}_r - \mathbf{H}_r \mathbf{s}_r \right\|^2 . \tag{2.30}$$

If $d = \infty$, this algorithm, which is exactly the same as exhaustive search, does not reduce the complexity. The complexity can be actually reduced if radius d is appropriately chosen. d should be small enough to limit the number of candidate vectors but not so small that the hypersphere is empty. The lattice point that achieve the smallest value of $\|\mathbf{y}_r - \mathbf{H}_r\mathbf{s}_r\|^2$ inside the hypersphere is the ML solution. If the hypersphere is empty, the initial search radius d should be increased, and the search continued with the new radius. The worst-case complexity for the sphere decoding is still exponential [23]. However, the performances, i.e., the time complexity of this algorithm are largely dependent on the search radius, and the complexity can become large when the SNR is low or when the problem dimension is high. Thus, the variability of its time complexity can be undesirably high. These considerations motivate the development of alternate near optimal detectors with constant time complexity.

2.4 Summary

In this chapter, some MUD strategies for DS-CDMA were analyzed. The ML criteria for synchronous DS-CDMA, MC-CDMA and MIMO systems were developed. The computational complexity of the ML detection grows exponentially with the problem size. This thesis aims to find low-complexity high-performance detectors.

Chapter 3

Metaheuristic Algorithms

This chapter discusses metaheuristic algorithms with an emphasis on EC algorithms for CO problems. Several EC algorithms are applied to ML detection for synchronous DS-CDMA, MC-CDMA and MIMO systems. An introduction to metaheuristics is given in Section 3.1. In Section 3.2, local search methods are described. Section 3.3 presents different types of EC algorithms. A new type of EC algorithm called EDA is introduced in Section 3.4, and the hybridization of metaheuristics is discussed in Section 3.5.

3.1 Introduction

Chapter 2 mentions that the optimum ML detection problem is an NP-hard CO problem. The complete methods for solving it often lead to computation times too high for practical purposes, even with the advent of new computer technologies and parallel processing. Thus, the use of approximate methods has received much attention in the last 30 years. Metaheuristics are the most recent development in approximate search methods for CO problems. The field of metaheuristics has become an important and rapidly growing area of research and applications. New technologies in telecommunications networks lead to NP-hard problems of large size, and metaheuristics can play an important role in their solutions [60].

3.1.1 Combinatorial Optimization

The domain of CO is optimization problems where the set of feasible solutions is discrete or can be reduced to discrete ones. The objective is to find a solution $s^* \in \mathcal{S}$ with a minimum objective function value; i.e., $f(s^*) \leq f(s)$ for all $s \in \mathcal{S}$ where $\mathcal{S}$ is the set of all feasible solutions.

3.1.2 Metaheuristics

Metaheuristics are approximate algorithms that guide and modify a subordinate heuristic to efficiently produce high-quality solutions. The term *heuristic* is defined by Reeves as follows: "A heuristic is a technique which seeks good (i.e. near optimal) solutions at a reasonable computational cost without being able to guarantee either feasibility or optimality, or even in many cases to state how close to optimality a particular feasible solution is" [31].

Metaheuristic is defined by Osman and Laporte as follows: "A metaheuristic is formally defined as an iterative generation process which guides a subordinate heuristic by combining intelligently different concepts for exploring and exploiting the search space, learning strategies are used to structure information in order to find efficiently near-optimal solutions" [30]. In short, metaheuristics are high level approaches for guiding search processes by using concepts derived from artificial intelligence, biological, mathematical, neural and physical sciences. An overview of metaheuristics and conceptual comparison is given in [61].

Two very important concepts in metaheuristics are intensification and diversification, which enable metaheuristic applications to achieve a high performance. Diversification generally refers to the investigation of the search space; i.e., it allows the process to search other parts of the solution space whenever search is being trapped in a local optimum. Intensification refers to the utilization of the accumulated search experiences, so that diversification and intensification are both contrary and complementary. A dynamic balance between diversification and intensification is essential since on one side intensification helps to quickly identify regions in the search space with high-quality solutions and on the other side diversification does not waste too much time in regions of the search space which either have been already explored or which do not provide high-quality solutions.

Metaheuristics can be classified according to the number of solutions used per instance: a search can be either single-point- or population-based. A single-point search, known as the trajectory method [61], considers a single solution at a time and encompasses local search-based metaheuristics, such as Iterative Improvement, Simulated Annealing (SA), TS, ILS, Guided Local Search (GLS), Variable Neighborhood Search (VNS), and Greedy Randomized Adaptive Search Procedure (GRASP). Population-based metaheuristics consider a set of solutions as a population concurrently and encompass EC algorithms such as GA, EP, ES, and Genetic Programming (GP), ACO, Particle Swarm Optimization (PSO), and Scatter Search (SS). EC algorithms are artificial intelligence methods for optimization which uses mechanisms based on biological evolution: selection, recombination, mutation, reproduction and others.

3.2 Trajectory Methods

In trajectory methods, the search process is characterized by a trajectory in the search space. The algorithm starts from an initial state and moves in a state space trajectory. In this section, two trajectory methods used in the thesis are explained.

3.2.1 Iterative Improvement

Iterative improvement is the basic local search algorithm where a subset of the feasible solutions is discovered by continually moving from the current solution to a neighborhood solution. Neighborhood solutions are generated by a move strategy.

A solution can be specified by a vector $\mathbf{s}$. The set of all feasible solutions is denoted by $\mathcal{S}$, and the cost of the solution is denoted by $f(\mathbf{s})$, which is often called as objective function. Each solution $\mathbf{s} \in \mathcal{S}$ has an associated set of neighbors $\mathcal{N}(\mathbf{s}) \subset \mathcal{S}$ that are in the vicinity of $\mathcal{S}$. The set $\mathcal{N}(\mathbf{s})$ is called the neighborhood of $\mathbf{s}$, and each solution $\tilde{\mathbf{s}} \in \mathcal{N}(\mathbf{s})$ is called a neighbor of $\mathbf{s}$ that can be reached directly from $\mathbf{s}$ by a move operation [62]. A local search procedure starts from a feasible solution $\mathbf{s}_1 \in \mathcal{S}$. At each step p, a new solution $\mathbf{s}_{p+1} \in \mathcal{N}(\mathbf{s}_p)$ can be reached directly from $\mathbf{s}_p$ by a move operation. Generally, two types of move strategies can be used in the local search: first improvement and best improvement [61]. The former evaluates the neighborhood $\mathcal{N}(\mathbf{s}_p)$ and takes the first improved solution $\mathbf{s}_{p+1} \in \mathcal{N}(\mathbf{s}_p)$ that is better than $\mathbf{s}_p$. The latter one evaluates all neighbors of $\mathbf{s}_p$ and selects the improved neighborhood solution $\mathbf{s}_{p+1}$ with the best objective function value.

For a minimization problem, the traditional form of local search is equivalent to a steepest descent strategy, in which each move is performed only if the resulting solution has a lower cost than that of the current solution. An optimal solution could be found as a solution or a set of solutions at the minimum possible cost in a feasible neighborhood solution space of the problem. This form of local search is also known as hill-climbing. The most general way for selecting the next solution $\mathbf{s}_{p+1}$ is to pick the best one in the neighborhood of $\mathbf{s}_p$,

$$f(\mathbf{s}_{p+1}) \leq f(\mathbf{s}_p), \qquad \forall \mathbf{s} \in \mathcal{N}(\mathbf{s}_p). \tag{3.1}$$

The search process terminates when no better solution is found in the neighborhood. Both strategies of moving often result in a convergence to a local rather than a global optimum. The performance of this search method depends on the defining criterion of the neighborhood $\mathcal{N}(\mathbf{s})$, the move strategy, the speed of evaluation of the objective function and the starting solution.

The neighborhood of a solution characterized by binary variables can be defined on the basis of the Hamming distance (d_H) between two binary vectors. Therefore, the neighborhood of a solution symbolized by a binary vector can be defined by the solutions that can be obtained by flipping a single or multiple elements in the binary vector simultaneously.

1-*opt* local search

The simplest form of local search is the 1-*opt* local search. In each step of the search process, a solution with a better fitness value in the neighborhood of the current solution is obtained. The 1-*opt* neighborhood of ($\mathbf{s}$) contains all solutions with a Hamming distance of 1 to ($\mathbf{s}$), i.e.,

$$\mathcal{N}_{1-opt}(\mathbf{s}) = \{\tilde{\mathbf{s}} \in \mathcal{S} | d_H(\mathbf{s}, \tilde{\mathbf{s}}) = 1\}, \tag{3.2}$$

where $d_H(\mathbf{s}, \tilde{\mathbf{s}})$ denotes the hamming distance between $\mathbf{s}$ and $\tilde{\mathbf{s}}$. For example, if $\mathbf{s} \in \{-1, +1\}^K$, $K = 4$, and $\mathbf{s} = (+1, +1, +1, +1)$, the $\mathcal{N}_{1-opt}(\mathbf{s})$ are $(-1, +1, +1, +1)$, $(+1, -1, +1, +1)$, $(+1, +1, -1, +1)$ and $(+1, +1, +1, -1)$. The pseudocode of the 1-opt local search algorithm for the minimization problem is given in Fig. 3.1.

Procedure: 1-opt local search ()
$\mathbf{s} \leftarrow$ Generate Initial Solution
begin
  repeat
    choose $\tilde{\mathbf{s}} \in N_{1-opt}(\mathbf{s})$
    if $f(\tilde{\mathbf{s}}) < f(\mathbf{s})$
    then $\mathbf{s} \leftarrow \tilde{\mathbf{s}}$
    endif
  until $f(\tilde{\mathbf{s}}) \geq f(\mathbf{s}), \forall \tilde{\mathbf{s}} \in N_{1-opt}(\mathbf{s})$
  return s
end

Fig. 3.1. Algorithm: 1-opt local search.

A larger neighborhood can be obtained by flipping up to k elements in the current solution vector simultaneously; this search process is known as the k-*opt* local search. The neighborhood of size k can be defined as

$$\mathcal{N}_{k-opt}(\mathbf{s}) = \{\tilde{\mathbf{s}} \in \mathcal{S} | d_H(\mathbf{s}, \tilde{\mathbf{s}}) \leq k\}, \tag{3.3}$$

where $1 < k < K$, and K is the length of $\mathbf{s}$. The size of the neighborhood grows exponentially with k and so does the complexity.

3.2.2 Simulated Annealing

Local search methods run the risk of being trapped in local optima. Then numerous approaches suggested to avoid this problem are known as global search techniques. One of them is SA, which is capable of escaping local optima by accepting up-hill/down-hill moves.

SA is an effective heuristic approach to solve a large number of problems where a neighbor of current solution is continually selected and then the difference of cost function of these solutions is compared to a threshold in order to identify any improvement. If the cost difference is below the threshold, the current solution is replaced by the improved neighbor solution, and the process is repeated. Otherwise, the search continues with the current solution being considered as an approximation of the optimum. SA uses a uniform randomized threshold. Each neighbor solution can be accepted with a positive probability to substitute the current solution [63].

The difference between SA and the descent algorithm is that here, the neighbors which give rise to an increase in the cost function may be accepted, and this acceptance depends on a control parameter and the magnitude of increase of the cost function. The idea of SA derives from an analogy with the physical annealing process. Annealing is a thermodynamic process by which solids are heated to a high temperature and cooled gradually until they solidify into a low-energy state. The control parameter of the SA algorithm is equivalent to the temperature of the annealing process [63].

At each step p of the SA process, a solution $\tilde{\mathbf{s}}$ in $\mathcal{N}(\mathbf{s})$ is generated from the current solution $\mathbf{s}$. If $\tilde{\mathbf{s}}$ has a lower cost function than $\mathbf{s}$, a new solution is accepted unconditionally, but even if $\tilde{\mathbf{s}}$ has a higher cost function value, then $\tilde{\mathbf{s}}$ will be accepted with a probability which is a function of the control parameter; the temperature, denoted as T; and the difference between two cost functions, $f(\tilde{\mathbf{s}}) - f(\mathbf{s})$. The probability is usually computed by following the Boltzman distribution. At step p, the probability

$Pr(p)$ is chosen as

$$Pr(p) = \exp\left[-\frac{\Delta f_p}{T(p)}\right], \tag{3.4}$$

where $\Delta f_p = |f(\tilde{\mathbf{s}}) - f(\mathbf{s})|$, and $T(p)$ is the temperature at step p.

At first, the temperature is kept high to increase the probability of accepting uphill moves, so that almost any move is accepted, allowing us to explore the solution space. The higher the temperature, the higher the probability of accepting the move. Thus, SA allows diversification. Hence, the probability decreases as the temperature decreases. Then, gradually, the temperature is decreased to make the process more selective in accepting new solutions. The temperature is reduced according to a "cooling schedule", which specifies the initial temperature and the rate at which the temperature decreases. The most common cooling schedule is the geometric function used in this thesis to reduce the temperature T by a constant factor α by using $T = \alpha T$ after each iteration where $0 < \alpha < 1$. The pseudocode of the standard SA algorithm for minimization problem is given in Fig. 3.2.

Procedure: Simulated Annealing ()
$\mathbf{s} \leftarrow$ Generate Initial Solution
$T \leftarrow T_0$ (Initial Temperature)
while termination conditions not met **do**
   choose $\tilde{\mathbf{s}} \in N(\mathbf{s})$ randomly
   if $f(\tilde{\mathbf{s}}) < f(\mathbf{s})$
   then $\mathbf{s} \leftarrow \tilde{\mathbf{s}}$
   else accept $\tilde{\mathbf{s}}$ as new solution with probability $\Pr(T, \mathbf{s}, \tilde{\mathbf{s}})$
   endif
   Update (T)
endwhile

Fig. 3.2. Algorithm: Simulated Annealing.

The SA algorithm was originally used to solve a unconstrained binary quadratic programming problem. In this thesis, we apply the SA based metaheuristic [64], which is a simple extension of standard SA. The 1-*opt* local search is applied once at the end of the standard SA in order to provide a better performance.

3.3 Evolutionary Computation

EC algorithms use population-based metaheuristic optimization techniques to solve a large class of problems. These techniques originate from the natural process of biological evolution, by mimicking some theories on how species evolve and adapt to their environment. Although the convergence of EC algorithms is possible to a global optimum only in a weak probabilistic sense, they have successfully been used to solve various complex CO problems.

EC algorithms maintain a population of structures that evolve by using operators such as selection, recombination and mutation. Each individual in the population is evaluated by using a fitness function, and individuals with high fitness values are selected. The recombination operator recombines two or more individuals to produce new individuals, and mutation causes a self-adaption of individuals. In the EC algorithm, the use of a recombination operator is the intensification strategy that explicitly guides the search areas of elite individuals, i.e., that intensively explores areas of the search space with high-quality solutions. As well the use of a mutation operator is the diversification strategy that performs the perturbation of an individual to move to unexplored areas of the search space. The search method is terminated by using a predefined criterion. The main advantages of EC algorithms are their simplicity, as they are relatively cheap and quick to implement, and their ability to cope well with noisy, inaccurate data in general [65]. For the ML detection problem, the $(1 + \lambda)$ ES algorithm is shown to perform better than GA, EP in [29]. In this thesis, we consider $(1 + \lambda)$ ES.

3.3.1 Evolutionary Strategy

ES is a probabilistic optimization technique with several variations such as $(1+1)$ ES, $(1+\lambda)$ ES, $(\mu+\lambda)$ ES, (μ,λ) ES [66]. ES emphasizes the behavioral change at the level of individuals. $(1+\lambda)$ ES uses a parent population size of 1 and creates λ individuals/offsprings in each generation by mutation. Reproductive selection or recombination is not used in this algorithm, making ES simpler than other EC algorithms, but reproductive variation is accomplished via a standard bit flip mutation operator that flips each bit of an individual independently of the other bits with some mutation probability. The basic $(1+\lambda)$ ES [67] has two main steps:

1. Mutation of the current solution to produce offsprings.
2. Selection of the best offspring for the next generation by using the fitness function.

For the minimization problem, in the selection step, the parent is replaced by an offspring with minimum fitness if and only if the offspring's minimum fitness is less than or equal to the parent's fitness. The process is repeated to get better and better solutions and until some stopping condition is fulfilled. The pseudocode of the $(1+\lambda)$ ES algorithm for the minimization problem is given in Fig. 3.3.

3.4 Estimation of Distribution Algorithms

The parameters of EC algorithms (crossover and mutation operators, probabilities of crossover and mutation, size of population etc., which control the creation of new individuals) need to be tuned correctly to obtain a good result. The task of making the best choice of those parameters is itself an optimization problem [68]. Moreover, the prediction of the movements of the populations in the search space of the EC algorithm is extremely difficult. In [69], the author mentions that considering the interactions among the variables that represent individuals can be useful for an intelligent search

Procedure: $(1+\lambda)$ Evolutionary Strategy ()
$\mathbf{s} \leftarrow$ Generate Initial Solution (parent)
while termination conditions not met **do**
For each $i \in \{1,...,\lambda\}$: do mutation to generate λ individuals $\{\tilde{\mathbf{s}}_i\}$ with mutation probability p_m.
if $\min\{f(\tilde{\mathbf{s}}_1),..., f(\tilde{\mathbf{s}}_\lambda)\} \le f(\mathbf{s})$
then $\mathbf{s} \leftarrow \tilde{\mathbf{s}}_{i,\min}$
endif
endwhile

Fig. 3.3. Algorithm: $(1 + \lambda)$ Evolutionary Strategy.

through the solution space. This idea, together with the drawbacks of typical EC algorithms, inspired the development of a new type of EC algorithms named EDAs.

EDAs were introduced in [38] and require no mutation or crossover operations. Instead, better solutions are selected from the current population, and global statistical information about the search space is extracted explicitly from the selected solutions. A probabilistic model of the promising solutions is developed. New solutions are sampled from the model thus built. After each iteration, the probability model is updated until the stopping conditions are met. Several EDAs have been proposed for global optimization problems [68]. One of the first EDAs is Population-based Incremental Learning (PBIL), which was proposed for solving CO problems [70]. We develop two PBIL-based detectors.

3.4.1 Population-based Incremental Learning

The PBIL algorithm is a combination of evolutionary optimization and hill climbing. The objective is defined in the binary space $\Omega = \{0, 1\}^n$, where n is the size of the problem. PBIL generates a real-valued probability vector to sample the search space. The probabilistic vector is initialized with entries set to 0.5. A number of random solution vectors are generated by using this probability vector. The set of solutions is evaluated according to the problem-specific fitness function. The probability vector is pushed towards the selected solutions depending on the parameter of the learning rate. After the probability vector is updated, the cycle is repeated. The search stops when a termination condition is satisfied. The PBIL pseudocode for the minimization problem is given in Fig. 3.4.

Procedure: Population-based Incremental Learning ()
$p_0(\mathbf{s})$: Generate Initial Probability Vector
while termination conditions not met **do**
  Using $p_g(\mathbf{s})$, obtain λ individuals as $\mathbf{s}_1^g, \ldots, \mathbf{s}_k^g, \ldots, \mathbf{s}_\lambda^g$
  Evaluate and rank individuals according to ascending order
  Select the $N(N \leq \lambda)$ best individuals: $\mathbf{s}_{1,\lambda}^g, \ldots, \mathbf{s}_{k,\lambda}^g, \ldots, \mathbf{s}_{N,\lambda}^g$
  Update the probability vector:

$$p_{g+1}(\mathbf{s}) = (1-\alpha) p_g(\mathbf{s}) + \alpha \frac{1}{N} \sum_{k=1}^{N} \mathbf{s}_{k:\lambda}^g$$

  where $\alpha \in (0,1]$ is the learning rate of the algorithm
endwhile

Fig. 3.4. Algorithm: Population-based Incremental Learning.

3.5 Hybrid Metaheuristics

Although EC is a tremendously growing field, some form of domain knowledge must be incorporated into EC algorithms to make them competitive with other domain-specific optimization techniques [71]. There are several ways to achieve this incorporation. A simple and promising way is hybridization with other domain-specific heuristics. One of the most popular methods of hybridization involves the use of local search-based methods with population-based methods. EC algorithms that apply a local search algorithm to each individual of a population are called *memetic* algorithms [72]. To identify a highly effective search space, the combination of EC algorithms with a local search has been shown to be promising.

The strength of EC algorithms is based mainly on the concept of recombining or mutating solutions, which allows for guided steps in the search space, which are usually larger than the steps made by local search-based methods. The advantage of using local search methods results from how they explore a promising region in the search space. In short, EC methods are better in identifying promising areas in the search space, whereas local search methods are better in exploring the promising areas in the search space. The hybridization of EC and local search algorithms are thus successful, as with these methods, the local search is the driving component, and a promising region in the search space is searched in a more structured way. The use of local search techniques in the EC algorithms is an intensification strategy which helps to quickly identify "good" areas in the search space. Thus, combining a local search with EC algorithms increases the efficiency of EC, since a problem's attributes can be exploited in the local search to speed up the neighborhood search process. In this thesis, several hybridized algorithms are developed to solve the ML detection problem efficiently.

3.6 Summary

An overview of metaheuristic algorithms, focusing on EC, were provided in this chapter. Three metaheuristics, iterative improvement, SA, and $(1+\lambda)$ ES, for CO problems were described. A new type of EC algorithm, EDA was introduced. PBIL, one of the first EDA approaches to CO, was described. The hybridization of metaheuristics was studied. This promising approach is designed to overcome a typical problem in metaheuristics: being trapped in local optima.

Chapter 4

Metaheuristics for Maximum Likelihood Detection

In this chapter, hybrid $(1 + \lambda)$ ES, SA, PBIL, Modified PBIL are proposed for ML detection. These algorithms are then evaluated through numerical experiments based on synchronous DS-CDMA, MC-CDMA and spatial multiplexing MIMO systems. These algorithm-based detectors provide near-optimal or optimal BER performances. For large systems, all these algorithms also perform better in terms of computational complexity compared to existing detectors.

4.1 Hybrid $(1 + \lambda)$ ES for ML Detection

Since the $(1 + \lambda)$ ES algorithm has no recombination operator, we develop a hybrid $(1 + \lambda)$ ES that uses an intensification strategy in the search process. This hybrid algorithm employs $(1 + \lambda)$ ES for the basic search which directs the search process towards an elitist solution space. A simple 1-*opt* local search is applied for a thorough search in the elitist region. When this algorithm is used for ML-MUD for DS-CDMA, the following fitness/objective function is used to evaluate offsprings:

$$f(\mathbf{b}) = \mathbf{b}^T \mathbf{\Sigma}^T \mathbf{R} \mathbf{\Sigma} \mathbf{b} - 2\mathbf{y}^T \mathbf{\Sigma} \mathbf{b}. \tag{4.1}$$

For a K-user synchronous DS-CDMA system using BPSK modulation, the hybrid ES MUD algorithm detector can be described as follows:

- The output of the single-user MF receiver is taken as the initial solution (parent): $\mathbf{b}_p = \mathbf{b}_{initial} = \hat{\mathbf{b}}_{CD} = \text{sign}(\mathbf{y})$.
- The population size of the offspring is set as $\lambda = \lfloor K \ln K \rfloor$, where $\lfloor x \rfloor$ is the largest integer less than x.
- For iteration $m = 1, 2, ..., N_g$; N_g is the number of generations,
  1. For each $l = 1, 2, ..., \lambda$, $\acute{\mathbf{b}}_l \in \{-1, +1\}^K$ is created by copying $\mathbf{b}_p$ and independently flipping each bit with mutation probability P_m.
  2. For each individual of the λ population, the 1-*opt* local search is performed, representing all individuals in the population as local minima. For each $\acute{\mathbf{b}}_l, l = 1, 2, ..., \lambda$, the best neighbor $\acute{\mathbf{b}}_{l,best}$ in the 1-*opt* neighborhood $N(\acute{\mathbf{b}}_l)$ is searched by evaluating the objective function. If $f(\acute{\mathbf{b}}_{l,best}) < f(\acute{\mathbf{b}}_l)$, then $\acute{\mathbf{b}}_l \leftarrow \acute{\mathbf{b}}_{l,best}$ else $\acute{\mathbf{b}}_l \leftarrow \acute{\mathbf{b}}_l$.
  3. Each locally minimized individual of the current population is then evaluated by using the fitness function, and the individual with the minimum fitness value is determined as $\acute{\mathbf{b}}_{l,\min}$.
  4. If $f(\acute{\mathbf{b}}_{l,\min}) \leq f(\mathbf{b}_p)$, then $\mathbf{b}_p \leftarrow \acute{\mathbf{b}}_{l,\min}$ else $\mathbf{b}_p \leftarrow \mathbf{b}_p$.
- The whole search process is terminated after N_g generations, giving $\mathbf{b}_p$ as the best solution.

When the hybrid $(1 + \lambda)$ ES is applied to ML-MUD for MC-CDMA, the following fitness/objective function is used to evaluate the offsprings:

$$f(\mathbf{b}_r) = \|\mathbf{y}_r - \mathbf{D}_r \mathbf{b}_r\|^2 . \tag{4.2}$$

For a K-user synchronous MC-CDMA system using 4-QAM, the hybrid ES MUD algorithm can be described as follows:

- The output of the matched filter is taken as the initial solution which is the parent: $\mathbf{b}_{r(p)} = \hat{\mathbf{b}}_{r(ZF)} = \text{sign}((\mathbf{D}_r^T\mathbf{D}_r)^{-1}\mathbf{D}_r^T\mathbf{y}_r)$.

- The population size of the offspring is set as: $\lambda = \lfloor 2K \ln 2K \rfloor$.

- For iteration $m = 1, 2, ..., N_g$; N_g is the number of generations,

  1. For each $l = 1, 2, ..., \lambda$, $\acute{\mathbf{b}}_{r(l)} \in \{-1, +1\}^{2K}$ is created by copying $\mathbf{b}_{r(p)}$ and independently flipping each bit with mutation probability P_m.
  2. For each individual of the λ population, the 1-*opt* local search is performed, representing all individuals in the population as local minima. For each $\acute{\mathbf{b}}_{r(l)}, l = 1, 2, ..., \lambda$, the best neighbor $\acute{\mathbf{b}}_{r(l,best)}$ in the 1-*opt* neighborhood $\mathcal{N}(\acute{\mathbf{b}}_{r(l)})$ is searched by evaluating the objective function. If $f(\acute{\mathbf{b}}_{r(l,best)}) < f(\acute{\mathbf{b}}_{r(l)})$, then $\acute{\mathbf{b}}_{r(l)} \leftarrow \acute{\mathbf{b}}_{r(l,best)}$ else $\acute{\mathbf{b}}_{r(l)} \leftarrow \acute{\mathbf{b}}_{r(l)}$.
  3. Each locally minimized individual of the current population is then evaluated by using the fitness function, and the individual with the minimum fitness value is determined as $\acute{\mathbf{b}}_{r(l,\min)}$.
  4. If $f(\acute{\mathbf{b}}_{r(l,\min)}) \leq f(\mathbf{b}_{r(p)})$, then $\mathbf{b}_{r(p)} \leftarrow \acute{\mathbf{b}}_{r(l,\min)}$ else $\mathbf{b}_{r(p)} \leftarrow \mathbf{b}_{r(p)}$.

- The whole search process is terminated after N_g generations, giving $\mathbf{b}_{r(p)}$ as the best solution.

When the hybrid $(1 + \lambda)$ ES is applied to ML problem for a spatial multiplexing MIMO system, the following fitness/objective function is used to evaluate the offsprings:

$$f(\mathbf{s}_r) = \|\mathbf{y}_r - \mathbf{H}_r\mathbf{s}_r\|^2 . \tag{4.3}$$

For an $M \times M$ single-user MIMO system using 4-QAM, the hybrid ES algorithm for the ML detector can be described in the same way as for the MC-CDMA system, where K should be replaced by M, $\mathbf{b}_r$ should be replaced by $\mathbf{s}_r$ and $\mathbf{D}_r$ should be replaced by $\mathbf{H}_r$.

Janson, Jong and Wegner [67] analyzed the effect of the offspring population size λ. The influence of this parameter on the ES search performance is high because the computation cost of the fitness evaluation depends largely on this parameter and function evaluation is the main computation cost for the ML detection problem. If λ is too small, the convergence is slow, and if λ is too high, the computational complexity is high. The number of fitness function evaluations in different instances for ML-MUD is also investigated in [29]. In [67] and [29], it is found that the computation cost is the least if λ is approximately $K \ln K$, providing a better convergence rate, where K is the number of users when mutation is the only operator exploring the search space. Although several papers have suggested that mutation probability for the ES algorithm should be $1/n$, where n is the problem size, mutation probability P_m is chosen in this thesis for simulation, as recommended in [29], where it is shown that setting $P_m = 0.2$ makes the ES algorithm escape from local optima efficiently.

4.2 Simulated Annealing for ML Detection

The SA-based MUD algorithm for a K-user synchronous DS-CDMA system using BPSK modulation can be outlined as follows:

- The output of the single-user matched filter is taken as the initial solution: $\mathbf{b}_1 = \hat{\mathbf{b}}_{CD} = \text{sign}(\mathbf{y})$. Temperature T is initialized to a suitably high value, and the constant factor α is chosen as $0 < \alpha < 1$ for temperature reduction.

- The best objective value f^* of f and the corresponding solution $\mathbf{b}^*$ are initialized as $f^* \leftarrow f(\mathbf{b}_1)$ and $\mathbf{b}^* \leftarrow \mathbf{b}_1$, where

$$f(\mathbf{b}_1) = \mathbf{b}_1^T \mathbf{\Sigma}^T \mathbf{R} \mathbf{\Sigma} \mathbf{b}_1 - 2\mathbf{y}^T \mathbf{\Sigma} \mathbf{b}_1. \tag{4.4}$$

- For step $p = 1, 2, ...$; $\mathbf{b}_p$ denotes the current solution, and $\acute{\mathbf{b}}$ denotes the solution in $\mathcal{N}_{1-opt}(\mathbf{b}_p)$ for which $f(\acute{\mathbf{b}}) = \acute{f}$.

1. A position k is chosen randomly from the current solution vector such that it has not been picked up before, and then the bit of that position is flipped.

2. If $f(\acute{\mathbf{b}}) < f(\mathbf{b}_p)$, then $\mathbf{b}_p \leftarrow \acute{\mathbf{b}}$.

3. If $f(\acute{\mathbf{b}}) < f(\mathbf{b}^*)$, then $\mathbf{b}^* \leftarrow \acute{\mathbf{b}}$ and $f^* \leftarrow \acute{f}$.

4. If $f(\acute{\mathbf{b}}) > f(\mathbf{b}_p)$, a random number r is drawn from $[0, 1]$. If $r < Pr(p)$, then $\mathbf{b}_p \leftarrow \acute{\mathbf{b}}$, or else, the bit of kth from the solution vector is again flipped to reset.

5. If all positions of the solution vector have been chosen for flipping, and a better new solution is found, then $\mathbf{b}_{p+1} \leftarrow \mathbf{b}_p$, else $\mathbf{b}_p \leftarrow \mathbf{b}_p$ and T is reduced as $T = \alpha T$, which is a common cooling schedule.

- The whole search process is terminated after the stopping condition is fulfilled, giving $\mathbf{b}^*$ as the best solution.

- Then simple 1-*opt* local search is performed with this best solution $\mathbf{b}^*$.

Here, for the neighborhood structure, a simple 1-*opt* neighborhood is used to reduce the complexity of function evaluation, and local search is used to intensify the search process.

The SA-based MUD algorithm for a K-user synchronous MC-CDMA system using 4-QAM can be outlined as follows:

- The output of the matched filter is taken as the initial solution: $\mathbf{b}_{r(1)} = \hat{\mathbf{b}}_{r(ZF)} = \mathrm{sign}((\mathbf{D}_r^T \mathbf{D}_r)^{-1} \mathbf{D}_r^T \mathbf{y}_r)$. Temperature T is initialized to a suitably high value, and the constant factor α is chosen as $0 < \alpha < 1$ for temperature reduction.

- The best objective value f^* of f and the corresponding solution $\mathbf{b}_r^*$ are initialized as $f^* \leftarrow f(\mathbf{b}_{r(1)})$ and $\mathbf{b}_r^* \leftarrow \mathbf{b}_{r(1)}$, where,

$$f(\mathbf{b}_{r(1)}) = \left\| \mathbf{y}_{r(1)} - \mathbf{D}_{r(1)} \mathbf{b}_{r(1)} \right\|^2 . \tag{4.5}$$

- For step $p = 1, 2, ...;$ $\mathbf{b}_{r(p)}$, where $\mathbf{b}_{r(p)} \in \{-1, +1\}^{2K}$ denotes the current solution, $\acute{\mathbf{b}}_r$ denotes the solution in $\mathcal{N}_{1-opt}(\mathbf{b}_{r(p)})$ for which $f(\acute{\mathbf{b}}_r) = \acute{f}$.

  1. A position k is chosen randomly from the current solution vector such that it has not been picked up before, and then the bit of that position is flipped.
  2. If $f(\acute{\mathbf{b}}_r) < f(\mathbf{b}_{r(p)})$, then $\mathbf{b}_{r(p)} \leftarrow \acute{\mathbf{b}}_r$.
  3. If $f(\acute{\mathbf{b}}_r) < f(\mathbf{b}_r^*)$, then $\mathbf{b}_r^* \leftarrow \acute{\mathbf{b}}_r$ and $f^* \leftarrow \acute{f}$.
  4. If $f(\acute{\mathbf{b}}_r) > f(\mathbf{b}_{r(p)})$, a random number r is drawn from $[0, 1]$. If $r < Pr(p)$, then $\mathbf{b}_{r(p)} \leftarrow \acute{\mathbf{b}}_r$, or else, the bit of kth from the solution vector is again flipped to reset.
  5. If all positions of the solution vector have been chosen for flipping and better new solution is found, then $\mathbf{b}_{r(p+1)} \leftarrow \mathbf{b}_{r(p)}$ or else, $\mathbf{b}_{r(p)} \leftarrow \mathbf{b}_{r(p)}$, and T is reduced as $T = \alpha T$.

- The whole search process is terminated after the stopping condition is fulfilled, giving $\mathbf{b}_r^*$ as the best solution.

- A simple 1-*opt* local search is performed with this best solution $\mathbf{b}_r^*$.

When SA is applied to MIMO ML detection, the following fitness/objective function is used to evaluate the offsprings:

$$f(\mathbf{s}_{r(1)}) = \left\|\mathbf{y}_{r(1)} - \mathbf{H}_{r(1)}\mathbf{s}_{r(1)}\right\|^2. \tag{4.6}$$

For an $M \times M$ single-user MIMO system using 4-QAM, the SA algorithm for the ML detector can be described in the same way as for the MC-CDMA system, where $\mathbf{b}_r$ should be replaced by $\mathbf{s}_r$ and $\mathbf{D}_r$ should be replaced by $\mathbf{H}_r$.

The performance of SA depends on some implementation choices such as the initial temperature, the cooling schedule to reduce the temperature, the stopping criterion and the conditions for reaching the *thermal equilibrium* at each temperature. The initial temperature should be large enough in order to guarantee the independence of the final solution from the starting solution. This thesis uses $T_{initial} = 0.3K$, where K is the number of user, as recommended in [63]. The temperature reduction rate α is set to 0.99 as in [63], but it could be varied in the range from 0.8 to 0.99 for different optimization problems [31]. The stopping criteria is chosen as the number of iterations exceeding a given maximum.

4.3 PBIL for ML Detection

When the PBIL algorithm is applied to the ML-MUD problem for a K user synchronous DS-CDMA system using BPSK modulation, this algorithm uses the following real-valued probability vector to represent the attributes of the population:

$$p_g(\mathbf{b}) = [p_g(b_1)...p_g(b_m)...p_g(b_K)], \tag{4.7}$$

where $p_g(b_m)$ refers to the probability of getting a value of 1 in the m^{th} variable of the g^{th} generation of the population. The PBIL MUD algorithm can be described as follows:

1. The probability vector is initialized as $p_0(\mathbf{b}) = [0.5...0.5...0.5]$.
2. The population size of individual is set as $\lambda = \lfloor K \ln K \rfloor$.
3. For iteration $g = 1, 2, ..., N_g$; N_g is the number of generations.
   (a) By using $p_g(\mathbf{b})$, λ individuals are generated: $\mathbf{b}_1^g, ..., \mathbf{b}_k^g, ..., \mathbf{b}_\lambda^g$. Here, individuals are represented by $\{0, 1\}$ instead of $\{-1, 1\}$.
   (b) Each individual of the current population is then evaluated by using (4.1), and the individuals are ranked according to ascending order of fitness value.
   (c) N $(N \leq \lambda)$ best individuals are selected from the ranked population: $\mathbf{b}_{1:\lambda}^g, ..., \mathbf{b}_{k:\lambda}^g, ..., \mathbf{b}_{N:\lambda}^g$.

(d) The probability vector is updated based on these selected individuals by using a Hebbian-inspired rule:

$$p_{g+1}(\mathbf{b}) = (1-\alpha)p_g(\mathbf{b}) + \alpha \frac{1}{N}\sum_{k=1}^{N} \mathbf{b}_{k:\lambda}^{g}, \tag{4.8}$$

where $\alpha \in (0, 1]$ is the learning rate of the algorithm.

(e) A new population is generated by sampling using the updated probability vector.

4. As the search advances, the entries in the probability vector converge to either 0.0 or 1.0.

5. The whole search process is terminated after N_g generations.

6. With the last updated probability vector, the best solution $\mathbf{b}$ is generated.

When the PBIL algorithm is applied to ML detection for an $M \times M$ MIMO system using 4-QAM, the following real-valued probability vector represents the attributes of the population:

$$p_g(\mathbf{s}_r) = [p_g(s_{r(1)})...p_g(s_{r(m)})...p_g(s_{r(2M)})], \tag{4.9}$$

where $p_g(s_{r(m)})$ refers to the probability of getting a value of 1 in the m^{th} variable of the g^{th} generation of the population. The PBIL algorithm for the MIMO ML detector can thus be described as follows:

1. The probability vector is initialized as $p_0(\mathbf{s}_r) = [0.5...0.5...0.5]$.

2. The population size of individual is set as $\lambda = \lfloor 2M \ln 2M \rfloor$.

3. For iteration $g = 1, 2, ..., N_g; N_g$ is the number of generations.

   (a) By using $p_g(\mathbf{s}_r)$, λ individuals are generated: $\mathbf{s}_{r(1)}^g, ..., \mathbf{s}_{r(k)}^g, ..., \mathbf{s}_{r(\lambda)}^g$. Here, individuals are represented by $\{0, 1\}$ instead of $\{-1, 1\}$.

(b) Each individual of the current population is then evaluated by using (4.3), and the individuals are ranked according to ascending order of fitness value.

(c) N ($N \leq \lambda$) best individuals are selected from the ranked population: $\mathbf{s}^g_{r(1:\lambda)}, \ldots, \mathbf{s}^g_{r(k:\lambda)}, \ldots, \mathbf{s}^g_{r(N:\lambda)}$.

(d) The probability vector is updated based on these selected individuals by using a Hebbian-inspired rule:

$$p_{g+1}(\mathbf{s}_r) = (1-\alpha)p_g(\mathbf{s}_r) + \alpha\frac{1}{N}\sum_{k=1}^{N}\mathbf{s}^g_{r(k:\lambda)}, \tag{4.10}$$

where $\alpha \in (0, 1]$ is the learning rate of the algorithm.

(e) A new population is generated by sampling using the updated probability vector.

4. As the search advances, the entries in the probability vector converge to either 0.0 or 1.0.

5. The whole search process is terminated after N_g generations.

6. With the last updated probability vector, the best solution $\mathbf{s}_r$ is generated.

When PBIL is applied to a K user synchronous MC-CDMA system, the objective function which will be used for evaluating the offsprings is (4.2). The whole process is the same as for the MIMO system, where M should be replaced by K, $\mathbf{s}_r$ should be replaced by $\mathbf{b}_r$ and $\mathbf{H}_r$ should be replaced by $\mathbf{D}_r$.

The population size and learning rate are used to control PBIL algorithms. The size of the offspring population λ has the same impact on the search process as in $(1+\lambda)$ ES. Increasing the population size will increase the chance of finding the global optimum solution while increasing the number of function evaluations required. So λ is kept same as in hybrid $(1+\lambda)$ ES. The effect of the learning rate α is analyzed by Baluja [70]. If α is high, the algorithm will not fully explore the search space and may converge to a local optimum; i.e., the probability of premature algorithm convergence increases. A lower learning rate will allow for greater exploration and escape from local optima. The learning rate lies in the range 0 to 1. In this thesis, we set $\alpha = 0.15$ for simulation. This parameter is chosen empirically.

4.4 Modified PBIL for ML Detection

An efficient EC algorithm must utilize both the local and global information. Global information can guide the search for exploring promising areas whereas the local information can be useful for exploiting the search. The search in PBIL is based mainly on global information, so to enhance performance, we propose a modified PBIL-based MIMO detector by combining PBIL as in [73] with a simple 1-*opt* local search [35]. In [73], the probability vector is learnt, i.e., pushed towards only the best solution instead of towards a set of better solutions. Thus, the algorithm keeps searching in only the most promising region. And 1-*opt* local search helps to make a thorough search in this elitist region. Here, local search performs as an intensification strategy of metaheuristic.

For the modified PBIL algorithm, steps (1) and (2) and steps (4), (5), and (6) are the same as for the PBIL-based multiuser detector for a synchronous K user DS-CDMA system using BPSK modulation. Only step (3) is modified. In step (3), steps (a), (b), and (c) are the same as for the PBIL-based ML-MUD detector. The modified PBIL ML-MUD detector can be described as follows:

3. For iteration $g = 1, 2, ..., N_g$; N_g is the number of generations.

   (d) For each individual of the selected N population, 1-*opt* local search is performed representing all individuals in the population as local minima. For each $\acute{\mathbf{b}}_k^g, k = 1, 2, ..., N$, the best neighbor $\acute{\mathbf{b}}_{k,best}^g$ in the 1-*opt* neighborhood $\mathcal{N}(\acute{\mathbf{b}}_k^g)$ is searched by evaluating the fitness function of (4.1). If $f(\acute{\mathbf{b}}_{k,best}^g) < f(\acute{\mathbf{b}}_k^g)$, then $\acute{\mathbf{b}}_k^g \leftarrow \acute{\mathbf{b}}_{k,best}^g$ or else, $\acute{\mathbf{b}}_k^g \leftarrow \acute{\mathbf{b}}_k^g$.

   (e) Each locally minimized individual of the current selected population of size

N is then evaluated by using (4.1), and the individual with the minimum fitness value is determined as $\acute{\mathbf{b}}^{g}_{k,\min}$.

(f) The probability vector is updated based on this best solution $\acute{\mathbf{b}}^{g}_{k,\min}$:

$$p_{g+1}(\mathbf{b}) = (1-\alpha)p_g(\mathbf{b}) + \alpha\acute{\mathbf{b}}^{g}_{k,\min}, \tag{4.11}$$

where $\alpha \in (0,1]$ is the learning rate of the algorithm.

(g) A new population is generated by sampling using the updated probability vector.

For the modified PBIL algorithm, steps (1) and (2) and steps (4), (5), and (6) are the same as for an $M \times M$ PBIL-based MIMO detector using 4-QAM. Only step (3) is modified. In step (3), steps (a), (b), and (c) are the same as for the PBIL-based MIMO detector. The modified PBIL MIMO detector can be described as follows:

3. For iteration $g = 1, 2, \ldots\ldots, N_g$; N_g is the number of generations.

(d) For each individual of the selected N population, 1-*opt* local search is performed representing all individuals in the population as local minima. For each $\acute{\mathbf{s}}^{g}_{r(k)}, k = 1, 2, \ldots\ldots, N$, the best neighbor $\acute{\mathbf{s}}^{g}_{r(k,best)}$ in the 1-*opt* neighborhood $\mathcal{N}(\acute{\mathbf{s}}^{g}_{r(k)})$ is searched by evaluating the fitness function of (4.3). If $f(\acute{\mathbf{s}}^{g}_{r(k,best)}) < f(\acute{\mathbf{s}}^{g}_{r(k)})$, then $\acute{\mathbf{s}}^{g}_{r(k)} \leftarrow \acute{\mathbf{s}}^{g}_{r(k,best)}$ or else, $\acute{\mathbf{s}}^{g}_{r(k)} \leftarrow \acute{\mathbf{s}}^{g}_{r(k)}$.

(e) Each locally minimized individual of the current selected population of size N is then evaluated by using (4.3), and the individual with the minimum fitness value is determined as $\acute{\mathbf{s}}^{g}_{r(k,\min)}$.

(f) The probability vector is updated based on this best solution $\acute{\mathbf{s}}^{g}_{r(k,\min)}$:

$$p_{g+1}(\mathbf{s}_r) = (1-\alpha)p_g(\mathbf{s}_r) + \alpha\acute{\mathbf{s}}^{g}_{r(k,\min)}, \tag{4.12}$$

where $\alpha \in (0,1]$ is the learning rate of the algorithm.

(g) A new population is generated by sampling using the updated probability vector.

When the modified PBIL is applied to a K user synchronous MC-CDMA system, the objective function used for evaluating offsprings is as in (4.2). The whole process is the same as for the MIMO system where M should be replaced by K, $\mathbf{s}_r$ should be replaced by $\mathbf{b}_r$ and $\mathbf{H}_r$ should be replaced by $\mathbf{D}_r$.

4.5 Performance Evaluation

This section studies the BER performance and complexity of metaheuristic detectors. Numerical experiments are conducted for DS-CDMA, MC-CDMA and MIMO systems. We compare the performances of the proposed detectors to several existing detectors. The simulations are done in a MATLAB environment on a 2.4 GHz Intel (R) Xeon (TM) personal computer with 2 Gb of RAM.

In these simulations, a K-user synchronous DS-CDMA system with perfect power control using BPSK transmission over the AWGN channel is considered. All users are assumed to have equal average signal energy. Randomly generated binary signature sequences of length 31 are used for the DS-CDMA system. A K-user synchronous MC-CDMA system using 4-QAM transmission over a Rayleigh fading channel is also considered. Orthogonal Walsh-Hadamard signature codes are used, and the number of subcarriers is kept the same as the number of users for simulations of the MC-CDMA system. Finally, an $M \times M$ MIMO system using 4-QAM transmission over the Rayleigh fading channel is also considered.

The stopping condition for the metaheuristic detectors is chosen as the maximum number of iteration (N_g). For $(1 + \lambda)$ ES, 1-*opt* local search, $(1 + \lambda)$ ES with 1-*opt* local search, SA, PBIL and PBIL with 1-*opt* local search, the maximum number of iteration is kept the same as the problem size, so that for the DS-CDMA system, $N_g = K$; for the MC-CDMA syste, $N_g = 2K$; and for the MIMO system, $N_g = 2M$.

4.5.1 BER Performance Comparison

For the 20-user DS-CDMA system, the average BER versus SNR of the proposed hybrid $(1 + \lambda)$ ES detector is plotted in Fig. 4.1. The BER performances of the CD, the 1-*opt* local search [35], the SA [64] and the optimal detector [26] are also plotted. The BER of the proposed hybrid ES detector offers near optimal performance and also outperforms CD, 1-*opt* local search, $(1 + \lambda)$ ES, and SA detectors. SA shows a worse performance than that of 1-*opt* local search and $(1+\lambda)$ ES. This result may be due to accepting uphill climbing in the search space of SA algorithm. This acceptance makes the SA algorithm getting solution more far away from the optimal solution. Fig. 4.2 shows the same comparison of the BER performance for a 25-user system.

For the 20-user DS-CDMA system, the average BER versus SNR of the proposed PBIL and the modified PBIL detector is plotted in Fig. 4.3. The BER performances of the CD, the 1-*opt* local search, and the optimal detector are also plotted. The proposed modified PBIL detector offers an optimal performance and also outperforms the CD, the PBIL and the 1-*opt* local search detectors. Original PBIL algorithm performs poorly compared to even the 1-*opt* local search. Fig. 4.4 shows the same comparison of the BER performance for a 25-user system.

The BER performances of the proposed $(1 + \lambda)$ ES, the hybrid $(1 + \lambda)$ ES and the SA detectors for a 12-user synchronous MC-CDMA system with 12 subcarriers are presented in Fig. 4.5, which includes comparisons to those of the ZF, the 1-*opt* local search detector, and the sphere decoder (SD). The proposed $(1 + \lambda)$ ES with 1-*opt* local search detector achieves a worse performance for a MC-CDMA system than that of a DS-CDMA system. This result may be due to the structure of the code sequences used for each user. In a DS-CDMA system, random binary signature sequences are used, whereas in a MC-CDMA system, a orthogonal Walsh-Hadamard code is used. Fig. 4.6 shows the same comparison of the BER performances for a 16-user system with 16 subcarriers.

The BER performances of the proposed PBIL and the modified PBIL detectors for

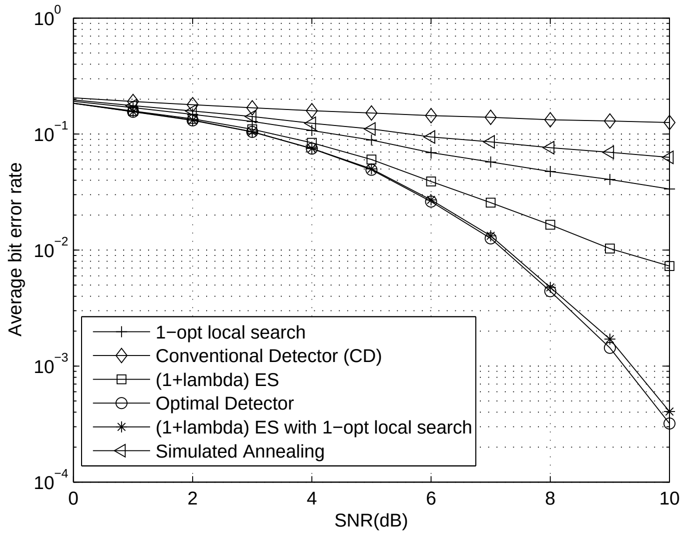

Fig. 4.1. Average BER of CD, Optimal Detector, 1-*opt* local search, $(1 + \lambda)$ ES, $(1 + \lambda)$ ES with 1-*opt* local search, SA detectors for synchronous DS-CDMA system of 20 users.

a 12-user synchronous MC-CDMA system with 12 subcarriers are presented in Fig. 4.7, which includes comparisons to those of the ZF, the 1-*opt* local search detector, and the SD. The PBIL detector performs worse than even the ZF. Fig. 4.8 shows the same comparison of the BER performances for a 16-user system with 16 subcarriers.

The simulation in Fig. 4.9 considers a 12×12 uncoded MIMO system. The average symbol error rate (SER) versus the SNR of the proposed $(1 + \lambda)$ ES and $(1 + \lambda)$ ES with 1-*opt* local search and SA detectors is plotted. The SER performances of the ZF, 1-*opt* local search detector, and the optimal SD are also given. The hybrid $(1+\lambda)$ ES detector shows a better performance than those of all other proposed algorithms. The SA detector also outperforms the $(1 + \lambda)$ ES detector for the MIMO system. Fig. 4.10 shows the same comparison of the SER performance for a 16×16 MIMO system. For a large MIMO system, the hybrid $(1 + \lambda)$ ES detector offers almost the same performance as that of the SA detector.

The simulation in Fig. 4.11 considers the SER performances of the proposed PBIL and modified PBIL-based detector for a 12×12 uncoded MIMO system, and includes

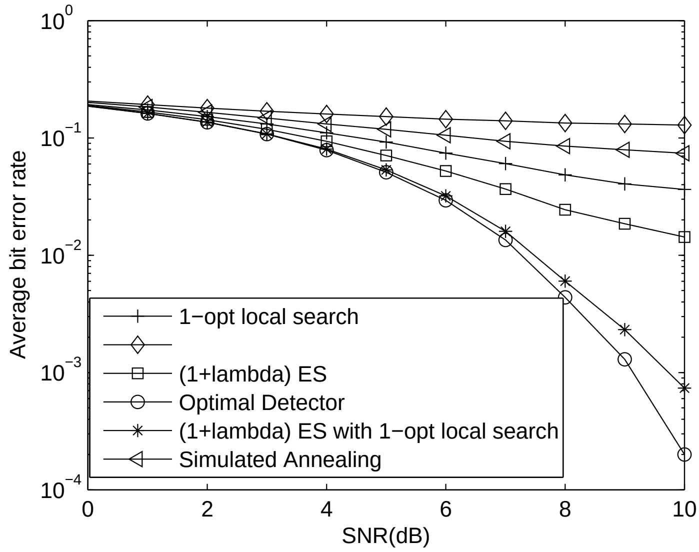

Fig. 4.2. Average BER of CD, Optimal Detector, 1-*opt* local search, $(1+\lambda)$ ES, $(1+\lambda)$ ES with 1-*opt* local search, SA detectors for synchronous DS-CDMA system of 25 users.

comparison to those of the ZF, the 1-*opt* local search detector, and the optimal SD. Although the PBIL algorithm performs poorly, the SER performance of the modified PBIL, i.e., the PBIL with 1-*opt* local search is very close to that of the SD and also outperforms the ZF and 1-*opt* local search detectors. The same performance comparison is shown in Fig. 4.12 for a 16×16 MIMO system.

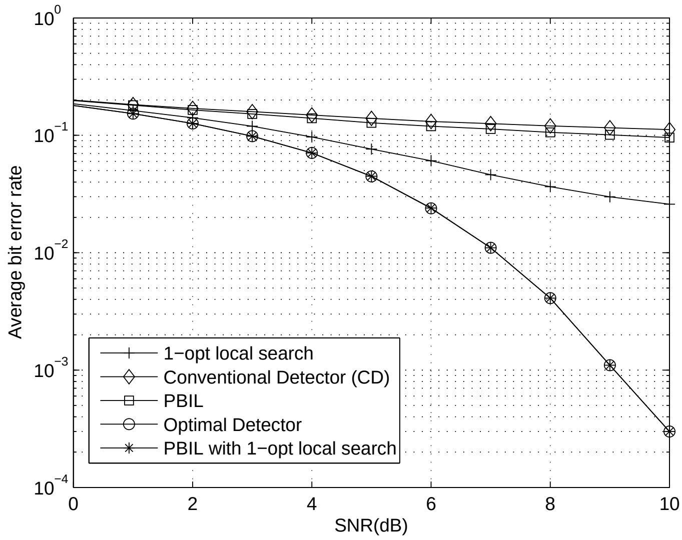

Fig. 4.3. Average BER of CD, Optimal Detector, 1-*opt* local search, PBIL, PBIL with 1-*opt* local search detectors for synchronous DS-CDMA system of 20 users.

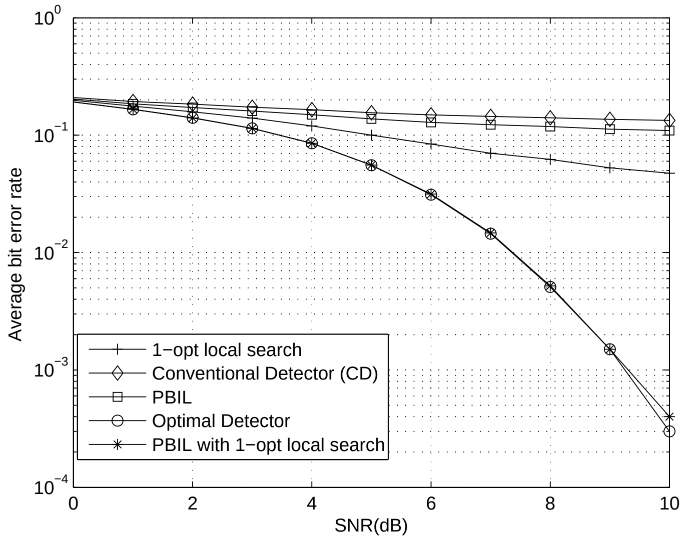

Fig. 4.4. Average BER of CD, Optimal Detector, 1-*opt* local search, PBIL, PBIL with 1-*opt* local search detectors for synchronous DS-CDMA system of 25 users.

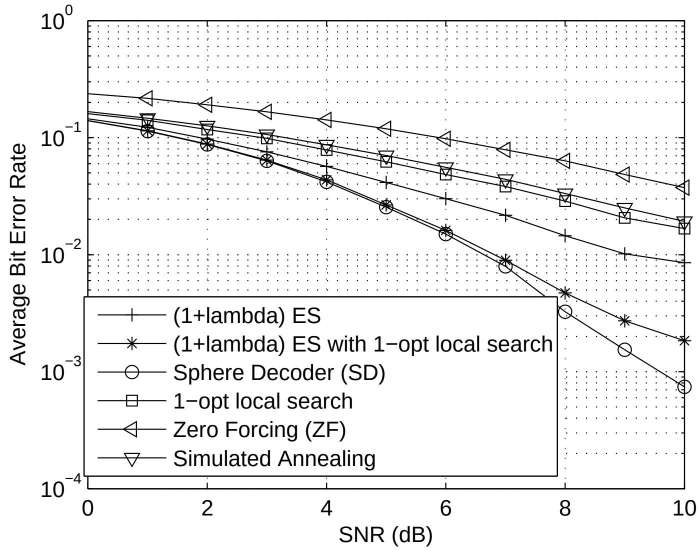

Fig. 4.5. Average BER of ZF, SD, 1-*opt* local search, $(1 + \lambda)$ ES, $(1 + \lambda)$ ES with 1-*opt* local search, SA detectors for synchronous MC-CDMA system of 12 users with 12 subcarriers.

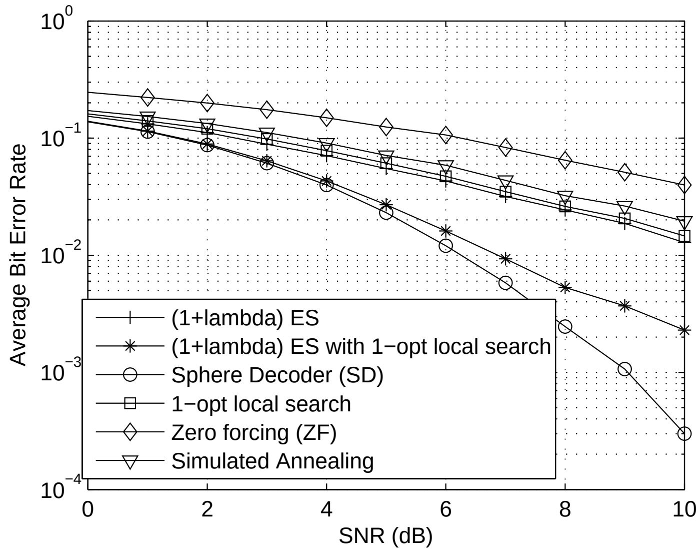

Fig. 4.6. Average BER of ZF, SD, 1-*opt* local search, $(1 + \lambda)$ ES, $(1 + \lambda)$ ES with 1-*opt* local search, SA detectors for synchronous MC-CDMA system of 16 users with 16 subcarriers.

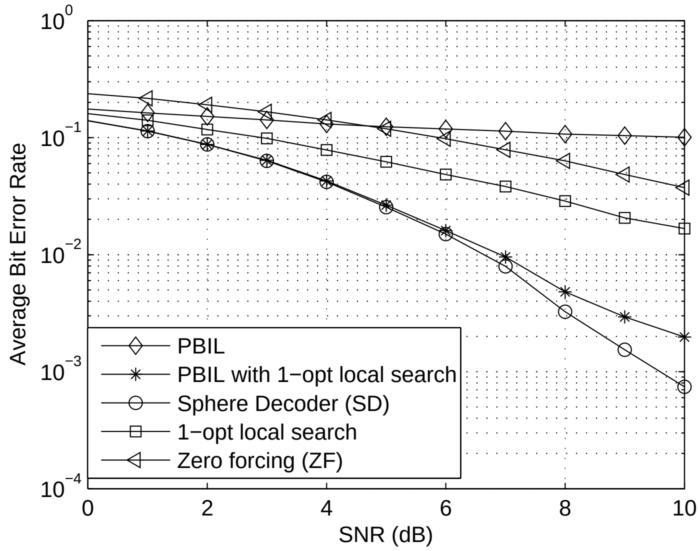

Fig. 4.7. Average BER of ZF, SD, 1-*opt* local search, PBIL, PBIL with 1-*opt* local search detectors for synchronous MC-CDMA system of 12 users with 12 subcarriers.

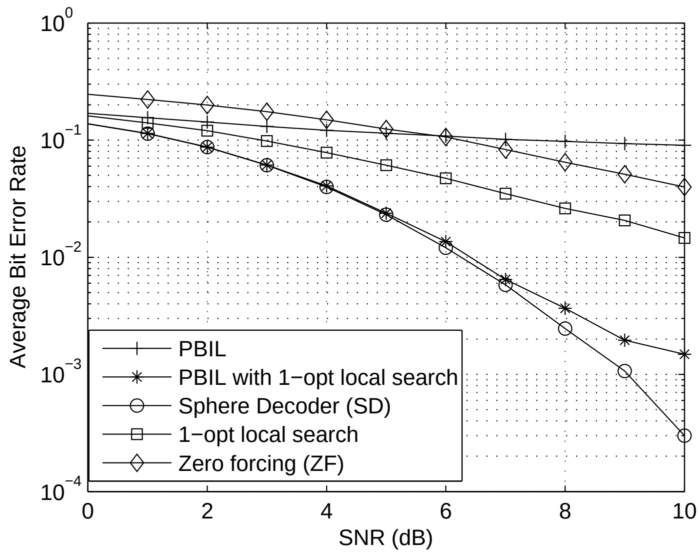

Fig. 4.8. Average BER of ZF, SD, 1-*opt* local search, PBIL, PBIL with 1-*opt* local search detectors for synchronous MC-CDMA system of 16 users with 16 subcarrierss.

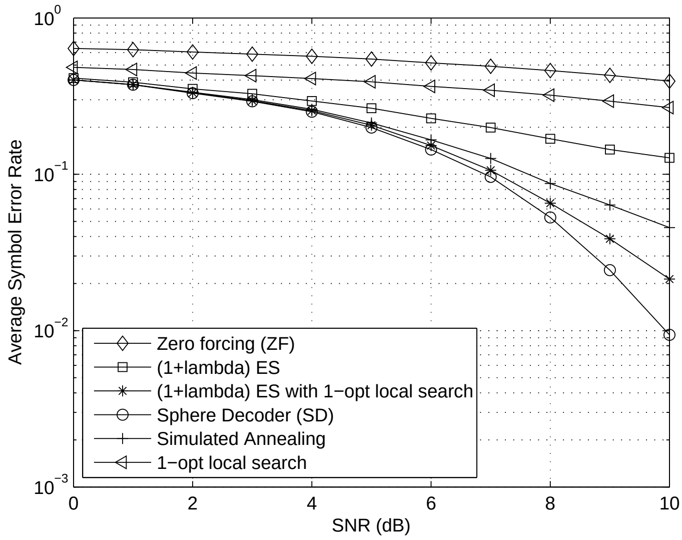

Fig. 4.9. Average BER of ZF, SD, 1-*opt* local search, $(1 + \lambda)$ ES, $(1 + \lambda)$ ES with 1-*opt* local search, SA detectors for 12×12 MIMO system.

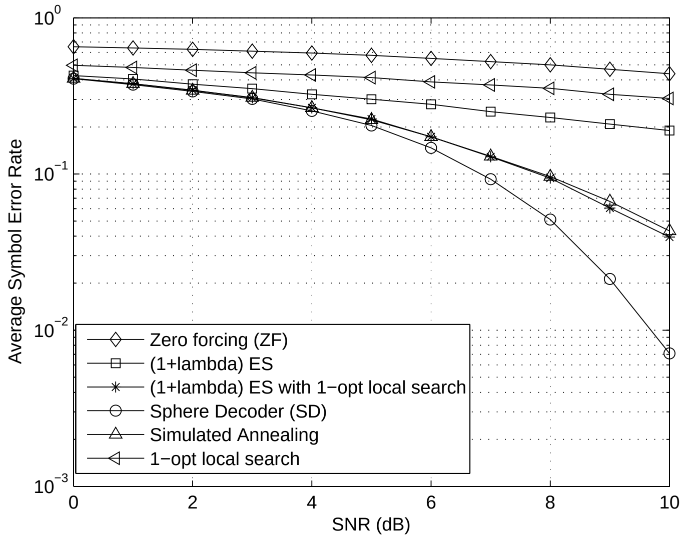

Fig. 4.10. Average BER of ZF, SD, 1-*opt* local search, $(1 + \lambda)$ ES, $(1 + \lambda)$ ES with 1-*opt* local search, SA detectors for 16×16 MIMO system.

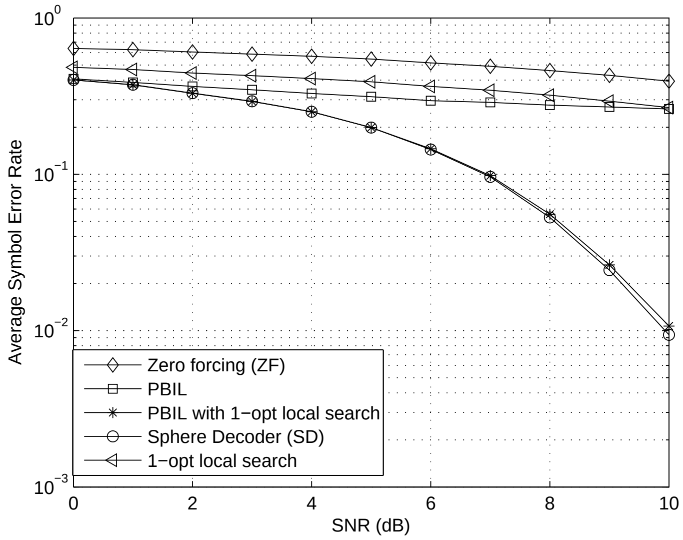

Fig. 4.11. Average BER of ZF, SD, 1-*opt* local search, PBIL, PBIL with 1-*opt* local search detectors for 12×12 MIMO system.

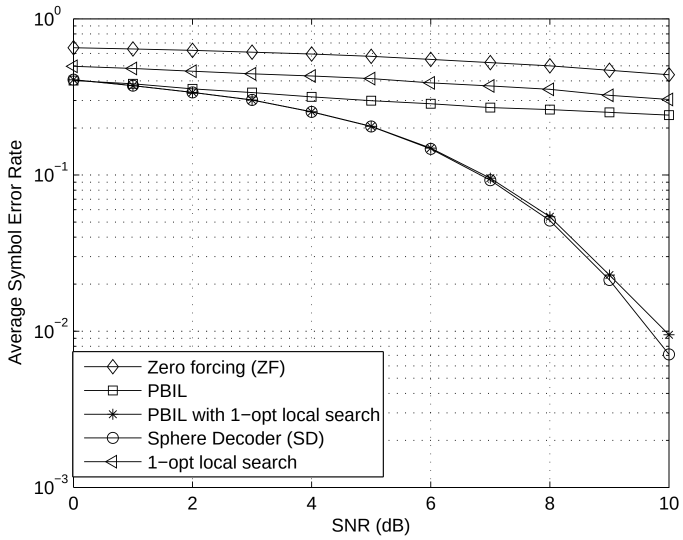

Fig. 4.12. Average BER of ZF, SD, 1-*opt* local search, PBIL, PBIL with 1-*opt* local search detectors for 16×16 MIMO system.

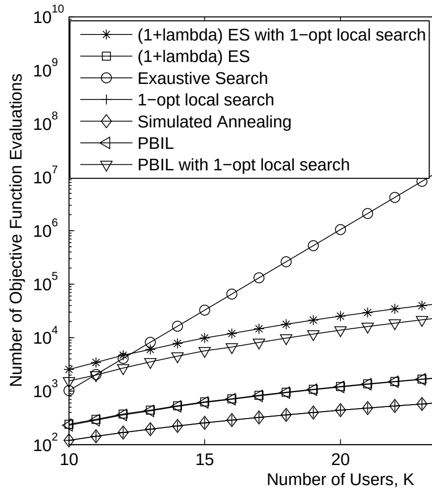

Fig. 4.13. Number of objective function evaluations required to implement the algorithms for exhaustive search, $(1+\lambda)$ ES, $(1+\lambda)$ ES with 1-*opt* local search, PBIL, PBIL with 1-*opt* local search, SA, 1-*opt* local search for different number of users in synchronous DS-CDMA system.

4.5.2 Complexity Analysis

The computational cost for evaluating the objective function as in (4.1) is considered for a DS-CDMA system. The $(1 + \lambda)$ ES-based detector evaluates the objective function (4.1) $(\lambda+1)$ times; the hybrid $(1+\lambda)$ ES evaluates (4.1) $K(\lambda(K+1)+1)$ times; the 1-*opt* local search evaluates (4.1) $K(K+2)$ times; the SA evaluates $K(K+2)+K$ times; the PBIL evaluates (4.1) $K\lambda(K+1)$ times; and the modified PBIL evaluates $K(\lambda + N(K+1))$ times where $N = \left\lfloor \frac{\lambda}{2} \right\rfloor$. All these algorithms are in polynomial time complexity in the number of users, K. An exhaustive search requires 2^K times function evaluation (4.1). Fig. 4.13 shows that the proposed detectors reduce the complexity of the optimum ML detector efficiently for large number of users. The SA and 1-*opt* local search algorithms have lower complexity than that of the other methods.

The complexity is quantified for the MC-CDMA and MIMO systems by counting

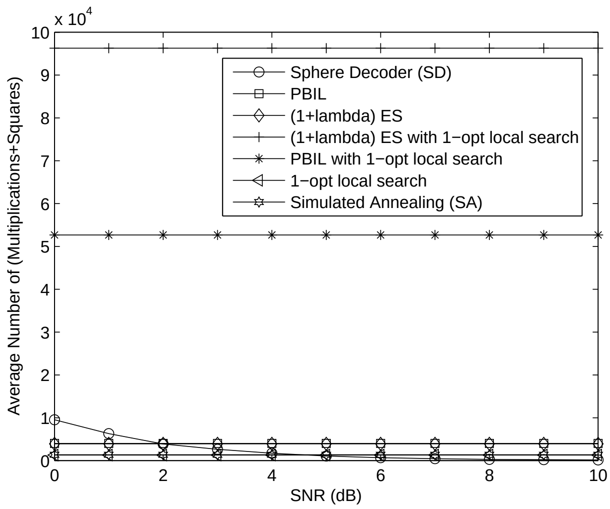

Fig. 4.14. Average Number of (Multiplications+Squares) required per symbol detection to implement the algorithms for SD, $(1+\lambda)$ ES, $(1+\lambda)$ ES with 1-*opt* local search, PBIL, PBIL with 1-*opt* local search, SA, 1-*opt* local search detectors for 12 user synchronous MC-CDMA system with 12 subcarriers.

the number of multiplications and squaring operations required per symbol detection. Fig. 4.14 compares the complexities of the SD and the proposed detectors for the 12-user MC-CDMA system with 12 subcarriers. Fig. 4.15 performs the same comparison a for 16-user MC-CDMA system with 16 subcarriers. The average number of multiplications and squaring operations for the hybrid $(1+\lambda)$ ES and modified PBIL detectors are always higher compared to that of the SD for all SNR. For the SA and 1-*opt* local search detectors, the average number of multiplication and squares are always lower than that of the SD.

Fig. 4.16 shows that up to 3 dB SNR, the modified PBIL-based detector is less complex than the SD, and that at 0 dB, the complexity in SD is 2.9 times higher than that of the modified PBIL detector for a 12×12 MIMO system. The hybrid $(1+\lambda)$ ES detector has less complexity than the SD up to only 1 dB SNR. All other algorithm-based detectors are less complex than the SD for all SNR. Fig. 4.17 shows the complexity comparison curve for a 16×16 MIMO system. Fig. 4.17 shows that

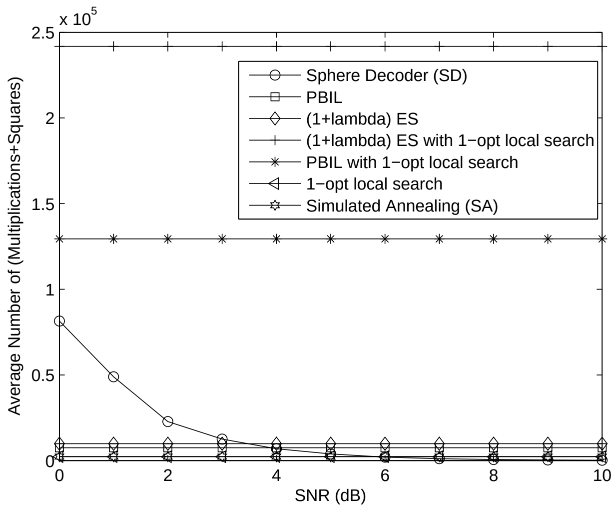

Fig. 4.15. Average Number of (Multiplications+Squares) required per symbol detection to implement the algorithms for SD, $(1+\lambda)$ ES, $(1+\lambda)$ ES with 1-*opt* local search, PBIL, PBIL with 1-*opt* local search, SA, 1-*opt* local search detectors for 16 user synchronous MC-CDMA system with 16 subcarriers.

up to 5 dB SNR and 7 dB SNR, the proposed hybrid $(1 + \lambda)$ ES detector and the modified PBIL detector, respectively, have less complexity than the SD, and that at 0 dB, the modified PBIL is about 21 times less complex than the SD. All the other algorithms always have less complexity than the SD.

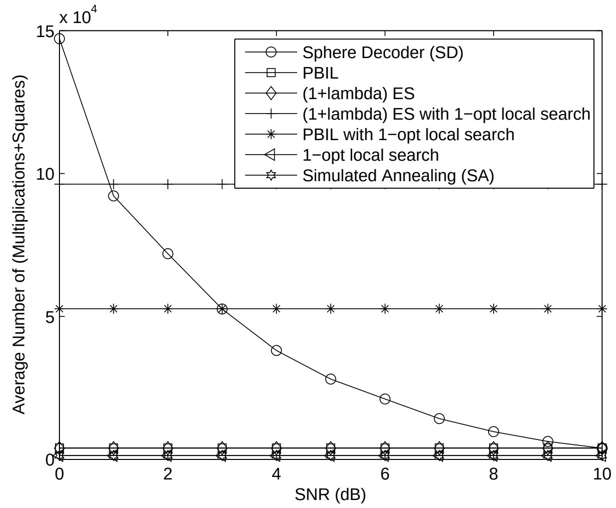

Fig. 4.16. Average Number of (Multiplications+Squares) required per symbol detection to implement the algorithms for SD, $(1+\lambda)$ ES, $(1+\lambda)$ ES with 1-*opt* local search, PBIL, PBIL with 1-*opt* local search, SA, 1-*opt* local search detectors for 12×12 MIMO system.

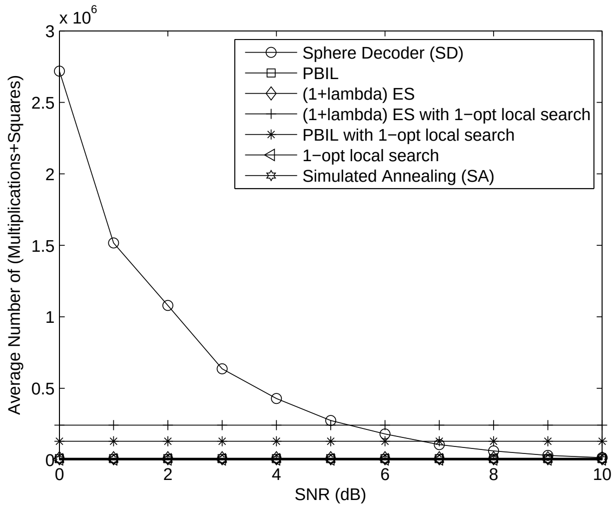

Fig. 4.17. Average Number of (Multiplications+Squares) required per symbol detection to implement the algorithms for SD, $(1+\lambda)$ ES, $(1+\lambda)$ ES with 1-*opt* local search, PBIL, PBIL with 1-*opt* local search, SA, 1-*opt* local search detectors for 16×16 MIMO system.

4.6 Summary

In this chapter, local search based SA algorithm was described for MUD for a DS-CDMA system, and ML detection for MC-CDMA and MIMO systems. A new detector based on PBIL was also introduced for those systems. To obtain better BER performances, the hybridization of $(1 + \lambda)$ ES and PBIL was performed. Hybrid algorithms employ a simple local search with 1-*opt* neighborhood. The BER/SER performances and computational complexity of all these metaheuristic detectors were analyzed for comparison with optimal and conventional detectors. All the metaheuristic detectors have polynomial time complexity in the size of problem. The simulation results showed the superiority of the modified PBIL approach for approximating the BER/SER performance of the ML detector. As well in a low SNR region, this approach outperformed the SD in terms of computational complexity.

Chapter 5

Conclusion

It is well known that the ML detection problem is NP-hard. Most of the research has focused on developing an improved suboptimal detection strategy that would be feasible to implement. The research into efficient detector design has played an important role in the communications industry. Thus, this thesis has been motivated by the need to develop low-complexity detectors, concentrating on developing algorithms able to find near-optimal solutions with moderate computational complexity. This chapter includes the summary and key contributions of this thesis and also provides suggestions for potential future research.

5.1 Thesis Summary

Several metaheuristic algorithms were proposed for ML detection. Local search-based metaheuristic named the SA algorithm and EC-based metaheuristic were applied in DS-CDMA, MC-CDMA and MIMO systems. Although metaheuristics cannot guarantee an optimal solution, these algorithms show much promise. Metaheuristics provide significantly lower computational complexity for large size of problems and produce high-quality solutions. To obtain a better performance, the hybridization of EC-based metaheuristics was proposed. An introduction to the ML detection problem for DS-CDMA, MC-CDMA and MIMO systems was provided in Chapter 2. Chapter

3 provided the background of metaheuristic algorithms, and applications of these methods in ML detection was investigated for different systems in Chapter 4.

The numerical results in Chapter 4 were used to evaluate the performances of the proposed algorithms applied in ML detection for different systems. A hybrid evolutionary strategy (ES) employing 1-*opt* local search in the $(1+\lambda)$ ES was proposed for solving the ML problem. Hybrid ES algorithm-based detector was found to offer the near-optimal BER performance for a DS-CDMA system, but for MC-CDMA and MIMO systems, some performance gaps were found in the high SNR region.

The hybrid $(1 + \lambda)$ ES detector was able to significantly reduce the detection complexity in terms of function evaluation compared to the optimum ML detector when the number of users wass higher than 20. The gain in complexity reduction increases with the number of users. The complexity in terms of the average number multiplications and squaring operations required per symbol detection to implement the algorithm was considered for MC-CDMA and MIMO systems. For both systems, the hybrid $(1 + \lambda)$ ES algorithm performed poorly in terms of complexity compared to the SD and other metaheuristics.

The proposed SA and PBIL algorithms for all systems offered inferior BER/SER performances compared to other metaheuristics, but in terms of complexity they showed better performances.

The modified PBIL algorithm employing PBIL with 1-*opt* local search showed a superior performance for all systems. This algorithm-based detector achievd almost ML BER/SER performances for all systems. In terms of complexity, this algorithm is better than the SD in the low SNR region for the MIMO system, but for the MC-CDMA system, none of the metaheuristics was able to reduce complexity, whereas the SD was able to do so.

For large systems, all the proposed metaheuristic algorithms outperformed an exhaustive search in terms of complexity. This gain in complexity reduction increased with the size of problem. The simulation results showed that the modified PBIL algorithm was competitive and better for all systems in terms of error rate performance and computational complexity.

The major contributions of this thesis are summarized below:

1. Extended the $(1+\lambda)$ ES-based detector for the synchronous MC-CDMA system and the MIMO system and also proposed a hybrid $(1+\lambda)$ ES algorithm for the synchronous DS-CDMA, MC-CDMA, MIMO systems.
2. Proposed the SA algorithm for ML detection in synchronous DS-CDMA, MC-CDMA and MIMO systems.
3. Developed a new ML detection scheme based on the PBIL algorithm for DS-CDMA, MC-CDMA and MIMO systems and also proposed a modified PBIL algorithm to provide a better performance.

5.2 Future Work

Several issues and possible directions for further research are considered here.

1. This thesis assumed that the receiver has perfect knowledge of the channel. In practice, the receiver must estimate the channel, resulting in estimation errors. The effect of imperfect channel estimation on detection performance could be studied.
2. This thesis discusses efficient data detection for uncoded systems only. All the proposed detectors may be extended for coded systems.
3. Multipath fading is a fundamental factor that limits the performance of wireless systems. How the proposed detectors perform in fading channels can be investigated.

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