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PERFORMANCE OF DUAL HOP RELAY SYSTEMS WITH IMPERFECT CSI

by

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To my parents

Abstract

Wireless relays offer benefits such as coverage extension and improving robustness. Selection of relays may provide further benefits. The best relays are selected based on the channel state information. However, due to feedback or scheduling delays, in practise, the relay and/or the source node can have outdated channel state information. This outdated information will cause a performance degradation, and in this thesis its impact on dual-hop relay systems is investigated. The performance of amplify-and-forward (AF) relays under partial relay selection and opportunistic relay selection is analyzed. Both variable gain AF and fixed gain AF schemes are considered. Expressions for the outage probability and the average bit error rate and simplified high signal-to-noise ratio approximations are derived. The effect of parameters such as the number of relays, the rank of chosen relay, and the correlation between the delayed and current channel state information are analyzed.

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List of Symbols

$\mathbf{Z}^+$	Set of positive integers
$\mathbf{E}(X)$	Statistical average of random variable X
$f_X(x)$	Probability density function of random variable X
$\Pr$	Probability
${}_2F_1$	Gauss hypergeometric function
$\Gamma(x)$	Gamma function
$\Gamma(a,x)$	Upper incomplete gamma function
$x\,!$	Factorial
$\binom{N}{k}$	Combinations of k elements from N elements
$Q(x)$	Standard Gaussian Q-function
$J_\nu(x)$	νth order Bessel function of the first kind
$I_\nu(x)$	νth order modified Bessel function of the first kind
$K_\nu(x)$	νth order modified Bessel function of the second kind
$\mathrm{Ei}(x)$	Exponential integral function
$\mathcal{W}_{\mu,\nu}(x)$	Whittaker hypergeometric function W
$\mathcal{CN}(\mu,\sigma)$	Complex Gaussian distribution with mean μ and variance σ
$\mathcal{G}(\lambda,\theta)$	Gamma distribution with shape parameter θ and scale parameter λ.

Acronyms

AF Amplify-and-Forward

APR All Participating Relaying

AWGN Additive White Gaussian Noise

BER bit error rate

BPSK Binary Phase Shift Keying

CF Code-and-Forward

CCDF Complementary Cumulative Distribution Function

CDF Cumulative Distribution Function

CSI Channel State Information

DF Decode-and-Forward

e2e end-to-end

FDD Frequency Division Duplexing

FG Fixed Gain

i.i.d. Independent and identically distributed

IP Internet Protocol

LOS line-of-sight

LTE Long Term Evolution

LTE-A Long Term Evolution-Advanced

M-PSK M-ary Phase Shift Keying

MRS Multiple Relay Selection

ORS Opportunistic Relay Selection

PDF Probability Density Function

PRS Partial Relay Selection

QAM Quadrature Amplitude Modulation

QoS Quality of Service

QPSK Quadrature Phase Shift Keying

RV Random Variable

SNR Signal to Noise Ratio

SRS Single Relay Selection

TDD Time Division Duplexing

VG Variable Gain

WiMAX Worldwide Interoperability for Microwave Access

WLAN Wireless Local Area Network

WMAN Wireless Metropolitan Area Networks

WPAN Wireless Personal Area Networks

WWAN Wireless Wide Area Networks

Chapter 1

Introduction

1.1 Wireless Communications

Wireless communication field has its roots dated back in the late 19^{th} century in the times of Alexander Graham Bell, David E. Hughes, T.A. Edison and others. Wireless systems are now ubiquitous and can be categorized based on the coverage area of the network [1].

- Wireless Local Area Network (WLAN)

  WLAN interconnects wireless devices in an area of a room or a building and includes a distribution system, access points and portal entities [2]. IEEE 802.11 standard governs WLANs.

- Wireless Personal Area Networks (WPAN)

  WPANs connect a private group of devices over short distances, and usually do not have links to the outside world [3]. WPAN devices are standardized by IEEE 802.15 standards.

- Wireless Metropolitan Area Networks (WMAN)

  WMANs interconnect several Wireless LANs. Broadband WMANs are covered by the IEEE 802.16 standard, which is also known as WiMax.

- Wireless Wide Area Networks (WWAN)

WWANs are wireless networks which cover large areas in the range of cities and towns.

Wireless cellular systems have gained widespread adaption globally, and International Telecommunication Union reported more than 5.3 billion mobile phone subscribers were present in 2010, about 78 % of the world's population [4]. This strong growth is facilitated by the improvements in digital electronics, making smaller devices with higher computational powers affordable for most users.

This chapter introduces basic concepts of wireless communications, including fading, diversity and statistical models for wireless channels. Furthermore, an introduction to relay systems, relay categories, relay selection and performance metrics are presented. This chapter concludes with a summary of the contribution of this work and an outline of the thesis.

1.2 Wireless Channel Basics

Understanding wireless channel behaviour is fundamental to performance analysis. The wireless channel behaviour is dependent on multipath fading, the rate of time variation and frequency selectivity.

Multipath Propagation

The wireless channel can be modeled as a linear time varying system [5]. The time-varying behaviour arises due to the movement of the wireless terminals. Moreover, due to reflections and scattering in the path of the signal, multiple copies of the transmit signal will be received at the destination, with different delays and different attenuation levels.

Due to these multiple copies, the transmitted signal $x(t)$ and the signal received over the wireless channel $y(t)$ are related by [5, Eq. 2.37],

$$y(t) = \sum_i a_i(t)x(t - \tau_i(t)) + w(t), \tag{1.1}$$

where $a_i(t)$ is the attenuation factor from the ith path, $\tau_i(t)$ is the delay associated with it, and $w(t)$ is the additive noise.

For narrow band signals, $a_i(t)$s and $\tau_i(t)$s can be assumed to be frequency independent. But the channel response would be frequency dependent due to the phase differences of each path associated with the different delays. These phase differences will also form constructive or destructive interferences of the copies of the signal, creating rapid fluctuations in the received signal level [1].

Doppler effect and Channel Coherence Time

Doppler effect is a shift in the received signal frequency (from the original transmit frequency), caused by the relative movement between the source and the destination nodes. If they are stationary, the channel would be time invariant and the Doppler shift would be zero. i.e. the $a_i(t)$s and $\tau_i(t)$s in (1.1) will be constants.

To be precise, if the relative velocity of the source with respect to the destination is v (towards the destination) and the frequency of the signal at the source is f, there will be a Doppler shift (i.e a frequency change) of fv/c, where c is the speed of light. i.e. the component of the signal with frequency f at the source would be of frequency $f + fv/c$ at the receiver. The range of Doppler shifts across the bandwidth of the signal is called the Doppler spread (D_s).

Due to the Doppler effect, the channel varies with time. The coherence time of a channel denotes the period of time within which the channel fading remains correlated above a predetermined threshold. The coherence time is inversely proportional to the Doppler spread of the wireless channel [1, Sec. 3.3.3]. In [5], coherence time (T_c) is expressed as following,

$$T_c = \frac{1}{4D_s}. \tag{1.2}$$

According to the uniform scattering model presented by Jakes in [6], the received signal (given in (1.1)) in a time varying channel would have the following autocorrelation [1]:

$$R_y = E[y(t)y(t+\tau)] = P_y J_0\left(\pi D_s \tau\right), \tag{1.3}$$

where P_y is the received signal power, D_s is the Doppler spread, and J_0 is the zeroth order Bessel function of the first kind [7, Eq. 9.1.18]. Within the coherence time (i.e. $\tau < T_c$), the channel would remain highly correlated, having a correlation coefficient of above 0.85.

1.2.1 Small Scale Fading Models

Small scale fading refers to the rapid fluctuations in signal amplitude and phase as a result of small changes (i.e. on the scale of half wavelength or more) in the distance between the transmitter and receiver. These signal fluctuations are modelled by several statistical distributions, which are described next.

Rayleigh Fading

Rayleigh fading occurs when there are multiple scattered paths from the source to the destination and there is no direct (line-of-sight) path. The Probability Density Function (PDF) of the received Signal to Noise Ratio (SNR) (γ) in this case is given by [8],

$$f_\gamma(x) = \frac{x}{\bar{\gamma}} e^{-\frac{x}{\bar{\gamma}}} \quad , \qquad x \geq 0 \tag{1.4}$$

where $\bar{\gamma}$ is the average SNR.

Rician Fading

The Rician fading model is employed when there is a dominant non-fading path between the source and the destination, in addition to the scattered paths. The PDF of the SNR under Rician fading is as follows [8]:

$$f_\gamma(x) = \frac{2(1+K)}{\bar{\gamma}} e^{-\left(K + \frac{(1+K)x}{\bar{\gamma}}\right)} I_0 \left(2\sqrt{\frac{K(1+K)x}{\bar{\gamma}}} \right) \quad , \qquad x \geq 0 \tag{1.5}$$

where K, known as the 'Rice factor', is the ratio between the power of line-of-sight (LOS) component to the power of multipath components, and $\bar{\gamma}$ is the average SNR. K is a measure of the fading in the channel, and the smaller the K, the more severe the fading. $K = 0$ is when there is no LOS path and $K = \infty$ corresponds to the case where only the LOS component is present (i.e. no fading). In a mixed urban-suburban environment, at 915 MHz, the Rice factor was found to be between 2-4 dB in a 1.23 MHz bandwidth and 8-12 dB in a 20 MHz bandwidth [9].

Nakagami-m Fading

Nakagami-m fading, first proposed in [10], is a more general distribution that can be employed to model small scale fading in wireless channels. The PDF of the SNR under Nakagami-m fading is given by

$$f_\gamma(x) = \frac{x^{m-1}}{\Gamma(m)} \left(\frac{m}{\bar{\gamma}}\right)^m e^{-\frac{mx}{\bar{\gamma}}} \quad , \qquad x \geq 0, m \geq 0.5 \tag{1.6}$$

where m is the fading parameter . Rayleigh fading is a special case of Nakagami-m fading, obtained when $m = 1$. $m \to \infty$ corresponds to a non-fading Additive White Gaussian Noise (AWGN) channel [8].

Other fading models developed to model different propagation environments include Nakagami-q fading and Weibull fading. These fading models have been generalized as the $\kappa - \mu$ distribution, $\eta - \mu$ distribution [11] and $\alpha - \mu$ distribution [12].

1.2.2 Performance Metrics

Performance analysis of wireless systems requires the averaging of performance metrics over these fading models. In this work, the main performance metrics are the average BER (P_b) and the outage probability (P_o).

The outage probability is the probability that the instantaneous SNR will fall below a predetermined threshold, below which the system performance is unacceptable [1]. For example, in a mobile voice channel, this threshold would relate to the minimum received SNR level, below which the sound from one node is imperceptible at the other, or in a video streaming channel, the minimum SNR required to maintain a seamless video. Hence, the outage probability is an important Quality of Service (QoS) measure.

The average BER is the probability that a received bit would be in error at the destination. The BER depends on the instantaneous SNR at the destination and the modulation and coding schemes used for transmission. The average BER can be improved using adaptive modulation schemes, which is recommended in the 4G standards. In the WiMAX standard, depending on the SNR level, modulation schemes from Binary Phase Shift Keying (BPSK) with a $\frac{1}{2}$ code rate to 64-Quadrature Amplitude Modulation (QAM) with a $\frac{3}{4}$ code rate are recommended [13]. In Long Term Evolution (LTE) standard, conversational voice and conversational video has maximum BERs of 10^{-2} and 10^{-3} respectively [14].

High SNR Approximations

Typically, the outage probability and the average BER expressions take complicated forms, which hinder quick insights. Hence, simpler high SNR approximations may show the impact of certain parameters on the performance of wireless systems. Furthermore, these approximations are quite useful to compare different wireless systems in the high SNR range or to analyze the influence of some parameters on the system performance. They are sometimes an alternative to, or a validation device of system simulations, which require a large number of simulation points to estimate performance in the high SNR regime.

High SNR approximations are simplified functions of the average SNR, derived taking the limits of the expression as the average SNR goes to infinity and keeping the dominant term. For example, the average BER for BPSK modulation under Rayleigh fading is given by [1, Eq. 6.58],

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right) \tag{1.7}$$

where $\bar{\gamma}$ is the average SNR. The high SNR approximation (P_b^{∞}) is obtained as follows,

$$P_b^{\infty} = \lim_{\bar{\gamma}\to\infty} \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right) = \frac{1}{4\bar{\gamma}}. \tag{1.8}$$

It is instructive to compare (1.8) with the average BER of BPSK in an unfaded AWGN channel:

$$P_b = Q\left(\sqrt{2\bar{\gamma}}\right) \leq e^{-\bar{\gamma}}, \tag{1.9}$$

where $Q(\cdot)$ is the Gaussian Q-function and $\bar{\gamma}$ is the SNR. To obtain the inequality, Chernoff bound is used [15]. It can be seen that, due to Rayleigh fading, the BER has increased from $e^{-\bar{\gamma}}$ to $\frac{1}{\bar{\gamma}}$. This dramatic increase shows the difficulty of achieving reliable communication over fading channels.

Diversity Gain

Diversity gain (G_d) is defined as [16],

$$G_d = -\lim_{\bar{\gamma}\to\infty} \frac{\log(P_b(\bar{\gamma}))}{\log(\bar{\gamma})}. \tag{1.10}$$

G_d is a measure of how many independently fading signal copies are combined at the receiver [5]. The higher the diversity gain, the more robust the system would be against the fading effects.

Coding Gain

Coding gain is a measure of the increase of SNR achieved due to coherent combining of multiple copies of the signal at the destination. Unlike the diversity gain, this can be achieved even in a channel without fading [1]. Coding gain (G_c) is defined as [17],

$$G_c = \lim_{\bar{\gamma}\to\infty} (\bar{\gamma}^{G_d} P_b)^{-\frac{1}{G_d}}. \tag{1.11}$$

The high SNR approximation for the average BER can be written in terms of the coding gain and the diversity gain as follows [17],

$$P_b^{\infty} = (G_c\bar{\gamma})^{-G_d}. \tag{1.12}$$

For the example of the average BER of BPSK under Rayleigh fading considered under high SNR approximations, the coding gain $G_c = 4$ and the diversity order $G_d = 1$, as can be seen in (1.8).

1.3 Cooperative Diversity

As shown in (1.8), fading significantly degrades the reliability of communication. One popular technique to mitigate this, is to send several copies of the data signal over independent fading channels. This is called diversity. If the signal is transmitted in two sufficiently separated time slots, time diversity is created. In the same manner, frequency diversity or spatial diversity can be created, using separation in frequency or space, respectively.

Another approach is cooperative diversity, which was proposed in [18–20]. Cooperative diversity is achieved by using relays that receive the source transmission and forward to the destination. Such systems typically operate over two phases. In the first phase signal transmission from source is received by multiple relays and the destination. In the second phase, relays can use orthogonal (non-interfering) channels. The destination combines the relayed signals and the direct signal in the first phase to improve the SNR. This process creates multiple fading channels from the source to the destination. Because the relays are generally located in different physical locations, the spatial separation of the relays would enforce independent fading, thus creating a diversity gain.

Cooperative diversity will be useful in cellular systems and wireless ad-hoc networks [19]. In addition to the benefits of increased diversity, cooperative networks offer better coverage and lower power consumption than conventional networks [21].

1.3.1 Relay Categories

Relays can be user nodes or fixed terminals known as infrastructure relays. Relays, according to their processing functionality, can be divided in to three major categories [22], Amplify-and-Forward (AF) [18], Decode-and-Forward (DF) and Code-and-Forward (CF).

Amplify-and-Forward

Amplify-and-Forward relay is the simplest form of the relays. As the name suggests, it amplifies the received signal and retransmits [21]. Hence, there will be an amplification of the noise as well. AF relays can be further categorized depending on how the amplification gain is chosen:

- **Variable Gain Relay**
  In Variable Gain (VG) relaying, the relay uses instantaneous Channel State Information (CSI) of the received path, to choose the amplification gain [23]. Variable gain relaying generally performs better than fixed gain relaying sys-

tems. However, the requirement of instantaneous channel information will increase the cost and complexity of the relay.

- **Fixed Gain Relay**

  In Fixed Gain (FG) relaying, the relay uses long term statistics of the inward channel when choosing the amplification gain [24]. Hence, the instantaneous CSI is not required for this. This is also known as 'semi-blind' relay. The advantage of fixed gain relaying is the lack of need for the relay to measure the channel from the source, and hence has less system complexity and overhead.

Decode-and-Forward

Decode-and-Forward relays will sample, demodulate and decode the received signal [25]. The decoded and regenerated signal is then transmitted to the destination. These relays do not have the drawback of noise amplification. Moreover, adaptive DF relaying offer path loss savings over conventional relaying and diversity gains [26]. Reference [27] proposed a framework for maximum likelihood detection in decode-and-forward relay systems and [28] analyzed the performance of a DF relay selection system under Rayleigh fading.

If the signal is decoded incorrectly at the relay, error propagation will result. The effect of error propagation can be mitigated if the DF relays participate in communication only under conditions that ensure high probability of correct decoding [29], e.g. with SNR above a predetermined threshold. In [30, 31], such DF relay performance was analyzed. Cyclic redundancy check was used in these systems to determine if the relays decoded the signal correctly.

Code-and-Forward

Code-and-Forward relays offer added functionality. These relays will decode the received signal and will re-encode by using a network code before retransmission to the destination [22]. Coded cooperation is an example of CF relaying [21], where the relay, which is also a communicating user with the same destination, decodes the signal from the source, and encodes it's signal so that it contains information

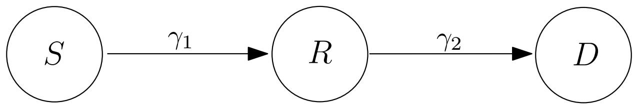

Figure 1.1: A relay (R) connecting the source (S) and the destination (D)

about the codeword from the source.

1.4 Relay Selection

The ability of a relay to improve the overall communications performance depends on the quality (i.e. the SNR) of the inward and outward communication channels to and from the relay. Consider a dual hop network with a source (S), destination (D) and a relay (R) in between (Figure 1.1). A low source-relay ($S - R$) link SNR (γ_1) would cause either decoding errors in DF relays or noise enhancement in the AF relays. A low relay-destination ($R - D$) link SNR (γ_2) would increase the error rate at the destination. Since forwarding relays require power and bandwidth resources, it is imperative to select relays with good link SNRs to participate in the communication.

Another reason for relay selection is to increase the spectral efficiency. For example, all participate relaying (APR) (shown in Figure 1.2) gives the best performance in terms of the diversity gain and the coding gain [29]. But this requires $N_r + 1$ (where N_r is the number of relays) orthogonal channels. The destination can combine the N_r signals to obtain a diversity gain of N_r. But the use of N_r orthogonal channels lowers bandwidth resource utilization.

By selecting only a subset of all available relays, the number of orthogonal channels required for communication is reduced, thus increasing the spectral efficiency. Hence methods for selecting relays become of interest. There are several methods for the selection of relays discussed in the literature. Few such schemes are discussed in the following subsections.

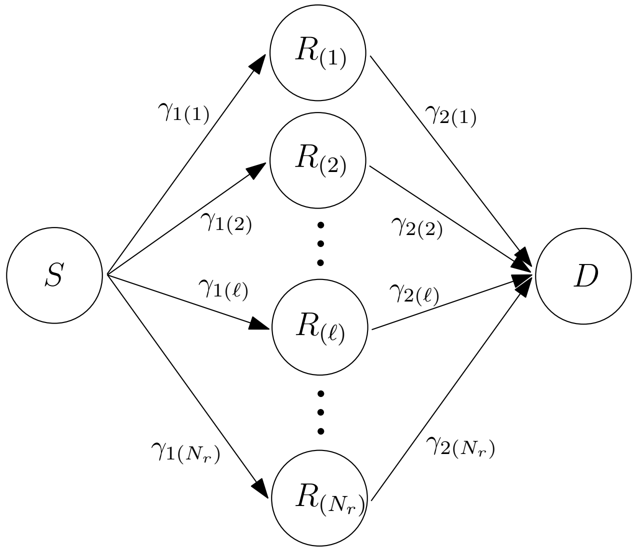

Figure 1.2: All Participate Relaying (APR) system

1.4.1 Single Relay Selection

Consider a dual-hop system (shown in Figure 1.2) with a single Source (S), a single destination (D) and N_r relays. $\gamma_{1(\ell)}$, $\gamma_{2(\ell)}$ are the $S-R$ link SNR and the $R-D$ link SNR of the ℓth path respectively. In Single Relay Selection (SRS) systems, only a single relay out of the available relays would be selected for communication.

Best Relay Selection

Best relay selection [32], is based on the end-to-end (e2e) SNR of the system. The e2e SNR of the ℓth relay path in a AF VG system, depends on $\gamma_{1(\ell)}, \gamma_{2(\ell)}$. The relay that offers the highest e2e SNR is selected for communication. This achieves the optimal performance out of all the SRS schemes.

Opportunistic Relay Selection

Opportunistic Relay Selection (ORS) criteria from [30] is also known as best-worse channel selection [33]. This scheme targets to maximize the minimum link SNR of the system. As the link with the lowest SNR is the bottleneck in the system which limits the performance, maximizing the minimum link SNR is desired. The selected relay path under ORS will maximize $\min\left(\gamma_{1(\ell)}, \gamma_{2(\ell)}\right)$.

The ORS scheme approaches the optimal performance at high SNR and this is a simpler scheme than best relay selection. This can be applied to higher number of hops without the complexity being significantly increased .

Partial Relay Selection

In [34], a new reduced complexity Partial Relay Selection (PRS) scheme using AF VG relays was introduced. This selection method is based on the link SNR of the $S-R$ hop. The relay path selected is given by $\underset{\ell\in\{1,2,\ldots,N_r\}}{\arg\max}\ \gamma_{1(\ell)}$. This relay selection scheme is discussed in detail in Chapter 2.

The performance of PRS is inferior to that of the ORS scheme, because the channel information of the $R-D$ links are not employed in the former. But, since this requires only the local hop channel information, only the local channels need to be estimated, which can be done by broadcasting the pilots from the source node. The relays do no need to send pilots to estimate their channel with the destination, unlike in the case of ORS. Hence, the CSI overhead in PRS system is much less than that of the ORS system. This is more significant when the number of relaying nodes increase in the network. Therefore the simple PRS approach finds wide applicability especially in low complexity ad-hoc and sensor networks since such networks may not have significant resources to implement complex relay selection protocols [34]. Moreover, since only the local hop channel measurements are required for PRS, the relay selection process would take a shorter time than the ORS process. This would ensure the channels would be less likely to vary significantly during the relay selection process.

1.4.2 Multiple Relay Selection

In Multiple Relay Selection (MRS) systems, a subset of relays out of the total N_r would participate in the communication for each data transmission (Figure 1.2), thus aiming to improve the performance while keeping the system complexity and resource utilization in acceptable levels. When the number of relays chosen for communication is increased, the performance improves, but the system complexity

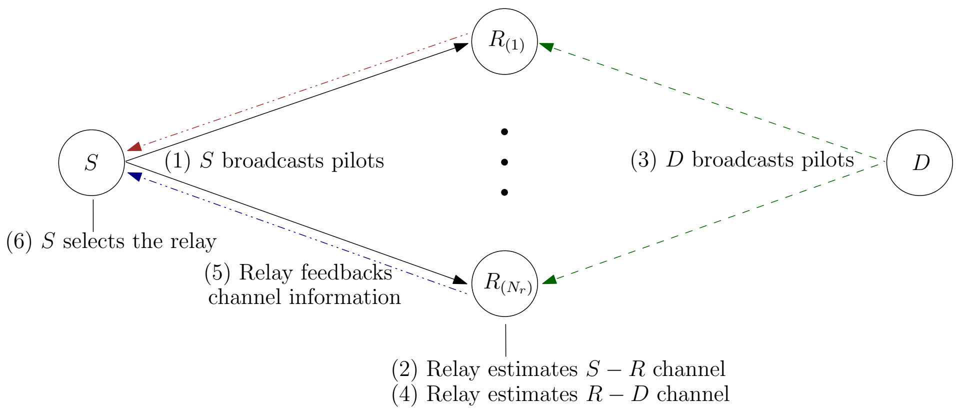

Figure 1.3: Relay selection process

increases as well. An optimal multiple relay selection scheme in the presence of orthogonal channels was proposed in [35], and optimal and suboptimal MRS systems for shared channels was analyzed in [33]. A generalized selection combining based MRS system was proposed in [36].

1.4.3 Practical issues in relay selection

Relay selection is based on the SNR of different relay paths. Hence, prior to relay selection, the channels need to be estimated and the CSI need to be communicated to a control node. The control node will select the relay and notify it. This thesis does not focus on proposing an implementation mechanism for relay selection. But, the following are possible processes for relay selection in PRS and ORS systems.

For ORS, both the $S-R$ and $R-D$ channels need to be estimated. The process shown in Figure 1.3 may be used to select the relays. In this scheme, the source first broadcasts pilots, and each relay estimates its $S-R$ channel. Then the destination broadcasts pilots and the relays estimates the $R-D$ channel, assuming reciprocity in the uplink and downlink. Relays then communicate the minimum of the two link SNRs to the source. The source selects the relay, and broadcast the ID of the relay chosen.

For PRS systems, only the $S-R$ channel needs to be estimated in the relay selection process. A process similar to that of Figure 1.3, excluding the steps (3) and (4), can be used for PRS. In this scheme, the source initially broadcasts pilots.

Each relay estimates the $S - R$ channel and sends this information to the source. Source selects the relay, and broadcasts the ID of the relay chosen. Alternatively, in the PRS scheme, the relays can first broadcast the pilots. The source can measure each $S-R$ channel, select the relay and broadcast the ID of the relay. This would be particularly useful in FG relay systems, where the relay does not need instantaneous CSI of the $S - R$ channel.

1.5 Relays in 4G standards

The fourth generation(4G) of cellular standards support speeds of up to 1Gbps and Internet Protocol (IP) based mobile broadband for users. The two main candidates for the upcoming 4G standards are Long Term Evolution-Advanced (LTE-A) and Wireless MAN-Advanced (IEEE 802.16m) [37].

Relays in both these candidate standards can be divided in to two categories, based on whether or not the relay has the control over managing resources. IEEE 802.16m classifies the former as 'non-transparent relays' and the latter as 'transparent relays', while LTE-A treats them as 'type-1 relays' and 'type-2 relays' respectively.

Relays in IEEE 802.16m are stationary, decode-and-forward, non-transparent relays [37]. They are limited to dual-hop and support both Time Division Duplexing (TDD) and Frequency Division Duplexing (FDD).

In the LTE-A standard, type-1 relays are proposed for the purpose of coverage extension. Two categories of relays are included in the standard, type-1a and type-1b. Type-1b relays operates in the same frequency band as the base stations while type-1a relays operate out-of the frequency band.

1.6 Motivation

Relay selection discussed above requires CSI to be available at the source, relays and destination. To motivate the problem of outdated CSI, we first consider a simple point-to-point channel between a source (S) and a destination (D) (Figure. 1.4). To

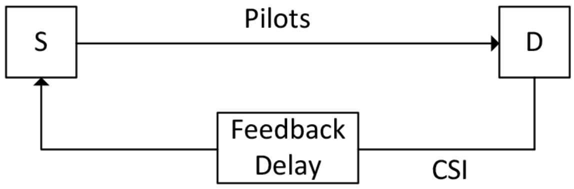

Figure 1.4: Feedback process of CSI

estimate the channel between S and D, initially pilots are sent from the source. The destination will use the received signal of these pilots to estimate the channel. This estimation (i.e. CSI) is communicated back to the source via a feedback channel.

Assume this process of CSI estimation and feedback to the source takes a time T_d. The channel, due to its time varying nature caused by the Doppler and other effects, would change during this time. Hence the CSI at the source would be outdated. In this work, it is assumed that, the channel SNR calculated using the outdated CSI, is correlated with the actual SNR of the channel. The central focus of this thesis is to quantify the performance degradations due to outdated CSI.

1.7 Contributions and Outline of the Thesis

As discussed above, in relay selection systems, the source and relays may have outdated CSI. Due to the estimation time of the $S - R$ and $R - D$ links, and feedback delays in communicating this information back to source and relays, CSI at the time of selection differs from the CSI at the time of actual data transmission. This problem may be further enhanced by additional effects like scheduling delays. This issue of outdated CSI has not been comprehensively investigated in the open literature. For practical relay systems, understanding this effect is important and is the main contribution of this work. The performance of dual hop AF relay systems is the main focus of this thesis.

Chapter 1

In this chapter, brief introductions to the basic concepts in wireless communication channels and small scale propagation effects were presented. The concepts of cooperative diversity and relay communication were also briefly discussed.

Chapter 2

In Chapter 2, the system model is introduced and analyzed under Rayleigh fading. To understand the effect of outdated CSI, two relay selection schemes, PRS and ORS are considered. PRS gives suboptimal performance under perfect CSI case. However, since relay selection process under PRS will take less time due to its lower complexity, it can be assumed that it will have less delay between relay selection and data transmission. Hence the CSI at the relay selection would be better correlated to the channel states at data transmission, and would have this as an advantage over ORS system.

Also in this chapter, under partial relaying, both variable gain and fixed gain relays are investigated. Outdated CSI at the relays will hinder the correct estimation of amplification gain of relay systems. To investigate the impact of these effects, in addition to the VG relaying systems which use instantaneous CSI to obtain the amplification gain, this work also considers the FG relay systems which has lower complexity. To quantify the performance loss only due to not having updated CSI at the relays to estimate the amplification gain, a VG system which has perfect CSI at the relays, but with relay selection based on outdated CSI is also analyzed.

Chapter 3

In this chapter, the analysis is extended to evaluate the performance under more general Nakagami-m fading. PRS systems with both FG and VG relays are investigated. The analysis is limited to Nakagami-m fading environments with an integer fading parameter m.

In Chapters 2 and 3, exact expressions for the system outage probability and average BER are derived. High SNR approximations of these expressions are also presented, to obtain further insights, in terms of the diversity order and the coding gain. These results enable the comparison of dual hop relay systems, under the impact of outdated CSI. The accuracy of the derived expressions are validated through system simulations.

Chapter 4

The final chapter presents the conclusions and suggest directions for future work.

Chapter 2

Amplify-and-Forward relaying with outdated CSI in Rayleigh fading

2.1 Introduction

In this chapter, the effect of outdated CSI on the performance of a dual hop relay system is analyzed under Rayleigh fading. The main contributions of this chapter can be summarized as follows:

- The performance of PRS systems in the presence of outdated CSI is investigated. Selection of the k-th worst (equivalent to choosing the $(N_r - k)$-th best relay) relay is analyzed.
  - Performance of FG relaying is examined with outdated CSI for relay selection.
  - VG relaying performance is investigated when the relay selection and amplification is done with outdated CSI.
  - VG relaying performance is analyzed when the relay selection is performed in the presence of outdated CSI, while perfect CSI is used for amplification
- The performance of ORS VG systems is examined when relay selection is based on outdated CSI. Two VG relaying systems are analyzed, where one system performs amplification based on outdated CSI, while the other uses perfect CSI for amplification.

- New outage probability and average BER expressions are derived and their accuracy is validated by using Monte Carlo simulations. These expressions are general in the sense that they characterize the performance due to the k-th worst relay selection and arbitrary correlation coefficient between the current and outdated CSI.

- High SNR approximations are derived to obtain further insights of the systems. The impact of outdated CSI on the diversity order of the systems is analyzed.

The performance of the ORS scheme in dual-hop transmissions has been analyzed in the literature (see for e.g. [38–42] and references therein). The outage performance is examined in [38], and the performance of ORS with VG AF relaying under Rayleigh and Rician fading is analyzed in [39] and [40].

Among many works that cover PRS, in [34], the performance with VG AF relaying has been analyzed. In [43] and [44], the performance of PRS with fixed-gain AF relaying has been studied. Very recently in [45], diversity and coding gains of PRS with FG relaying over Nakagami-m channels have been studied.

So far, only few papers have investigated the impact of outdated channel state information on the performance of ORS and PRS schemes [46,47]. In time-varying channels, outdated CSI could be used for relay selection due to feedback delay [48]. Moreover, outdated CSI may also be used for signal amplification at the relay. Although outdated CSI corresponds to several realistic scenarios, to the best of our knowledge, the existing literature has not considered these issues.

The rest of the chapter is organized as follows. Section II introduces the PRS system model with fixed and VG relaying schemes, and ORS system model with VG relaying. The performance of PRS systems is investigated in Section III. Section IV presents the performance analysis of ORS systems. Numerical and simulation examples are presented in Section V. Finally Section VI concludes the chapter with some remarks.

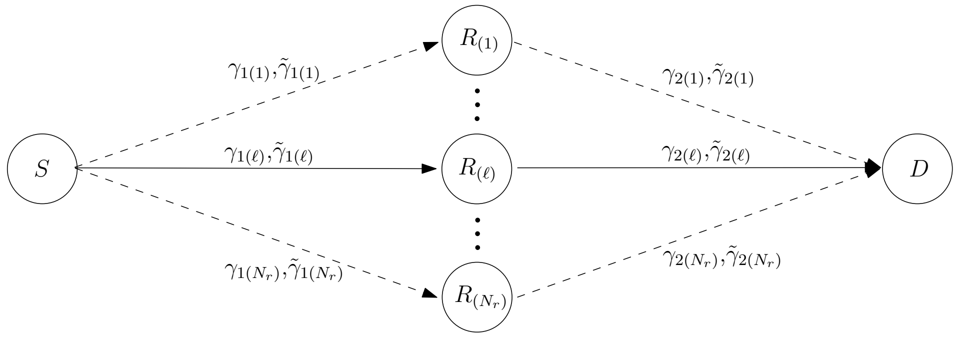

Figure 2.1: The system model. In PRS, $R_{(k)}$ feed backs CSI of $S - R_{(k)}$ link to S. In ORS, CSI of $R_{(k)} - D$ link is sent to S by D through $R_{(k)}$ in addition.

2.2 System Model

This work consider a dual-hop AF system, with a single source S, a single destination D and N_r relays. In this system (Fig. 2.1), it is assumed that S has no direct link to D, which for example may result from high shadowing between S and D. S periodically monitors the quality of its connectivity with the relays via transmission of a local feedback, and selects a single relay, $R_{(k)}$, with the k-th worst $S - R$ link. It is assumed that there is a delay (T_d) in the feedback. Hence this selection decision is based on outdated channel information. During the first time slot, S communicates with the selected $R_{(k)}$ relay. In the second time slot, the relay transmits its received signal to D. The received signal at the selected relay can be written as

$$y_{R_{(k)}}(t) = \sqrt{P_s} h_{S,R_{(k)}}(t) x(t) + n_{R_{(k)}}(t), \tag{2.1}$$

where P_s is the transmit power at S, the complex channel between S and $R_{(k)}$ is $h_{S,R_{(k)}}(t)$ and $n_{R_{(k)}}(t)$ is the additive white Gaussian noise (AWGN) satisfying $E\left(|n_{R_{(k)}}(t)|^2\right) = N_{01}$ with $E(\cdot)$ denoting the expectation. The relay multiplies $y_k(t)$ by a gain, G and the output is transmitted to D. The received signal at D is given by

$$y_D(t) = h_{R_{(k)},D}(t) G y_{R_{(k)}}(t) + n_D(t), \tag{2.2}$$

where $h_{R_{(k)},D}(t)$ is the complex channel between $R_{(k)}$ and D, and $n_D(t)$ is the AWGN satisfying $E\left(|n_D(t)|^2\right) = N_{02}$.

Let $\tilde{\gamma}_{1(k)} = |h_{S,R_{(k)}}(t)|^2\eta_1$ and $\tilde{\gamma}_{2(k)} = |h_{R_{(k)},D}(t)|^2\eta_2$, where $\eta_1 = \frac{P_s}{N_{01}}$, $\eta_2 = \frac{P_r}{N_{02}}$ and P_r is the average transmit power of $R_{(k)}$. Further, the link SNRs are defined as $\gamma_{1(k)} = |h_{S,R_{(k)}}(t-Td)|^2\eta_1$ and $\gamma_{2(k)} = |h_{R_{(k)},D}(t-T_d)|^2\eta_1$. It is assumed that the channels are Rayleigh fading channels, and $h_{S,R_{(k)}}(t), h_{R_{(k)},D}(t) \sim \mathcal{CN}(0,1)$ are complex Gaussian with zero mean and unit variance. Note that the relay selection would be based on $\gamma_{1(k)}$ and $\gamma_{2(k)}$. $\tilde{\gamma}_{1(k)}$ and $\tilde{\gamma}_{2(k)}$, the link SNRs experienced by the signal, are delayed versions of $\gamma_{1(k)}$ and $\gamma_{2(k)}$ respectively. We assume $\gamma_{1(k)}$ and $\tilde{\gamma}_{1(k)}$ are correlated with a correlation coefficient of ρ_1, while $\gamma_{2(k)}$ and $\tilde{\gamma}_{2(k)}$ have a correlation coefficient ρ_2.

2.2.1 Partial Relay Selection

Let $\gamma_{1(1)} \leq \gamma_{1(2)} \leq \cdots \leq \gamma_{1(N_r)}$ be the order statistics obtained by arranging $\gamma_{1(\ell)}$ for $\ell = 1, \ldots, N_r$ in an increasing order of magnitude. In PRS, the interest is in $\gamma_{1(k)}$ and the respective relay $R_{(k)}$ is selected for communication.

Fixed Gain Relaying

Consider a PRS system in which relays will amplify the received signal using a fixed gain factor [43, 44]. The relay does not require the instantaneous CSI of the $S-R$ link. Hence, this is a low complexity system, which offers practical relevance.

Assuming that $R_{(k)}$ knows the statistics of the $S-R$ channel, the relay can choose the fixed gain, $G = \sqrt{\frac{P_r}{P_s E\{|h_{S,R_{(k)}}(t)|^2\}+N_{01}}}$. Amplification gain calculated in this manner would remain constant when the instantaneous value of the $S-R$ channel coefficient varies. It would also make the average output power of the relay equal to P_r. It can be shown that the end-to-end (e2e) SNR is given by

$$\gamma_{eq1} = \frac{\tilde{\gamma}_{1(k)}\tilde{\gamma}_{2(k)}}{C + \tilde{\gamma}_{2(k)}}, \tag{2.3}$$

where $C = \frac{P_r}{G^2 N_{01}}$ [49]. From [46, Eq. (9)], it is known that the probability density function (PDF) of $\tilde{\gamma}_{1(k)}$ is given by

$$f_{\tilde{\gamma}_{1(k)}}(x) = k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m}{\eta_1}\binom{k-1}{m}\frac{1}{(N_r-k+m)(1-\rho_1)+1}e^{-\frac{(N_r-k+m+1)x}{((N_r-k+m)(1-\rho_1)+1)\eta_1}}, \tag{2.4}$$

where $0 \leq \rho_1 \leq 1$ is the correlation coefficient between $\tilde{\gamma}_{1(k)}$ and $\gamma_{1(k)}$, and $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ denotes the binomial coefficient. Using this result, the following expression can be obtained for C:

$$C = E\left\{\tilde{\gamma}_{1(k)} + 1\right\} = k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m}{\eta_1}\binom{k-1}{m}\frac{1}{(N_r-k+m)(1-\rho_1)+1} \tag{2.5}$$

$$\times \int_0^\infty (x+1)e^{-\frac{(N_r-k+m+1)x}{((N_r-k+m)(1-\rho_1)+1)\eta_1}}dx$$

$$= 1 + k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m\binom{k-1}{m}((N_r-k+m)(1-\rho_1)+1)\eta_1}{(N_r-k+m+1)^2}.$$

VG Relaying I

VG relaying I system, considered in this section, is a PRS system in which each relay only makes one channel measurement, based on which the selection of a relay is made at S. Variable gain relays calculate the amplification gain using the instantaneous CSI of $S-R$ link. Since the $S-R$ channel is measured during the relay selection process, the relay already have this information. But this CSI is outdated. The selected relay also uses this same outdated information to amplify the signal, $y_{R_{(k)}}(t)$. Hence the amplification gain factor at the relay can be expressed as [34]

$$G = \sqrt{\frac{P_r}{P_s|h_{S,R_{(k)}}(t-T_d)|^2 + N_{01}}}. \tag{2.6}$$

Plugging (2.6) into (2.2) and after some manipulations, the end-to-end SNR can be written as

$$\gamma_{eq2} = \frac{\tilde{\gamma}_{1(k)}\tilde{\gamma}_{2(k)}}{\gamma_{1(k)} + \tilde{\gamma}_{2(k)} + 1}. \tag{2.7}$$

VG Relaying II

Here, a variable gain relaying system with the assumption that the relays will have updated channel information to calculate the amplification gain is presented. Such information can be obtained using superimposed pilots. This system was analyzed in [46].

This scheme has a higher implementation complexity compared to the VG I presented in previous section, since this system requires the relay to estimate $h_{S,R_{(k)}}(t-T_d)$ as well as $h_{S,R_{(k)}}(t)$.

Now the VG factor at the relay,

$$G = \sqrt{\frac{P_r}{P_s|h_{S,R_{(k)}}(t)|^2 + N_{01}}}. \tag{2.8}$$

Substituting (2.8) into (2.2) and after some manipulations, the end-to-end SNR can be written as

$$\gamma_{eq3} = \frac{\tilde{\gamma}_{1(k)}\tilde{\gamma}_{2(k)}}{\tilde{\gamma}_{1(k)} + \tilde{\gamma}_{2(k)} + 1}. \tag{2.9}$$

It is noted that in order to study the performance metrics of this system a novel analysis is required. This is because the form of the instantaneous end-to-end SNR, γ_{eq3}, is different from γ_{eq2} in (2.7) and as a result new expressions must be derived.

2.2.2 Opportunistic Relay Selection

In this section a system with full relay selection, i.e. relay selection based on channel information on all $S-R$ and $R-D$ branches, is analyzed. It is assumed that the instantaneous channel measurements of each $S-R_{(l)}$ and $R_{(l)}-D$ links are transmitted back to the source, and these channel states would have changed by the time of actual communication.

VG Relaying I

Let $\hat{\gamma}_l = \min(\gamma_{1(l)}, \gamma_{2(l)})$, and let $\hat{\gamma}_k$ be the kth smallest among the $\hat{\gamma}_l$s. Then $S-R_{(k)}$and $R_{(k)}-D$ links will be chosen for the communication. The actual SNR experienced during communication on $S-R_{(k)}$ and $R_{(k)}-D$ links would be $\tilde{\gamma}_{1(k)}$ and $\tilde{\gamma}_{2(k)}$ respectively.

As VG relaying in this system is assumed, the gain G is given by (2.6), and the e2e SNR(γ_{eq4}) is given by $\gamma_{eq4} = \frac{\tilde{\gamma}_{1(k)}\tilde{\gamma}_{2(k)}}{\gamma_{1(k)}+\tilde{\gamma}_{2(k)}+1}$.

VG Relaying II

In this system, it is assumed that the relay has instantaneous CSI of the $S - R$ link through pilot assisted channel estimation using packets received from the source, and will select the amplification gain factor accordingly. Even in this case, there could be a delay in feedback and the relay selection could be done using outdated CSI. This is similar to the system analyzed in [46]. This analysis is undertaken in order to observe the benefit of having perfect CSI at the relays. For this case, the amplification gain factor, $G = \sqrt{\frac{P_r}{P_s|h_{S,R_{(k)}}(t)|^2+N_{01}}}$. The resulting e2e SNR can be expressed as $\gamma_{eq5} = \frac{\tilde{\gamma}_{1(k)}\tilde{\gamma}_{2(k)}}{\tilde{\gamma}_{1(k)}+\tilde{\gamma}_{2(k)}+1}$.

2.3 Analysis of Partial Relay Selection Systems

This section presents the derivations of important performance metrics; the outage probability and the average BER for the dual-hop partial relay selection system with fixed and VG relaying.

2.3.1 Fixed Gain Relaying

Outage Probability

The outage probability, P_o, defined as the probability that the end-to-end SNR drops below a predefined SNR threshold γ_T, is an important quality of service (QoS) measure. Mathematically, the outage probability can be evaluated using

$$F_{\gamma_{eq1}}(\gamma_T) = \Pr(\gamma_{eq1} < \gamma_T) = \Pr\left(\frac{\tilde{\gamma}_{1(k)}\tilde{\gamma}_{2(k)}}{C + \tilde{\gamma}_{2(k)}} < \gamma_T\right), \tag{2.10}$$

where $\Pr(\cdot)$ denotes the probability. Eq (2.10) can be simplified as

$$F_{\gamma_{eq1}}(\gamma_T) = 1 - \int_{\gamma_T}^{\infty} \Pr\left(\tilde{\gamma}_{2(k)} > \frac{C\gamma_T}{x - \gamma_T}\right) f_{\tilde{\gamma}_{1(k)}}(x)dx. \tag{2.11}$$

Using (2.4) and the complementary cumulative distribution function (CCDF) of $\tilde{\gamma}_{2(k)}$, with some algebraic manipulations, the following CDF is obtained.

$$F_{\gamma_{eq1}}(\gamma_T) = 1 - k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m}{\eta_1}\binom{k-1}{m}\frac{e^{-\frac{(N_r-k+m)\gamma_T}{((N_r-k+m)(1-\rho_1)+1)\eta_1}}}{(N_r-k+m)(1-\rho_1)+1} \tag{2.12}$$
$$\times\int_0^\infty e^{-\frac{C\gamma_T}{\eta_2 y}-\frac{(N_r-k+m+1)y}{((N_r-k+m)(1-\rho_1)+1)\eta_1}}dy.$$

Finally using [50, Eq. (4.5.25)], the outage probability can be expressed as

$$F_{\gamma_{eq1}}(\gamma_T) = 1 - 2k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m\binom{k-1}{m}}{((N_r-k+m)(1-\rho_1)+1)}e^{-\frac{(N_r-k+m+1)\gamma_T}{((N_r-k+m)(1-\rho_1)+1)\eta_1}} \tag{2.13}$$
$$\times\sqrt{\frac{((N_r-k+m)(1-\rho_1)+1)C\gamma_T}{(N_r-k+m+1)\eta_1\eta_2}}K_1\left(2\sqrt{\frac{(N_r-k+m+1)C\gamma_T}{((N_r-k+m)(1-\rho_1)+1)\eta_1\eta_2}}\right),$$

where $K_\nu(x)$ is the νth order modified Bessel function of the second kind [7, Sec. (9.6)]. The outage probability for the special case of $\rho_1 = 1$ and $N_r = k$ is given by [43, Eq. (5)]. Although the above result(2.13) gives the exact outage probability, a simpler high-SNR approximation is desirable in order to gain further insights, in terms of the diversity order and the coding gain. *Corollary 1* presents a simple result for the outage probability at high SNR.

Corollary 1 *The asymptotic outage probability for large η_1 and η_2 with fixed ratio, $\mu = \frac{\eta_2}{\eta_1}$ admits the first order approximation given by*

$$F_{\gamma_{eq1}}(\gamma_T)\approx\frac{\gamma_T}{\eta_1}\sum_{m=0}^{k-1}\frac{(-1)^m k\binom{N_r}{k}\binom{k-1}{m}}{(N_r-k+m)(1-\rho_1)+1}\left(\frac{\Lambda}{\mu}\ln\left(\frac{((N_r-k+m)(1-\rho_1)+1)\eta_1}{(N_r-k+m+1)}\right)+\phi\right), \tag{2.14}$$

with ϕ defined as

$$\phi = e^{-\Lambda} + \Lambda\left(1-\gamma+\mathit{Ei}(-\Lambda)-\ln(\Lambda)\right), \tag{2.15}$$

and $\Lambda = k\binom{N_r}{k}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}\frac{((N_r-k+m)(1-\rho_1)+1)}{(N_r-k+m+1)^2}$.

The proof is given in Appendix I.

In (2.15) $\gamma = 0.57721...$ is the Euler–Mascheroni constant and $\mathrm{Ei}(x)$ is the exponential integral function [7, Eq. (5.1.2)]. Note that the simpler form in (2.14) shows the impact of parameters such as k, ρ_1 and N_r on the outage probability. High SNR approximation for the special cases $\rho_1 = 0, 1$ are given below.

$$F_{\gamma_{eq1}}(\gamma_T) \approx \begin{cases} \frac{\gamma_T}{\eta_1}\left(\frac{\Lambda}{\mu}\ln(\eta_1)+\nu\right) & \rho_1 = 0, \\ \sum_{m=0}^{k-1}\frac{k\binom{N_r}{k}\gamma_T(-1)^m\binom{k-1}{m}}{\eta_1}\ln\!\left(\frac{\eta_1}{N_r-k+m+1}\right) & \rho_1 = 1. \end{cases} \tag{2.16}$$

Average BER

The analysis then proceeds to the system's average error performance. For many modulation formats used in wireless applications, the average BER can be expressed as

$$P_b = \alpha\, E[Q(\sqrt{\beta\gamma_{eq1}})] = \frac{\alpha}{\sqrt{2\pi}}\int_0^\infty F_{\gamma_{eq1}}\left(\frac{t^2}{\beta}\right)e^{-\frac{t^2}{2}}dt, \tag{2.17}$$

where $\alpha, \beta > 0$ are constants depending on the modulation scheme, and $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{y^2}{2}}dy$ is the Gaussian Q-function. Eq. (2.17) can be evaluated with the help of [50, Eq. (4.16.33)] and the following expression for the average BER is arrived at.

$$P_b = \frac{\alpha}{2} - \frac{\alpha\sqrt{\beta\eta_1}kC}{2\eta_2}\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m\binom{k-1}{m}}{((N_r-k+m)(1-\rho_1)+1)} \tag{2.18}$$
$$\times\left(\frac{(N_r-k+m+1)}{((N_r-k+m)(1-\rho_1)+1)}+\beta\eta_1\right)^{-\frac{3}{2}}e^{\varsigma_3}\left(K_1(\varsigma_3)-K_0(\varsigma_3)\right),$$

where $\varsigma_3 = \frac{C(N_r-k+m+1)}{\eta_2(2(N_r-k+m+1)+\beta\eta_1((N_r-k+m)(1-\rho_1)+1))}$. The average BER for the special case with $\rho_1 = 1$ and $N_r = k$ is given in [43, Eq. (12)].

Substituting (2.14) into (2.17) and solving the integral, the average BER at high SNR can be written as

$$P_b^\infty \approx \frac{\alpha k}{2\beta\eta_1}\sum_{m=0}^{k-1}\frac{(-1)^m\binom{N_r}{k}\binom{k-1}{m}}{(N_r-k+m)(1-\rho_1)+1}\left(\frac{\Lambda}{\mu}\ln\left(\frac{((N_r-k+m)(1-\rho_1)+1)\eta_1}{(N_r-k+m+1)}\right)+\phi\right). \tag{2.19}$$

2.3.2 Variable Gain Relaying I

Outage Probability

In order to derive the outage probability of VG relaying, it is convenient to obtain a statistical distribution formula for the general form:

$$Y = \frac{\tilde{\gamma}_{1(k)}\tilde{\gamma}_{2(k)}}{\gamma_{1(k)} + \tilde{\gamma}_{2(k)} + c}. \tag{2.20}$$

Note that $c = 1$ gives the exact expression for γ_{eq2} in (2.7), while $c = 0$ can be substituted to obtain an analytically feasible approximation. The cumulative distribution function (CDF) of the random variable (RV) Y can be written as

$$F_Y(\gamma_T) = \Pr\left(\frac{\tilde{\gamma}_{1(k)}\tilde{\gamma}_{2(k)}}{\gamma_{1(k)} + \tilde{\gamma}_{2(k)} + c} < \gamma_T\right). \tag{2.21}$$

After some mathematical manipulations, (2.21) can be written as follows

$$F_Y(\gamma_T) = 1 - \int_0^\infty \int_0^\infty \Pr\left(\tilde{\gamma}_{2(k)} > \frac{\gamma_T(y+c)}{w}\right) f_{\tilde{\gamma}_{1(k)},\gamma_{1(k)}}(w+\gamma_T, y)dwdy. \tag{2.22}$$

It is important to note that, $f_{\tilde{\gamma}_{1(k)}|\gamma_{1(k)}}(x|y) = f_{\tilde{\gamma}_{(\ell)}|\gamma_{(\ell)}}(x|y)$, where ℓ represents unordered relays. Hence the joint PDF of $\tilde{\gamma}_{1(k)}$ and $\gamma_{1(k)}$ can be established from

$$f_{\tilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y) = \frac{f_{\tilde{\gamma}_{1(\ell)},\gamma_{1(\ell)}}(x,y)}{f_{\gamma_{1(\ell)}(y)}} \times f_{\gamma_{1(k)}}(y). \tag{2.23}$$

Since $\tilde{\gamma}_{1(\ell)}$ and $\gamma_{1(\ell)}$ are two correlated exponentially distributed RVs, their joint PDF

$$f_{\tilde{\gamma}_{1(\ell)},\gamma_{1(\ell)}}(x,y) = \frac{e^{-\frac{x+y}{(1-\rho_1)\eta_1}}}{(1-\rho_1)\eta_1^2} I_0\left(\frac{2\sqrt{\rho_1 xy}}{(1-\rho_1)\eta_1}\right), \tag{2.24}$$

where $I_0(x)$ is the zeroth order modified Bessel function of the first kind. W e know that the PDF $f_{\gamma_{1(k)}}(y)$ is given by $f_{\gamma_{1(k)}}(y) = \frac{N_r!}{(k-1)!(N_r-k)!}[F_{\gamma_{1(\ell)}}(y)]^{k-1}[1 - F_{\gamma_{1(\ell)}}(y)]^{N_r-k} f_{\gamma_{1(\ell)}}(y)$ where $f_{\gamma_{1(\ell)}}(y) = \frac{1}{\eta_1}e^{-\frac{y}{\eta_1}}$ and $F_{\gamma_{1(\ell)}}(y) = 1 - e^{-\frac{y}{\eta_1}}$. Using the above results in (2.23) and after some simplifications, the joint PDF of $\tilde{\gamma}_{1(k)}$ and $\gamma_{1(k)}$, $f_{\tilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y)$, can be written as

$$f_{\tilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y) = \frac{k\binom{N_r}{k}e^{-\frac{x}{(1-\rho_1)\eta_1}}}{(1-\rho_1)\eta_1^2} I_0\left(\frac{2\sqrt{\rho_1 xy}}{(1-\rho_1)\eta_1}\right) \sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m} e^{-\frac{((N_r-k+m)(1-\rho_1)+1)y}{(1-\rho_1)\eta_1}}. \tag{2.25}$$

Substituting (2.25) and the CCDF of $\tilde{\gamma}_{2(k)}$ into (2.22), the following expression is obtained.

$$F_Y(\gamma_T) = 1 - \int_0^\infty \int_0^\infty e^{-\frac{\gamma_T(y+c)}{\eta_2 w}} \frac{k\binom{N_r}{k} e^{-\frac{(w+\gamma_T)}{(1-\rho_1)\eta_1}}}{(1-\rho_1)\eta_1^2} I_0\left(\frac{2\sqrt{\rho_1(w+\gamma_T)y}}{(1-\rho_1)\eta_1}\right) \quad (2.26)$$
$$\times \sum_{m=0}^{k-1}(-1)^m \binom{k-1}{m} e^{-\frac{((N_r-k+m)(1-\rho_1)+1)y}{(1-\rho_1)\eta_1}} dwdy.$$

Using the infinite series expansion $I_0(x) = \sum_{p=0}^{\infty} \frac{x^{2p}}{2^{2p}(p!)^2}$ from [51, Eq. (8.447.1)] in (2.26) and [50, Eq. (4.5.29)], (2.26) can be expressed as

$$F_Y(\gamma_T) = 1 - \sum_{p=0}^{\infty}\sum_{m=0}^{k-1}\sum_{n=0}^{p} \frac{(-1)^m\binom{k-1}{m}k\binom{N_r}{k}{\rho_1}^p e^{-\frac{\gamma_T}{(1-\rho_1)\eta_1}}}{(1-\rho_1)^{2p+1}(p!)^{2p}\eta_1^{2p+2}} \binom{p}{n}\gamma_T^{p-n} \quad (2.27)$$
$$\times \int_0^\infty y^p e^{-\frac{((N_r-k+m)(1-\rho_1)+1)y}{(1-\rho_1)\eta_1}} 2\left(\frac{\gamma_T(y+c)(1-\rho_1)\eta_1}{\eta_2}\right)^{\frac{n+1}{2}} K_{n+1}\left(\sqrt{\frac{4\gamma_T(y+c)}{\eta_1\eta_2(1-\rho_1)}}\right) dy.$$

To the best of authors' knowledge, the integral in (2.27) does not have a closed-form solution. Hence, a tight lower bound for (2.27) is obtained substituting $c = 0$ and using [50, Eq. (4.16.37)].

$$F_{\gamma_{eq2}}(\gamma_T) \geq 1 - \sum_{p=0}^{\infty}\sum_{m=0}^{k-1}\sum_{n=0}^{p} \frac{(-1)^m\binom{N_r}{k}\binom{k-1}{m}\binom{p}{n}k{\rho_1}^p\gamma_T^{p+1}(p+n+1)!}{(1-\rho_1)^{p-n-1}\eta_1^{p-n}\eta_2^{n+1}p!((N_r-k+m)(1-\rho_1)+1)^{p+n+2}} \quad (2.28)$$
$$\times \mathcal{U}\left(p+n+2, n+2; \frac{\gamma_T}{\eta_2((N_r-k+m)(1-\rho_1)+1)}\right) e^{-\frac{\gamma_T}{\eta_1(1-\rho_1)}},$$

where $\mathcal{U}(a, b; z)$ is the confluent hypergeometric function of the second kind [7, Eq. (13.1.3)].

In Section V, extensive simulation results to complement (2.28) are presented. The outage probability predicted from (2.28) and simulations match perfectly even at low SNRs as $\eta_1 = \eta_2 = 5$ dB. For the special case of $\rho_1 = 1$ and $N_r = k$, the outage probability is given in [34, Eq. (2)].

A high SNR approximation for the outage probability can be obtained and is presented as *Corollary 2* below.

Corollary 2 *The asymptotic outage probability for large η_1 and η_2 with fixed ratio, $\mu = \frac{\eta_2}{\eta_1}$ admits the first order approximation given by*

$$F_{\gamma_{eq2}}(\gamma_T) \approx k\binom{N_r}{k}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}\left(\frac{p_1+p_2}{N_r-k+m+1}+\frac{\ln(\eta_1/p_1)}{\mu\omega^2(1-\rho_1)}\right)\left(\frac{\gamma_T}{\eta_1}\right). \tag{2.29}$$

The proof is relegated to Appendix II with p_1, p_2 and ω defined.

Average BER

Using (2.17), and [50, Eq (4.22.16)], the average BER of the PRS system with VG relaying can be derived.

$$P_b \geq \frac{\alpha}{2} - \frac{\alpha k}{\sqrt{8\pi}}\sum_{p=0}^{\infty}\sum_{m=0}^{k-1}\sum_{n=0}^{p}\frac{(-1)^m\binom{N_r}{k}\binom{k-1}{m}\binom{p}{n}(p+n+1)!{\rho_1}^p\beta^{\frac{n}{2}-p}}{(1-\rho_1)^{p-n-1}\eta_1^{p-n}\eta_2^{\frac{n}{2}}p!((N_r-k+m)(1-\rho_1)+1)^{p+1+\frac{n}{2}}} \tag{2.30}$$

$$\times\frac{\Gamma(p+\frac{3}{2})\Gamma(p-n+\frac{1}{2})\varsigma_2^{\frac{n}{2}+1}}{\Gamma(2p+\frac{5}{2})(\varsigma_1+\frac{\varsigma_2}{2})^{p+\frac{3}{2}}}{}_2F_1\left(p+\frac{3}{2},p+n+2,2p+\frac{5}{2};\frac{\varsigma_1-\frac{\varsigma_2}{2}}{\varsigma_1+\frac{\varsigma_2}{2}}\right),$$

where $\varsigma_1 = \frac{1}{\beta\eta_1(1-\rho_1)} - \frac{1}{2\eta_2((N_r-k+m)(1-\rho_1)+1)\beta} + \frac{1}{2}$, $\varsigma_2 = \frac{1}{\beta\eta_2((N_r-k+m)(1-\rho_1)+1)}$ and ${}_2F_1(a,b;c;x)$ is the Gauss hypergeometric function [7, Eq. (15.1.1)]. The average BER for the special case with $\rho_1 = 1$, $N_r = k$ and BPSK modulation is given in [34, Eq. (14)].

Consider the average BER at high SNR. Following a similar approach as in the case of FG relaying, the average BER for the VG relaying, in the high SNR regime can be written as

$$P_b^{\infty} \approx \frac{\alpha k\binom{N_r}{k}}{2\beta\eta_1}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}\left(\frac{p_1+p_2}{N_r-k+m+1}+\frac{\ln(\eta_1/p_1)}{\mu\omega^2(1-\rho_1)}\right). \tag{2.31}$$

The average output power at the relay in the case of VG relaying would be different from P_r due to selecting the amplification gain factor using outdated CSI. Hence, for a fair comparison of the fixed and VG schemes, an average power normalization is made so that the average output power at the relay is equal to P_r. In order to do so, a modified amplification gain factor, $G = \sqrt{\frac{P_r/\xi}{P_s|h_{S,R_{(k)}}(t-T_d)|^2+N_{01}}}$ is introduced, where

$$\xi = E\left\{\frac{\tilde{\gamma}_{1(k)}+1}{\gamma_{1(k)}+1}\right\} = \frac{k\binom{N_r}{k}}{(1-\rho_1)\eta_1^2}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}\int_0^\infty\int_0^\infty\left(\frac{x+1}{y+1}\right) \times I_0\left(\frac{2\sqrt{\rho_1 xy}}{(1-\rho_1)\eta_1}\right)e^{-\left(\frac{x}{(1-\rho_1)\eta_1}+\frac{((N_r-k+m)(1-\rho_1)+1)y}{(1-\rho_1)\eta_1}\right)}dxdy. \tag{2.32}$$

Using [52, Eq. (9)] and [50, Eq. (4.2.6)], (2.32) can be evaluated to arrive at the expression for the scaling factor given by

$$\xi = k\binom{N_r}{k}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m} \times \left(\frac{\rho_1}{N_r-k+m+1}-(1-\rho_1)\left(1+\frac{1}{\eta_1}\right)e^{\frac{N_r-k+m+1}{\eta_1}}\mathrm{Ei}\left(-\frac{N_r-k+m+1}{\eta_1}\right)\right). \tag{2.33}$$

2.3.3 Variable Gain Relaying II

Outage Probability

The outage probability of the system can be derived by using the approach shown in Appendix III.

$$F_{\gamma_{eq3}}(\gamma_T) = 1-2k\binom{N_r}{k}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m} \times\sqrt{\frac{\gamma_T^2+c\gamma_T}{(N_r-k+m+1)((N_r-k+m)(1-\rho_1)+1)\eta_1\eta_2}} \times e^{-\left(\frac{N_r-k+m+1}{((N_r-k+m)(1-\rho_1)+1)\eta_1}+\frac{1}{\eta_2}\right)\gamma_T}K_1\left(2\sqrt{\frac{(N_r-k+m+1)(\gamma_T^2+c\gamma_T)}{((N_r-k+m)(1-\rho_1)+1)\eta_1\eta_2}}\right) \tag{2.34}$$

Outage Probability at High SNR

Although the outage probability in (2.34) is exact and valid for arbitrary SNRs, it is difficult to use (2.34) to get direct insights. For example, it is interesting to know how system and network parameters such as N_r, ρ_1 and SNR imbalance influence the system's outage performance. Since such insight can not be *directly* obtained from (2.34), a simple outage probability expression valid for high SNR is developed.

In the asymptotic SNR regime, $\eta_1, \eta_2 \to \infty$. Applying a Bessel function approximation for small arguments of x, $0 < x << \sqrt{\nu+1}$, given by

$$K_\nu(x) \simeq \frac{2^{\nu-1}\Gamma(\nu)}{x^\nu}, \tag{2.35}$$

using this in (2.34), it is seen that

$$F_{\gamma_{eq3}}(\gamma_T) \approx 1 - k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m}{N_r-k+m+1}\binom{k-1}{m}e^{-\left(\frac{N_r-k+m+1}{((N_r-k+m)(1-\rho_1)+1)\eta_1}+\frac{1}{\eta_2}\right)\gamma_T} \tag{2.36}$$

Let $\eta_2 = \mu\eta_1$ and $F_{\gamma_{eq3}}(\gamma_T)$ can be re-expressed as

$$F_{\gamma_{eq3}}(x) = 1 - k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m}{N_r-k+m+1}\binom{k-1}{m}e^{-\left(\frac{N_r-k+m+1}{(N_r-k+m)(1-\rho_1)+1}+\frac{1}{\mu}\right)x}, \tag{2.37}$$

where $x = \frac{\gamma_T}{\eta_1}$. Using the Maclaurin series representation for the exponential function in (2.37) one gets

$$\begin{aligned} F_{\gamma_{eq3}}(x) = 1 - k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m}{N_r-k+m+1}\binom{k-1}{m} \\ \times\sum_{p=0}^{\infty}(-1)^p\frac{\left(\frac{N_r-k+m+1}{(N_r-k+m)(1-\rho_1)+1}+\frac{1}{\mu}\right)^p}{p!}x^p. \end{aligned} \tag{2.38}$$

Simplifying further and collecting only the first order terms yield an outage probability approximation given by

$$F_{\gamma_{eq3}}(x) = k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m}{N_r-k+m+1}\binom{k-1}{m}\left(\frac{N_r-k+m+1}{(N_r-k+m)(1-\rho_1)+1}+\frac{1}{\mu}\right)x. \tag{2.39}$$

Finally, substituting $x = \frac{\gamma_T}{\eta_1}$, the outage probability can be written as

$$F_{\gamma_{eq3}}(\gamma_T) = k\binom{N_r}{k}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}\frac{\gamma_T}{((N_r-k+m)(1-\rho_1)+1)\eta_1}+\frac{\gamma_T}{\mu\eta_1}. \tag{2.40}$$

It follows from (2.40) that in the special cases of $\rho_1 = 0$ and $\rho_1 = 1$, for $k = N_r$, the outage becomes

$$F_{\gamma_{eq3}}(\gamma_T) = \begin{cases} \left(1 + \frac{1}{\mu}\right) \frac{\gamma_T}{\eta_1} & \rho_1 = 0, \\ \frac{\gamma_T}{\mu\eta_1} & \rho_1 = 1. \end{cases} \tag{2.41}$$

Average BER

We now derive expressions for the system's average BER. For many modulation formats (see below) used in wireless applications, the average BER can be expressed as

$$P_b = \alpha \, E[Q(\sqrt{\beta\gamma_{eq3}})], \tag{2.42}$$

where $\alpha, \beta > 0$, and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{y^2}{2}} dy$ is the Gaussian Q-function. For binary phase shift keying (BPSK) $(\alpha, \beta) = (1, 2)$, quadrature phase shift keying (QPSK) $(\alpha, \beta) = (1, 1)$ gives the exact BER and for M-PSK $(\alpha, \beta) = \Big(\frac{1}{\log_2 M},$ $\log_2 M \sin^2 \frac{\pi}{M}\Big)$ can be used to approximate the BER.

Using integration by parts it can be shown that (2.42) can be re-expressed as

$$P_b = \frac{\alpha}{\sqrt{2\pi}} \int_0^\infty F_{\gamma_{eq3}}\left(\frac{t^2}{\beta}\right) e^{-\frac{t^2}{2}} dt. \tag{2.43}$$

To the best of author's knowledge, (2.43) does not have a closed-form solution for $F_{\gamma_{eq3}}$ given in (2.34). To overcome this challenge, the following approximation is considered, $\gamma_{eq3} = \frac{\tilde{\gamma}_{1(k)}\gamma_2}{\tilde{\gamma}_{1(k)}+\gamma_2}$, since it is a tight upper bound for γ_{eq3} in the regimes of medium-to-high SNR. Extensive simulation results are provided in Section IV to complement the bounds. Thus, after substituting (2.34) with $c = 0$ into (2.43) we obtain (2.44).

$$P_b \approx \frac{\alpha}{2} - \frac{\alpha k}{\beta}\sqrt{\frac{2}{\pi\eta_1\eta_2}} \binom{N_r}{k} \sum_{m=0}^{k-1} \frac{(-1)^m \binom{k-1}{m}}{\sqrt{(N_r - k + m + 1)((N_r - k + m)(1 - \rho_1) + 1)}} \tag{2.44}$$

$$\times \int_0^\infty t^2 e^{-\left(\frac{N_r - k + m + 1}{((N_r - k + m)(1-\rho_1)+1)\beta\eta_1} + \frac{1}{\beta\eta_2} + \frac{1}{2}\right)t^2} K_1\left(\frac{2t^2}{\beta}\sqrt{\frac{(N_r - k + m + 1)}{((N_r - k + m)(1 - \rho_1) + 1)\eta_1\eta_2}}\right)$$

The integral in (2.44) can be solved with the help of [51, Eq. (6.621.3)] to yield (2.45).

$$P_b \approx \frac{\alpha}{2} - \frac{3\pi\alpha k}{\sqrt{2}\beta^2\eta_1\eta_2}\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{(-1)^m\binom{k-1}{m}}{((N_r-k+m)(1-\rho_1)+1)} \tag{2.45}$$
$$\times \frac{{}_2F_1\left(\frac{5}{2},\frac{3}{2},2;\frac{\frac{N_r-k+m+1}{((N_r-k+m)(1-\rho_1)+1)\beta\eta_1}+\frac{1}{\beta\eta_2}+\frac{1}{2}-\frac{2}{\beta}\sqrt{\frac{(N_r-k+m+1)}{((N_r-k+m)(1-\rho_1)+1)\eta_1\eta_2}}}{\frac{N_r-k+m+1}{((N_r-k+m)(1-\rho_1)+1)\beta\eta_1}+\frac{1}{\beta\eta_2}+\frac{1}{2}+\frac{2}{\beta}\sqrt{\frac{(N_r-k+m+1)}{((N_r-k+m)(1-\rho_1)+1)\eta_1\eta_2}}}\right)}{\left(\frac{N_r-k+m+1}{((N_r-k+m)(1-\rho_1)+1)\beta\eta_1}+\frac{1}{\beta\eta_2}+\frac{1}{2}+\frac{2}{\beta}\sqrt{\frac{(N_r-k+m+1)}{((N_r-k+m)(1-\rho_1)+1)\eta_1\eta_2}}\right)^{2.5}}$$

Average BER at High SNR

Substituting (2.40) into (2.43) the average BER at high SNR (P_b^∞) can be written as

$$P_b^\infty = \frac{\alpha}{\sqrt{2\pi}\eta_1\beta}\left(k\binom{N_r}{k}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}\right. \tag{2.46}$$
$$\left.\times\frac{1}{(N_r-k+m)(1-\rho_1)+1}+\mu^{-1}\right)\int_0^\infty w^2 e^{-\frac{w^2}{2}}dw$$

Simplifying the integral in (2.46) with the help of [51, Eq. (3.381.4)] yields

$$P_b^\infty \approx \frac{\alpha}{2}\left(k\binom{N_r}{k}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}\frac{1}{(N_r-k+m)(1-\rho_1)+1}+\mu^{-1}\right)(\beta\eta)^{-1} \tag{2.47}$$

2.4 Analysis of Opportunistic Relay Selection Systems

2.4.1 Variable Gain Relaying I

Outage Probability

To derive the outage probability, the distribution functions of $\tilde{\gamma}_{1(k)}$ and $\tilde{\gamma}_{2(k)}$ are required. In order to that, the distribution functions of $\gamma_{1(k)}$ and $\gamma_{1(k)}$ must be obtained first. The CDF of $\gamma_{1(k)}$ can be written as

$$
\begin{aligned}
&F_{\gamma_{1(k)}}(\gamma_T) = \Pr\left(\gamma_{1(l)} < \gamma_T \cap \gamma_{1(l)} > \gamma_{2(l)} \cap l = k\right) \\
&+ \Pr\left(\gamma_{1(l)} < \gamma_T \cap \gamma_{1(l)} < \gamma_{2(l)} \cap l = k\right) \\
&= \int_0^{\gamma_T} f_{\gamma_{1(l)}}(x) \int_0^{x} f_{\gamma_{2(l)}}(y) N_r \binom{N_r-1}{k-1} \left(F_{\hat{\gamma}_l}(y)\right)^{k-1} \left(1 - F_{\hat{\gamma}_l}(y)\right)^{Nr-k} \, dydx \\
&+ \int_0^{\gamma_T} f_{\gamma_{1(l)}}(x) N_r \binom{N_r-1}{k-1} \left(F_{\hat{\gamma}_l}(x)\right)^{k-1} \left(1 - F_{\hat{\gamma}_l}(x)\right)^{Nr-k} \int_x^{\infty} f_{\gamma_{2(l)}}(y) dydx.
\end{aligned}
\tag{2.48}
$$

Since $\gamma_{1(i)}$ and $\gamma_{2(i)}$ are exponential RVs, $F_{\hat{\gamma}_i}(x) = 1 - \Pr\left(\min(\gamma_{1(l)}, \gamma_{2(l)}) > x\right) = 1 - e^{-\frac{x}{\bar{\eta}}}$, with $\bar{\eta} = \frac{\eta_1\eta_2}{\eta_1+\eta_2}$. Employing the binomial expansion yields $\left(F_{\hat{\gamma}_l}(x)\right)^{k-1} = \sum_{m=0}^{k-1} \binom{k-1}{m}(-1)^m e^{-\frac{mx}{\bar{\eta}}}$. Using these results along with the exponential PDF in (2.48), the integrals in (2.48) can be evaluated. After further simplifications, $F_{\gamma_{1(k)}}(\gamma_T)$ can be expressed as

$$
\begin{aligned}
F_{\gamma_{1(k)}}(\gamma_T) = 1 - &\sum_{m=0}^{k-1} \frac{(-1)^m k\binom{N_r}{k}\binom{k-1}{m}}{1+\frac{\eta_2}{\bar{\eta}}(N_r-k+m)} \\
&\times \left(e^{-\frac{\gamma_T}{\eta_1}} + \frac{\eta_2(N_r-k+m)}{\eta_1(N_r-k+m+1)} e^{-\frac{\gamma_T}{\bar{\eta}}(N_r-k+m+1)}\right).
\end{aligned}
\tag{2.49}
$$

Taking the derivative with respect to γ_T of the above CDF, the PDF of $\gamma_{1(k)}$ can be obtained and is given by

$$
f_{\gamma_{1(k)}}(y) = \sum_{m=0}^{k-1} \frac{(-1)^m k\binom{N_r}{k}\binom{k-1}{m}}{1+\frac{\eta_2}{\bar{\eta}}(N_r-k+m)} \left(\frac{1}{\eta_1} e^{-\frac{y}{\eta_1}} + \frac{\eta_2(N_r-k+m)}{\eta_1\bar{\eta}} e^{-\frac{y}{\bar{\eta}}(N_r-k+m+1)}\right).
\tag{2.50}
$$

Using the relationship in (2.23), the joint PDF of $\tilde{\gamma}_{1(k)}$ and $\gamma_{1(k)}$ can be expressed as

$$
\begin{aligned}
f_{\tilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y) = &\frac{k\binom{N_r}{k} e^{-\frac{x}{(1-\rho_1)\eta_1}}}{(1-\rho_1)\eta_1^2} I_0\left(\frac{2\sqrt{\rho_1 xy}}{(1-\rho_1)\eta_1}\right) \sum_{m=0}^{k-1} \frac{\binom{k-1}{m}(-1)^m}{1+\frac{\eta_2}{\bar{\eta}}(N_r-k+m)} \\
&\times \left(e^{-\frac{y}{(1-\rho_1)\eta_1}} + \frac{\eta_2(N_r-k+m)}{\bar{\eta}} e^{-y\left(\frac{(N_r-k+m+1)}{\bar{\eta}} + \frac{\rho_1}{(1-\rho_1)\eta_1}\right)}\right).
\end{aligned}
\tag{2.51}
$$

Now the analysis proceeds to derive the PDF of $\tilde{\gamma}_{1(k)}$ as follows:

$$f_{\tilde{\gamma}_{1(k)}}(x) = \int_0^\infty f_{\tilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y)\,dy = \frac{k}{\eta_1}\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{\binom{k-1}{m}(-1)^m}{(1+\frac{\eta_2}{\bar{\eta}}(N_r-k+m))} \tag{2.52}$$
$$\times\left(e^{-\frac{x}{\eta_1}} + \frac{(N_r-k+m)\eta_2}{\rho_1\bar{\eta}+(1-\rho_1)(N_r-k+m+1)\eta_1}e^{-\frac{(N_r-k+m+1)x}{\rho_1\bar{\eta}+(1-\rho_1)\eta_1(N_r-k+m+1)}}\right).$$

Since the relay selection criteria is symmetric, by interchanging η_1 and ρ_1 with η_2 and ρ_2 respectively, the PDF of $\tilde{\gamma}_{2(k)}$ can be obtained. An expression for the CDF of $\tilde{\gamma}_{2(k)}$ can be obtained by integrating the PDF and is given by

$$F_{\tilde{\gamma}_{2(k)}}(y) = 1 - k\binom{N_r}{k}\sum_{m=0}^{k-1}p_m\sum_{i=1}^{2}q_{m,i}e^{-\frac{r_{m,i}}{\eta_1}y}, \tag{2.53}$$

where $p_m = \frac{\binom{k-1}{m}(-1)^m}{(1+\frac{\eta_1}{\bar{\eta}}(N_r-k+m))}$, $q_{m,1} = 1$, $q_{m,2} = \frac{(N_r-k+m)\eta_1}{(N_r-k+m+1)\eta_2}$, $r_{m,1} = \frac{\eta_1}{\eta_2}$ and $r_{m,2} = \frac{(N_r-k+m+1)\eta_1}{\rho_2\bar{\eta}+(1-\rho_2)\eta_2(N_r-k+m+1)}$. Observing that the above distribution is a sum of exponentials makes the further derivations much convenient. Having derived all the required distributions, the next step is deriving the analytic result for the outage probability, using $\gamma_{eq4} \approx \frac{\tilde{\gamma}_{1(k)}\tilde{\gamma}_{2(k)}}{\gamma_{1(k)}+\tilde{\gamma}_{2(k)}}$.

$$\begin{aligned} F_{\gamma_{eq4}}(\gamma_T) &\approx 1 - \Pr\left(\frac{\tilde{\gamma}_{1(k)}\tilde{\gamma}_{2(k)}}{\gamma_{1(k)}+\tilde{\gamma}_{2(k)}} > \gamma_T\right) \\ &= 1 - \int_{\gamma_T}^\infty\int_0^\infty f_{\tilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y)\left(1 - F_{\tilde{\gamma}_{2(k)}}\left(\frac{y\gamma_T}{x-\gamma_T}\right)\right)dydx. \end{aligned} \tag{2.54}$$

By modifying the limits of the outer integral, and substituting results from (2.51) and (2.53) $F_{\gamma_{eq4}}(\gamma_T)$ can be written as

$$F_{\gamma_{eq4}}(\gamma_T) \approx 1 - \frac{k^2}{(1-\rho_1)\eta_1^2}\binom{N_r}{k}^2\sum_{m=0}^{k-1}p_m\sum_{n=0}^{k-1}p'_n\sum_{i=1}^{2}q_{m,i}\sum_{j=1}^{2}q'_{n,j} \tag{2.55}$$
$$\times\int_0^\infty\int_0^\infty e^{-\frac{x+\gamma_T}{(1-\rho_1)\eta_1}}e^{-\frac{y}{\eta_1}(r'_{n,j}+\frac{r_{m,i}\gamma_T}{x})}I_0\left(\frac{2\sqrt{\rho_1(x+\gamma_T)y}}{(1-\rho_1)\eta_1}\right)dydx.$$

Simplifying (2.55) yields

$$F_{\gamma_{eq4}}(\gamma_T) \approx 1 - \frac{k^2\binom{N_r}{k}^2}{(1-\rho_1)\eta_1^2}\sum_{m=0}^{k-1}p_m\sum_{n=0}^{k-1}p'_n\sum_{i=1}^{2}q_{m,i}\sum_{j=1}^{2}q'_{n,j} \tag{2.56}$$
$$\times\int_0^\infty\frac{e^{-\frac{x+\gamma_T}{(1-\rho_1)\eta_1}\left(1-\frac{\rho_1}{(1-\rho_1)(r'_{n,j}+\frac{r_{m,i}\gamma_T}{x})}\right)}}{(\frac{r'_{n,j}}{\eta_1}+\frac{r_{m,i}\gamma_T}{\eta_1 x})}dx,$$

where $p'_n = \frac{\binom{k-1}{n}(-1)^n}{1+\frac{\eta_2}{\bar{\eta}}(N_r-k+n)}$, $q'_{n,1} = 1$, $q'_{n,2} = \frac{\eta_2(N_r-k+n)}{\bar{\eta}}$ $r'_{n,1} = \frac{1}{1-\rho_1}$ and $r'_{n,2} = \left(\frac{(N_r-k+n+1)\eta_1}{\bar{\eta}} + \frac{\rho_1}{(1-\rho_1)}\right)$. Since a closed-form solution with standard mathematical functions does not exist for the integral in (2.56), the analysis resorts to finding a high SNR approximation for $F_{\gamma_{eq4}}(\gamma_T)$. Substituting

$$\frac{x+\gamma_T}{(1-\rho_1)\eta_1} - \frac{\rho_1 x(x+\gamma_T)}{(1-\rho_1)^2\eta_1(r'_{n,j}x + r_{m,i}\gamma_T)} = x\left(\frac{1}{(1-\rho_1)\eta_1} - \frac{\rho_1}{(1-\rho_1)^2\eta_1 r'_{n,j}}\right) \tag{2.57}$$

$$+ \frac{\gamma_T}{(1-\rho_1)\eta_1}\left(\frac{\rho_1(r_{m,i}-r'_{n,j})}{(1-\rho_1)r'^2_{n,j}} + 1\right) + \frac{\gamma_T^2\rho_1 r_{m,i}(r'_{n,j}-r_{m,i})}{r'^2_{n,j}\eta_1(1-\rho_1)^2(\gamma_T r_{m,i} + r'_{n,j}x)},$$

using the Maclaurin series expansion of $e^{-\frac{\gamma_T^2\rho_1 r_{m,i}(r'_{n,j}-r_{m,i})}{r'^2_{n,j}\eta_1(1-\rho_1)^2(\gamma_T r_{m,i}+r'_{n,j}x)}}$ and ignoring higher order terms of $\frac{\gamma_T}{\eta_1}$ yields the following approximation given by

$$F_{\gamma_{eq4}}(\gamma_T) \approx 1 - \frac{k^2\binom{N_r}{k}^2}{(1-\rho_1)\eta_1}\sum_{m=0}^{k-1} p_m \sum_{n=0}^{k-1} p'_n \sum_{i=1}^{2} q_{m,i} \sum_{j=1}^{2} q'_{n,j} e^{-\frac{\gamma_T}{(1-\rho_1)\eta_1}\left(\frac{\rho_1(r_{m,i}-r'_{n,j})}{(1-\rho_1)r'^2_{n,j}}+1\right)} \tag{2.58}$$

$$\times \int_0^\infty e^{-x\left(\frac{1}{(1-\rho_1)\eta_1} - \frac{\rho_1}{(1-\rho_1)^2\eta_1 r'_{n,j}}\right)}\left(\frac{1}{r'_{n,j}} - \frac{\gamma_T r_{m,i}}{r'_{n,j}(r'_{n,j}x + r_{m,i}\gamma_T)}\right)\, dx + o\left(\left(\frac{\gamma_T}{\eta_1}\right)^2\right).$$

Using [49, Eq. (11)], $F_{\gamma_{eq4}}(\gamma_T)$ can be expressed as

$$F_{\gamma_{eq4}}(\gamma_T) \approx 1 - k^2\binom{N_r}{k}^2 \sum_{m=0}^{k-1} p_m \sum_{n=0}^{k-1} p'_n \sum_{i=1}^{2} q_{m,i} \sum_{j=1}^{2} q'_{n,j} e^{-\frac{\gamma_T}{(1-\rho_1)\eta_1}\left(\frac{\rho_1(r_{m,i}-r'_{n,j})}{(1-\rho_1)r'^2_{n,j}}+1\right)} \tag{2.59}$$

$$\times \left(\frac{1}{r'_{n,j}\left(1-\frac{\rho_1}{(1-\rho_1)r'_{n,j}}\right)} + \frac{\gamma_T r_{m,i}}{(1-\rho_1)\eta_1 r'^2_{n,j}} \ln\left(\frac{1}{(1-\rho_1)\eta_1} - \frac{\rho_1}{(1-\rho_1)^2\eta_1 r'_{n,j}}\right)\right).$$

Average BER

The average BER can be now derived substituting (2.59) in (2.17). Through simplifications after the integration, P_b can be expressed as

$$P_b \approx \frac{\alpha}{2} - \frac{\alpha k^2}{2}\binom{N_r}{k}^2 \sum_{m=0}^{k-1} p_m \sum_{n=0}^{k-1} p'_n \sum_{i=1}^{2} q_{m,i} \sum_{j=1}^{2} q'_{n,j} \sqrt{\frac{\beta}{\beta + \frac{2}{(1-\rho_1)\eta_1}\left(\frac{\rho_1(r_{m,i}-r'_{n,j})}{(1-\rho_1)r'^2_{n,j}} + 1\right)}} \tag{2.60}$$

$$\times \left(\frac{(1-\rho_1)}{\left(r'_{n,j}(1-\rho_1)-\rho_1\right)} - \frac{r_{m,i}\ln\left(\frac{1}{(1-\rho_1)\eta_1} - \frac{\rho_1}{(1-\rho_1)^2\eta_1 r'_{n,j}}\right)}{r'^2_{n,j}\beta(1-\rho_1)\eta_1 + 2\left(\frac{\rho_1(r_{m,i}-r'_{n,j})}{(1-\rho_1)} + r'^2_{n,j}\right)} \right).$$

The average output power at the relay will be different from P_r due to using outdated CSI to estimate the amplification gain. Hence, a modified amplification gain factor ξ_2 is introduced, for a fair comparison among the systems. Using a similar approach as earlier, ξ_2 can be expressed as

$$\begin{aligned} \xi_2 &= E\left\{\frac{\tilde{\gamma}_{1(k)}+1}{\gamma_{1(k)}+1}\right\} \\ &= \sum_{m=0}^{k-1} \frac{(-1)^m k\binom{N_r}{k}\binom{k-1}{m}}{1+\frac{\eta_2}{\bar{\eta}}(N_r-k+m)} \sum_{i=1}^{2} q'_{m,i}\left(\frac{\rho_1}{s_i} - (1-\rho_1)\left(1+\frac{1}{\eta_1}\right)e^{\frac{s_i}{\eta_1}}\,\mathrm{Ei}\left(-\frac{s_i}{\eta_1}\right)\right), \end{aligned} \tag{2.61}$$

where $s_1 = 1$ and $s_2 = \frac{(N_r-k+m+1)\eta_1}{\bar{\eta}}$.

2.4.2 Variable Gain Relaying II

Outage Probability

$$\begin{aligned} F_{\gamma_{eq5}}(\gamma_T) &= 1 - \int_0^\infty f_{\tilde{\gamma}_{1(k)}}(x+\gamma_T)\left(1 - F_{\gamma_{2(k)}}\left(\frac{\gamma_T(x+\gamma_T+1)}{x}\right)\right)dx \\ &= 1 - \frac{2k^2}{\eta_1}\binom{N_r}{k}^2 \sum_{m=0}^{k-1} p_m \sum_{n=0}^{k-1} p_{1n} \sum_{i=1}^{2} q_{m,i} \sum_{j=1}^{2} q_{n,1j} e^{-(r_{m,i}+r_{n,1j})\frac{\gamma_T}{\eta_1}} \\ &\quad \times \sqrt{\frac{r_{m,i}(\gamma_T+\gamma_T^2)}{r_{n,1j}}} K_1\left(\frac{2}{\eta_1}\sqrt{r_{m,i}r_{n,1j}(\gamma_T+\gamma_T^2)}\right), \end{aligned} \tag{2.62}$$

where $p_{1m} = \frac{\binom{k-1}{m}(-1)^m}{(1+\frac{\eta_2}{\bar{\eta}}(N_r-k+m))}$, $q_{n,11} = 1$, $q_{n,12} = \frac{(N_r-k+n)\eta_2}{\rho_1\bar{\eta}+(1-\rho_1)(N_r-k+n+1)\eta_1}$, $r_{n,11} = 1$ and $r_{n,12} = \frac{(N_r-k+n+1)\eta_1}{\rho_1\bar{\eta}+(1-\rho_1)\eta_1(N_r-k+n+1)}$. A simple high SNR approximation can be obtained using $K_1(x) \approx \frac{1}{x}$ as $x \to 0$ in the above result. Therefore, as η_1, η_2 tends to ∞, $F_{\gamma_{eq5}}(\gamma_T)$ can be expressed as

$$F_{\gamma_{eq5}}(\gamma_T) \approx \frac{\gamma_T k^2}{\eta_1} \binom{N_r}{k}^2 \sum_{m=0}^{k-1} p_m \sum_{n=0}^{k-1} p_{1n} \sum_{i=1}^{2} q_{m,i} \sum_{j=1}^{2} q_{n,1j} \left(1 + \frac{r_{m,i}}{r_{n,1j}}\right). \quad (2.63)$$

Average BER

The average BER can be now derived substituting (2.62) in (2.17). But as the resulting integral does not have a solution in standard mathematical functions, the approximation $\gamma_{eq5} \approx \frac{\tilde{\gamma}_{1(k)}\tilde{\gamma}_{2(k)}}{\tilde{\gamma}_{1(k)}+\tilde{\gamma}_{2(k)}}$ is used. Using [51, Eq 6.621.3] and further simplifications, the average BER is derived, which is given by

$$P_b \approx \frac{\alpha}{2} - \frac{3\pi\alpha k^2}{\sqrt{2}\beta^2\eta_1^2} \binom{N_r}{k}^2 \sum_{m=0}^{k-1} p_m \sum_{n=0}^{k-1} p_{1n} \sum_{i=1}^{2} q_{m,i} r_{m,i} \sum_{j=1}^{2} \frac{q_{n,1j}}{\left(\frac{1}{2} + \frac{(\sqrt{r_{m,i}}+\sqrt{r_{n,1j}})^2}{\beta\eta_1}\right)^{\frac{5}{2}}}$$
$$\times\ {}_2F_1\left(\frac{5}{2}, \frac{3}{2}; 2; \frac{\beta\eta_1 + 2(\sqrt{r_{m,i}} - \sqrt{r_{n,1j}})^2}{\beta\eta_1 + 2(\sqrt{r_{m,i}} + \sqrt{r_{n,1j}})^2}\right). \quad (2.64)$$

Using (2.63) in (2.17), as $\eta_1, \eta_2 \to \infty$ a high SNR approximation for the above BER can be obtained and is given by

$$P_b \approx \frac{\alpha k^2}{2\beta\eta_1} \binom{N_r}{k}^2 \sum_{m=0}^{k-1} p_m \sum_{n=0}^{k-1} p_{1n} \sum_{i=1}^{2} q_{m,i} \sum_{j=1}^{2} q_{n,1j} \left(1 + \frac{r_{m,i}}{r_{n,1j}}\right). \quad (2.65)$$

2.5 Numerical and Simulation Results

2.5.1 Partial Relay Selection

Figures 2.2-2.4 show the performance of the three systems investigated in Section III of this chapter. The variable gain relaying systems investigated in this chapter are labelled as VG I and VG II in the figures, and the fixed gain relaying system is labelled as FG. VG II system has perfect CSI at the relay to estimate the amplification gain, while the VG I system uses the outdated CSI for the same. All investigated cases revealed an excellent agreement between analytical and Monte Carlo simulation results.

Effect of relay rank on outage probability: Figure 2.2 shows the outage prob-

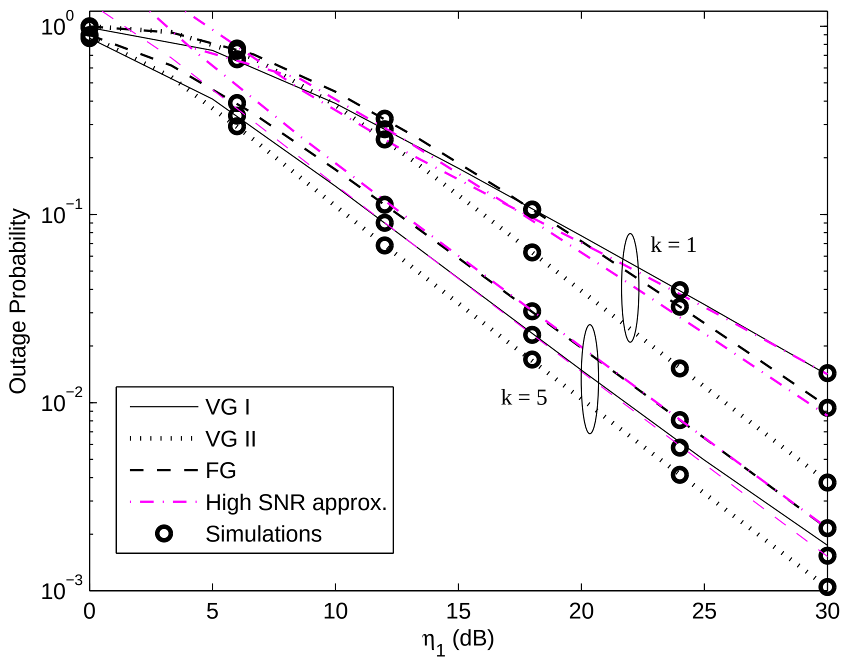

Figure 2.2: The outage probability with best and worst relay selection ($N_r = 5, \rho_1 = 0.8, \eta_1 = \eta_2$).

ability against η_1 in dB ($N_r = 5, \mu = 1, \rho_1 = 0.8$). Two cases where the best ($k = 5$) and worst ($k = 1$) relay is chosen is presented. It can be seen that, when the best relay is chosen, the VG relaying performs better than fixed relaying. In the case of worst relay selection, at low SNR the VG relaying outperforms FG relaying, while at high SNR it becomes worse. The simple high SNR approximations obtained shows good proximity to the exact results. Simulation results not included here showed for higher values of k (e.g. best relay) and ρ_1, the overlap of the high SNR approximation with the exact result happens as early as 15 dB, while for low k and ρ_1 values it happens around 40 dB.

Effect of correlation on outage probability: Figure 2.3 shows the influence of the correlation ρ_1 on the outage probability. When the best relay is chosen, i.e. $k = 5$, the performance improves with increasing ρ_1 in all systems, as expected. The performance of the VG systems is better than the FG system at high ρ_1 (> 0.3). In the case of worst relay selection, the FG relaying performs better than the "VG

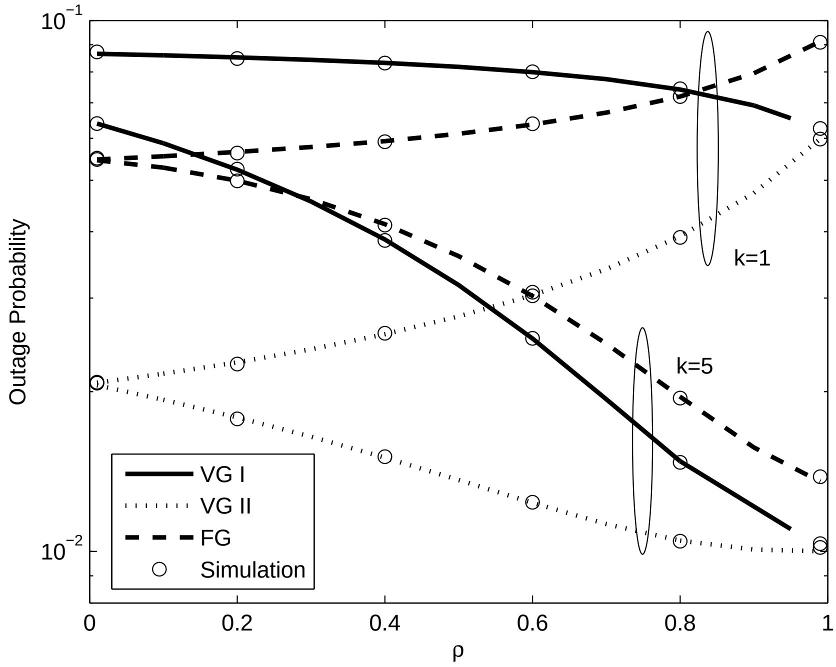

Figure 2.3: Outage probability versus ρ_1, for best ($k = 5$) and worst ($k = 1$) relay selection ($N_r = 5, \gamma_T = 1, \eta_1 = \eta_2 = 20$ dB).

I" system in $\rho_1 < 0.8$ region and the performance of the FG and VG II systems improve with decreasing ρ_1. The performance of 'VG I' under worst relay selection, in contrast, degrades with decreasing ρ_1, as the influence of incorrectly selecting the amplification gain factor at the relay becomes more significant. The performance gaps between best and worst selection curves in FG and VG II systems vanishes as $\rho_1 \to 0$. If the decision and actual link SNR values are not correlated, the ranking of relays would have no effect on the performance.

Effect of correlation on BER: Figure 2.4 presents the average BER with quadrature phase shift keying (QPSK) modulation ($\alpha = \beta = 1$), for two cases where the correlation between the outdated channel estimate and the actual channel is high ($\rho_1 = 0.8$) and low ($\rho_1 = 0.1$). The best relay ($k = 5$) out of all the ($N_r = 5$) relays was chosen and $\eta_1 = \eta_2$. The infinite series in (2.30) was truncated at 45 terms for calculations. In low SNR regions, the VG systems outperform the FG counterparts. When the correlation is high ($\rho_1 = 0.8$), VG relaying outperforms FG relaying. If

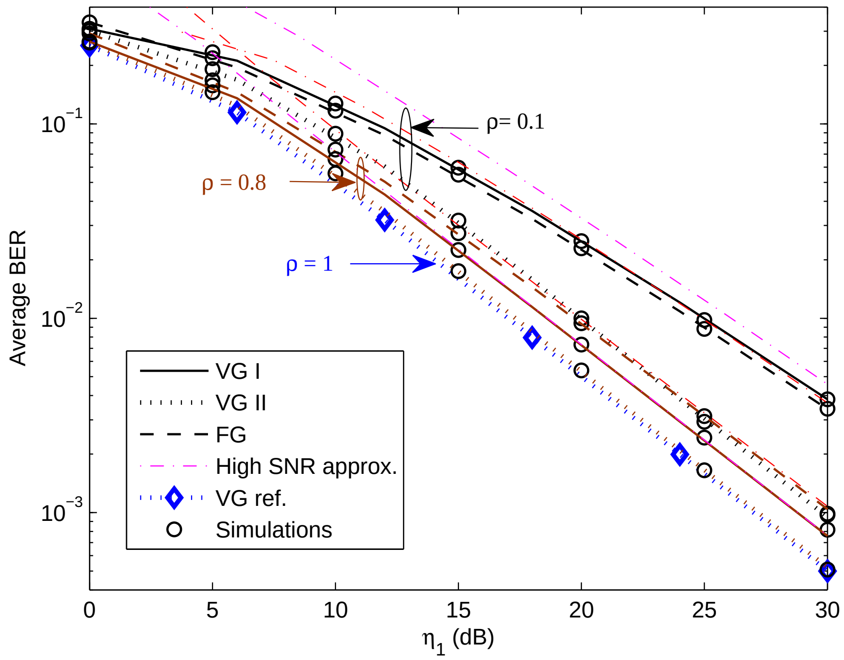

Figure 2.4: The average BER under different values of correlation ρ_1.($N_r = 5$, $k = 5$, $\eta_1 = \eta_2$)

the correlation is low ($\rho_1 = 0.1$), at high SNR, the FG system outperform VG system. In all cases, the reference "VG II" system demonstrates better performance. A reference curve for variable gain relaying with $\rho_1 = 1$ is plotted to observe the performance loss due to not having perfect information at the relay.

2.5.2 Opportunistic Relay Selection

Figures 2.5-2.7 show the performance of the two systems investigated in Section IV of this chapter. The figures include plots of the high SNR approximation for performance metrics of the system VG I, exact analytic results obtained for the system VG II and its high SNR approximations where appropriate labeled accordingly. The exact results offered excellent agreement with Monte Carlo simulation results in all investigated cases, while the approximations were admissible.

Effect of relay rank on outage probability: Figure 2.5 shows the outage probability of the opportunistic relay system for best($k = 5$) and worst($k = 1$) relay

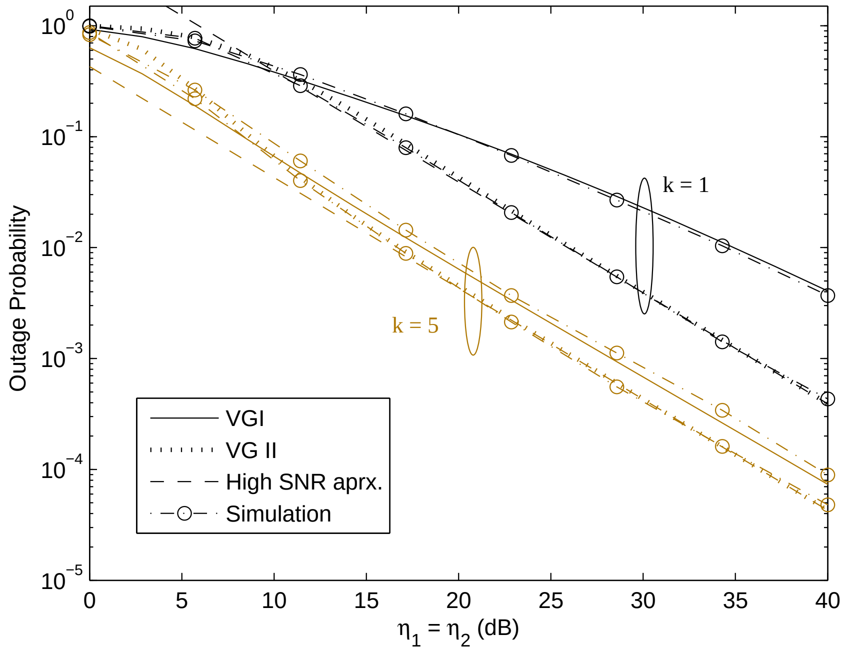

Figure 2.5: The outage probability of opportunistic relay selection with best and worst relay selection $(N_r = 5, \gamma_T = 1, \rho_1 = 0.8, \rho_2 = 0.7, \eta_1 = \eta_2)$.

selection, with $N_r = 5, \gamma_T = 1, \rho_1 = 0.8, \rho_2 = 0.7$ and $\eta_1 = \eta_2$. The VG I system with perfect information at the relay performs better than VG II, in both cases. However, in the case of $k = 1$, i.e. the worst relay, the performance loss due to imperfect information at the relay is higher. It is further observed that the diversity gain of the systems are one, except for VG II at $k = 1$, in which it is slightly less. The performance degradation is enhanced by the power factor correction ξ_2 (2.61) applied to maintain the average power at the relay constant. It is important to note that ξ_2 is higher for low k values and increases with η_1. One important thing to note is that, even though there are $N_r(= 5)$ relays, the diversity is not equal to N_r as it would be in the case of having perfect CSI at the source and the relays.

Effect of correlation on outage probability: Figure 2.6 shows the variation of outage probability with the correlation coefficients at $\eta_1 = \eta_2 = 20$ dB, for the best and worst relay selection scenarios. When the best relay is selected ($k = 5$), the outage probability of both VG I and II systems decreases as the correlation ρ_1 and ρ_2 increase, as one would expect. It is important to note that as $\rho \to 1$, there is a

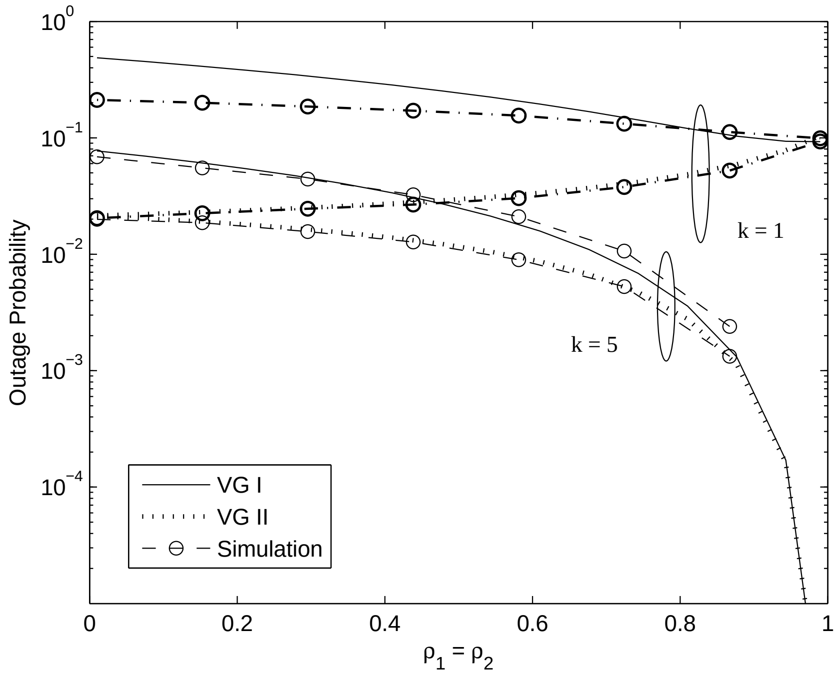

Figure 2.6: Outage probability versus ρ_1, for best ($k = 5$) and worst ($k = 1$) of opportunistic relay selection ($N_r = 5, \gamma_T = 1, \eta_1 = \eta_2 = 20$ dB).

significant change in outage performance. i.e., a small delay causing the decision link SNRs and actual link SNRs to be different, although highly correlated, there is a significant loss in the performance. If the worst relay was selected, an improvement of outage performance in VG II system is observed as the correlation decreases. This is as one would expect, because at low correlation there is high likelihood that the worst relay chosen is not the actual worst. But in contrast, the performance degrades in VG I system as the correlation decrease. This is caused by the increase of ξ_2 as $\rho_1 \to 0$, and the effect of incorrect selection of gain factor G is apparent. The performance gap between systems VG I and VG II narrows as $\rho_1, \rho_2 \to 1$ for both cases $k = 1$ and $k = 5$. As $\rho_1, \rho_2 \to 0$, in the VG II system, the performance gap between best and worst relay selection scenarios gets reduced. As expected, if the decision and actual link SNRs are not correlated, the ranking of the relays does not make a difference. But for the VG II system, this does not happen due to the effect of incorrect selection G.

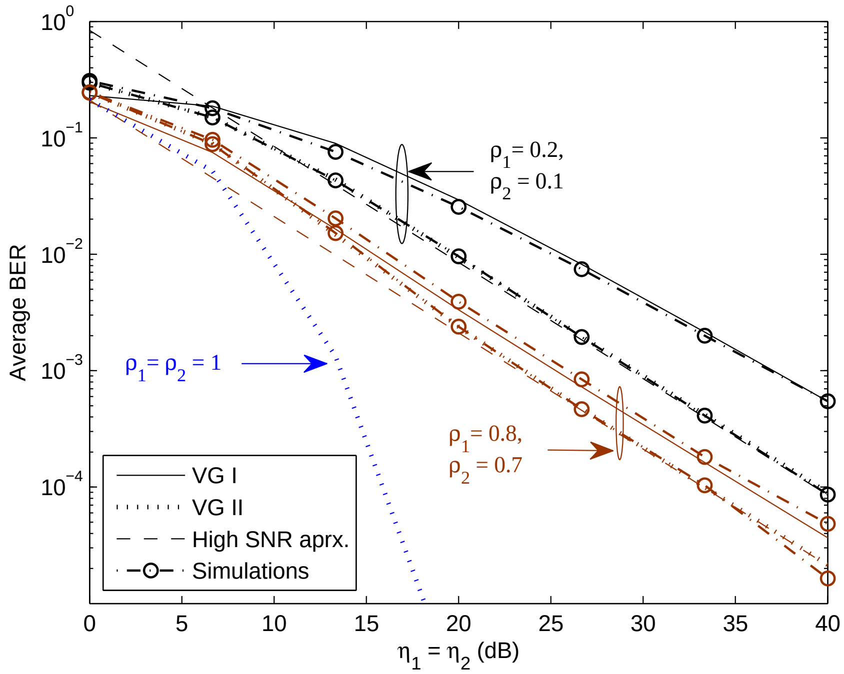

Figure 2.7: The average BER for QPSK under different values of correlation ρ_1, ρ_2 for opportunistic relay selection.($N_r = 5$, $k = 5$)

Effect of correlation on BER: Figure 2.7 has the plots of average BER vs the SNR of the links η_1 and η_2. Here situations of low correlation($\rho_1 = 0.2$, $\rho_2 = 0.1$) and high correlation($\rho_1 = 0.8$, $\rho_2 = 0.7$) are compared, selecting the best relay ($N_r = k = 5$) with QPSK modulation. As expected, the average BER of the VG II system is lower than that of VG I in both cases, and the performance gap between the systems reduces as ρ_1, ρ_2 increases. A reference curve for the case of $\rho_1 = \rho_2 = 1$ is plotted in the same figure, and from that the major performance degradation due to incorrect CSI becomes apparent. With perfect CSI, the diversity gain is equal to the number of relays, however with imperfect CSI, it is unity, irrespective of the number of relays.

PRS and ORS performance comparison: Comparing plots shown in Figs. 2.3 and 2.6, it is observed that the outage probability of ORS at 20 dB, shows higher variation with changing ρ than PRS. For the case of best relay selection, it can be seen that at low correlation ($\rho < 0.25$) the performance of PRS is slightly better than its ORS counterparts. However as correlation value increases, ORS systems show significantly better performance than PRS systems. With worst relay selection, PRS systems show better performance than respective ORS counterparts. The PRS FG system shows better performance than opportunistic relay VG I system, but is worse than VG II.

2.6 Conclusions

This chapter presents the analysis of the effect of outdated CSI at the source and the relay for relay selection in dual-hop systems. New analytical expressions and high SNR approximations for the outage probability and the average BER were derived for the case where the kth worse relay is selected. The high SNR approximations give a simple expression that provides quick insights on the influence of system parameters on the performance. It was found that in PRS systems, for low correlation values and with best relay selection, FG relaying gives better performance than VG relaying. However as correlation increases, VG relaying outperforms FG relaying, while VG relaying considered in [46] shows the best performance in all cases. Further insights obtained showed that, PRS schemes perform better than ORS counterparts, when the decision CSI and actual CSI has low correlation. However, as ρ increases, the opportunistic relay systems shows far superior performance. The diversity gain of ORS systems reduces to one with imperfect CSI, which shows the significance of focusing on the CSI errors in dual hop-relay systems.

Chapter 3

PRS Amplify-and-Forward Relaying with outdated CSI in Nakagami-m Fading

3.1 Introduction

In this chapter a dual hop relay system is analyzed under Nakagami-m fading. We employ a system model similar to that of Chapter 2. In Chapter 2, the system performance was analyzed under Rayleigh fading, and in this chapter we extend the analysis to the more general Nakagami-m fading.

The new contributions in this chapter can be summarized as follows:

- The performance of a PRS system when relay selection performed based on the outdated CSI is analytically investigated. Moreover, instead of only considering the best relay selection criterion, our analysis considers the most general case of kth worst relay selection [53]. Hence, the presented results can be directly applied to a large set of situations and fading scenarios.

- Exact closed-form expressions were derived for the outage probability and the average BER of relays systems equipped with either fixed gain or variable gain relays.

- The impact of outdated CSI on the performance is investigated, for both variable gain and fixed gain relaying using high SNR approximations. The achievable diversity order of PRS systems is investigated. This result proves that both cases relaying yield the same diversity order and is equal to either the first hop or the second hop Nakagami-m fading parameter, irrespective of the relay selection rank.

3.2 System Model

A system model similar to that of Chapter 2 (Fig 2.1) is used in this chapter as well.

It is assumed that each of the $S \to R_{(\ell)}$ and each of the $R_{(\ell)} \to D$, $\ell = 1, \ldots, N_r$ channels experience Nakagami-m fading with parameter $m_1 \in \mathbb{Z}^+$ and $m_2 \in \mathbb{Z}^+$ respectively.

The received signal at $R_{(k)}$ is given by

$$Y_{R_{(k)}}(t) = \sqrt{P_s} h_{S,R_{(k)}}(t) x(t) + n_{R_{(k)}}(t), \tag{3.1}$$

where P_s is the transmit power at S, the complex fading channel from S to $R_{(k)}$ is denoted by $h_{S,R_{(k)}}(t)$ and $n_{R_{(k)}}(t)$ is the additive white Gaussian noise (AWGN) at $R_{(k)}$ satisfying $E\left(|n_{R_{(k)}}(t)|^2\right) = N_{01}$ with $E(\cdot)$ denoting expectation. A scaling gain, G is applied by $R_{(k)}$ to $Y_{R_{(k)}}(t)$ and the output is re-transmitted to D. The received signal at D is given by

$$Y_D(t) = h_{R_{(k)},D}(t) \mathsf{G} Y_{R_{(k)}}(t) + n_D(t), \tag{3.2}$$

where $h_{R_{(k)},D}(t)$ is the complex channel between R and D, and $n_D(t)$ is the AWGN at D satisfying $E\left(|n_D(t)|^2\right) = N_{02}$.

Let $\widetilde{\gamma}_{1(k)} = |h_{S,R_{(k)}}(t)|^2 \eta_1$ and $\gamma_2 = |h_{R_{(k)},D}(t)|^2 \eta_2$, where $\eta_1 = \frac{P_s}{N_{01}}$, $\eta_2 = \frac{P_r}{N_{02}}$ and P_r is the average transmit power of R. The link SNRs are defined as $\gamma_{1(k)} = |h_{S,R_{(k)}}(t - T_d)|^2 \eta_1$. With the assumption of Nakagami-m fading, $\gamma_{1(\ell)} \sim \mathcal{G}\left(m_1, \frac{\eta_1}{m_1}\right)$ and $\gamma_2 \sim \mathcal{G}\left(m_2, \frac{\eta_2}{m_2}\right)$ where $\mathcal{G}(\lambda, \theta)$ is the gamma distribution with scale parameter θ and shape parameter λ and $\ell = 1 \ldots N_r$ represents unordered relays. For simplicity, it is assumed that $\gamma_{1(\ell)}$s are i.i.d distributed. A generalization to non-identical fading is straightforward. Note that the relay selection is based on $\gamma_{1(k)}$ while $\widetilde{\gamma}_{1(k)}$, the link SNR experienced by the signal, is a delayed version of $\gamma_{1(k)}$.

Fixed Gain Relaying: Consider the case where $R_{(k)}$ uses a fixed scaling gain such that a constant average transmit power at $R_{(k)}$ is maintained. Assuming that $R_{(k)}$ has the statistics of the $S \to R_{(k)}$ link, the following amplification gain can be selected.

$$\mathsf{G}_{\mathsf{F}} = \sqrt{\frac{P_r}{P_s E\{|h_{S,R_{(k)}}(t)|^2\} + N_{01}}}, \tag{3.3}$$

to apply to $Y_{R_{(k)}}(t)$. Therefore, at R, fixed gain relaying can avoid the task of continuous monitoring of the $S \to R$ link.

Variable Gain Relaying: At $R_{(k)}$ CSI-based variable gain relaying aims to maintain a constant instantaneous output power for the retransmitted signal. Assuming that $R_{(k)}$ has the $S \to R_{(k)}$ link instantaneous CSI knowledge, the following gain factor G_{V} is selected.

$$\mathsf{G}_{\mathsf{V}} = \sqrt{\frac{P_r}{P_s |h_{S,R_{(k)}}(t)|^2 + N_{01}}}. \tag{3.4}$$

3.3 Analysis of Fixed Gain Relaying

In this section the outage probability and the average BER using fixed gain relaying is analyzed.

It can be shown that the instantaneous end-to-end SNR is given by

$$\gamma_{eq1} = \frac{\widetilde{\gamma}_{1(k)}\gamma_2}{\gamma_2 + C}, \tag{3.5}$$

where $C = \frac{P_r}{\mathsf{G}_{\mathsf{F}}^2 N_{01}}$. After some manipulations, it is easy to show that, $C = E\left\{\gamma_{1(k)}\right\}$ $+1$.

Using a result from order statistics, and since the outdated CSI model adapted is similar to that used in [54], the PDF of $\widetilde{\gamma}_{1(k)}$ can be written as

$$f_{\widetilde{\gamma}_{1(k)}}(x) = \int_0^\infty f_{\widetilde{\gamma}_{1(k)}|\gamma_{1(k)}}(x|y) f_{\gamma_{1(k)}}(y)\, dy \tag{3.6}$$

$$= \int_0^\infty \frac{m_1 \left(\frac{x}{\rho y}\right)^{\frac{m_1-1}{2}} e^{-\frac{m_1(\rho y + x)}{(1-\rho)\eta_1}}}{(1-\rho)\eta_1} I_{m_1-1}\left(\frac{2m_1\sqrt{\rho x y}}{(1-\rho)\eta_1}\right) f_{\gamma_{1(k)}}(y) dy \tag{3.7}$$

where $I_\nu(z)$ denotes the νth order modified Bessel function of the first kind [7, Sec. (9.6)]. Since the relay with the kth lowest SNR is selected, the PDF of $\gamma_{1(k)}$ is given by

$$f_{\gamma_{1(k)}}(y) = k\binom{N_r}{k}[F_{\gamma_{1(\ell)}}(y)]^{k-1}[1-F_{\gamma_{1(\ell)}}(y)]^{N_r-k}f_{\gamma_{1(\ell)}}(y) \tag{3.8}$$

Since $\gamma_{1(\ell)} \sim \mathcal{G}\left(m_1, \frac{\eta_1}{m_1}\right)$, by substituting the respective PDF and CDF into (3.8) the following expression is obtained.

$$\begin{aligned} f_{\gamma_{1(k)}}(y) = k\binom{N_r}{k} &\sum_{p_1=N_r-k}^{N_r-1} \frac{(-1)^{p_1+k-N_r}\binom{k-1}{p_1+k-N_r}}{(m_1-1)!}e^{-\frac{m_1 y(p_1+1)}{\eta_1}} \\ &\times \sum_{r_1=0}^{p_1(m_1-1)} \phi_{p_1(r_1)}^{m_1-1}\left(\frac{m_1}{\eta_1}\right)^{r_1+m_1} y^{r_1+m_1-1}, \end{aligned} \tag{3.9}$$

where the coefficient $\phi_{b(t)}^a$ is defined as $\left(\sum_{t=0}^{a}\frac{x^t}{t!}\right)^b = \sum_{t=0}^{ab}\phi_{b(t)}^a x^t$. The value of $\phi_{b(t)}^a$ can be found recursively as [54]

$$\phi_{b(t)}^a = \sum_{\iota=\iota_1}^{\iota_2}\frac{\phi_{b-1(\iota)}^a}{(t-\iota)!}, \tag{3.10}$$

where $\iota_1 = \max(0, t-a)$ and $\iota_2 = \min(t, (b-1)(a-1))$. Using the infinite series expansion of for $I_{m_1-1}\left(\frac{2m_1\sqrt{\rho xy}}{(1-\rho)\eta_1}\right)$ [7, Eq. (9.6.10)], and following an approach similar to [54], an expression for $f_{\tilde{\gamma}_{1(k)}}$ is obtained, which is given by

$$\begin{aligned} f_{\tilde{\gamma}_{1(k)}}(x) = &\sum_{q_1=0}^{k-1}\frac{k\binom{N_r}{k}(-1)^{q_1}\binom{k-1}{q_1}}{(m_1-1)!}e^{\frac{-m_1(p_1+1)x}{(p_1(1-\rho)+1)\eta_1}}\sum_{r_1=0}^{p_1(m_1-1)}\phi_{p_1(r_1)}^{m_1-1} \\ &\times\sum_{s_1=0}^{r_1}\frac{(r_1+m_1-1)!\left(\frac{m_1}{\eta_1}\right)^{s_1+m_1}\binom{r_1}{s_1}\rho^{s_1}(1-\rho)^{r_1-s_1}x^{s_1+m_1-1}}{(p_1(1-\rho)+1)^{r_1+s_1+m_1}(s_1+m_1-1)!} \end{aligned} \tag{3.11}$$

where $p_1 = N_r - k + q_1$.

The next step in the analysis is calculating the value of C defined in (3.5).

$$\begin{aligned} C &= E\{\gamma_{1(k)}\}+1 \\ &= \sum_{q_1=0}^{k-1}\frac{k\binom{N_r}{k}(-1)^{q_1}\binom{k-1}{q_1}}{(m_1-1)!}e^{\frac{-m_1(p_1+1)x}{(p_1(1-\rho)+1)\eta_1}}\sum_{r_1=0}^{p_1(m_1-1)}\phi_{p_1(r_1)}^{m_1-1}(r_1+m_1-1)! \\ &\times\sum_{s_1=0}^{r_1}\binom{r_1}{s_1}\frac{\rho^{s_1}(1-\rho)^{r_1-s_1}(s_1+m_1)}{(p_1(1-\rho)+1)^{r_1-1}(p_1+1)^{s_1+m_1+1}}\left(\frac{m_1}{\eta_1}\right) \end{aligned} \tag{3.12}$$

3.3.1 Outage Probability

The outage probability, P_o, defined as the probability that the end-to-end SNR drops below a predefined threshold γ_T, is an important QoS measure. It is equal to the cumulative distribution function (CDF) value of the end-to-end SNR evaluated at γ_T, i.e., $P_o = F_{\gamma_{eq1}}(\gamma_T)$. Mathematically,

$$
\begin{aligned}
F_{\gamma_{eq1}}(\gamma_T) &= \text{Pr}\left(\frac{\widetilde{\gamma}_{1(k)}\gamma_2}{\gamma_2 + C} < \gamma_T\right) && (3.13)\\
&= 1 - \int_{\gamma_T}^{\infty} \text{Pr}\left(\gamma_2 > \frac{C\gamma_T}{x - \gamma_T}\right) f_{\widetilde{\gamma}_{1(k)}}(x)\,dx && (3.14)\\
&= 1 - \int_{\gamma_T}^{\infty} \overline{F}_{\gamma_2}\left(\frac{C\gamma_T}{x - \gamma_T}\right) f_{\widetilde{\gamma}_{1(k)}}(x)\,dx && (3.15)\\
&= 1 - \int_{0}^{\infty} \overline{F}_{\gamma_2}\left(\frac{C\gamma_T}{x}\right) f_{\widetilde{\gamma}_{1(k)}}(x + \gamma_T)\,dx && (3.16)
\end{aligned}
$$

where $\text{Pr}(\cdot)$ denotes probability and $\overline{F}_{\gamma_2}(x)$ is the complementary cumulative distribution function of γ_2 and $f_{\widetilde{\gamma}_{1(k)}}(x)$ is the probability density function of $\gamma_{1(k)}$. Since $\gamma_2 \sim \mathcal{G}\left(m_2, \frac{\eta_2}{m_2}\right)$,

$$
\overline{F}_{\gamma_2}\left(\frac{C\gamma_T}{x}\right) = \frac{\Gamma\left(m_2, \frac{m_2 C\gamma_T}{\eta_2 x}\right)}{\Gamma(m_2)}. \tag{3.17}
$$

Using [7, Sec.(6.5)] for $m_2 \in \mathbb{Z}^+$, (3.17) can be re-expressed as

$$
\overline{F}_{\gamma_2}\left(\frac{C\gamma_T}{x}\right) = e^{-\frac{m_2 C\gamma_T}{\eta_2 x}} \sum_{p_2=0}^{m_2-1} \frac{m_2^{p_2}\gamma_T^{p_2} C^{p_2}}{p_2!\,\eta_2^{p_2} x^{p_2}} \tag{3.18}
$$

Using (3.11), it can be seen that,

$$
\begin{aligned}
f_{\widetilde{\gamma}_{1(k)}}(x + \gamma_T) = & \sum_{q_1=0}^{k-1} \frac{k\binom{N_r}{k}(-1)^{q_1}\binom{k-1}{q_1}}{(m_1 - 1)!} e^{\frac{-m_1(p_1+1)(x+\gamma_T)}{(p_1(1-\rho)+1)\eta_1}} \sum_{r_1=0}^{p_1(m_1-1)} \phi_{p_1(r_1)}^{m_1-1} \qquad (3.19)\\
& \times \sum_{s_1=0}^{r_1} \frac{(r_1 + m_1 - 1)!\left(\frac{m_1}{\eta_1}\right)^{s_1+m_1} \binom{r_1}{s_1}\rho^{s_1}(1-\rho)^{r_1-s_1}}{(p_1(1-\rho)+1)^{r_1+s_1+m_1}(s_1 + m_1 - 1)!}\\
& \times \sum_{t_1=0}^{s_1+m_1-1} \binom{s_1 + m_1 - 1}{t_1} \gamma_T^{s_1+m_1-1-t_1} x^{t_1}
\end{aligned}
$$

Now substituting (3.18) and (3.19) into (3.16) and simplifying, the CDF can be obtained as,

$$F_{\gamma_{eq1}}(\gamma_T) = 1 - \sum_{q_1=0}^{k-1} \frac{k\binom{N_r}{k}(-1)^{q_1}\binom{k-1}{q_1}}{(m_1-1)!} e^{-\frac{m_1(p_1+1)\gamma_T}{(p_1(1-\rho)+1)\eta_1}} \sum_{r_1=0}^{p_1(m_1-1)} \phi_{p_1(r_1)}^{m_1-1} \tag{3.20}$$
$$\times \sum_{s_1=0}^{r_1} \frac{(r_1+m_1-1)!\left(\frac{m_1}{\eta_1}\right)^{s_1+m_1}\binom{r_1}{s_1}\rho^{s_1}(1-\rho)^{r_1-s_1}}{(p_1(1-\rho)+1)^{r_1+s_1+m_1}(s_1+m_1-1)!} \sum_{t_1=0}^{s_1+m_1-1} \binom{s_1+m_1-1}{t_1}$$
$$\times \gamma_T^{s_1+m_1-1-t_1} \sum_{p_2=0}^{m_2-1} \frac{m_2^{p_2}\gamma_T^{p_2}C^{p_2}}{p_2!\eta_2^{p_2}} \int_0^{\infty} e^{-\left(\frac{m_2 C\gamma_T}{\eta_2 x}+\frac{m_1(p_1+1)x}{(p_1(1-\rho)+1)\eta_1}\right)} x^{t_1-p_2}\, dx$$

Using result from [50, Eq. (4.5.29)] to perform the integration, the following expression is arrived at, for the outage probability.

$$F_{\gamma_{eq1}}(\gamma_T) = 1 - 2k\binom{N_r}{k} \sum_{q_1=0}^{k-1} \frac{(-1)^{q_1}\binom{k-1}{q_1}}{(m_1-1)!} e^{-\frac{m_1(p_1+1)\gamma_T}{(p_1(1-\rho)+1)\eta_1}} \sum_{r_1=0}^{p_1(m_1-1)} \phi_{p_1(r_1)}^{m_1-1} \tag{3.21}$$
$$\times \sum_{s_1=0}^{r_1} \frac{(r_1+m_1-1)!\left(\frac{m_1}{\eta_1}\right)^{s_1+m_1}\binom{r_1}{s_1}\rho^{s_1}(1-\rho)^{r_1-s_1}}{(p_1(1-\rho)+1)^{r_1+s_1+m_1}(s_1+m_1-1)!} \sum_{t_1=0}^{s_1+m_1-1} \binom{s_1+m_1-1}{t_1}$$
$$\times \sum_{p_2=0}^{m_2-1} \frac{m_2^{p_2}C^{p_2}\gamma_T^{s_1+m_1+\frac{p_2-t_1-1}{2}}}{p_2!\eta_2^{p_2}} \left(\frac{(p_1(1-\rho)+1)m_2 C\eta_1}{m_1(p_1+1)\eta_2}\right)^{\frac{t_1-p_2+1}{2}}$$
$$\times K_{t_1-p_2+1}\left(2\sqrt{\frac{(p_1+1)m_1 m_2 C\gamma_T}{(p_1(1-\rho)+1)\eta_1\eta_2}}\right),$$

where $K_\nu(z)$ is the νth order modified Bessel function of the second kind [7, Sec. (9.6)].

In the high SNR region with $\eta_1, \eta_2 \to \infty$, for $\rho < 1$, a power series expression for (3.20) can be obtained substituting the series expansion of $K_\nu(\cdot)$ [51, Eq. (8.446)] and the Maclaurin series expansion of the exponential function. After mathematical manipulations, an asymptotic approximation for $F_{\gamma_{eq1}}(\gamma_T)$ can be obtained as

$$F_{\gamma_{eq1}}(\gamma_T) = \begin{cases} \left(\tau_1 + \tau_2 \ln\left(\frac{m_1 m_2 C\gamma_T}{\eta_1\eta_2}\right)\right)\left(\frac{m_1 m_2 C}{\eta_1\eta_2}\right)^{m_1}\gamma_T^{m_1} & \text{(a)} \\ \tau_3\left(\frac{m_1 m_2 C}{\eta_1\eta_2}\right)^{m_2}\gamma_T^{m_2} & \text{(b)} \end{cases} \tag{3.22}$$

where (a) $= m_1 \leq m_2$ and (b) $= m_1 > m_2$. In (3.22), τ_1, τ_2 and τ_3 are given by

$$\begin{aligned}
\tau_1 = {} & \frac{k\binom{N_r}{k}}{(m_1-1)!}\sum_{q_1=0}^{k-1}\binom{k-1}{q_1}\sum_{r_1=0}^{p_1(m_1-1)}\phi_{p_1(r_1)}^{m_1-1}\sum_{s_1=0}^{r_1}\frac{(r_1+m_1-1)!}{(s_1+m_1-1)!} \\
& \times \frac{\binom{r_1}{s_1}\rho^{s_1}(1-\rho)^{r_1-s_1}}{(p_1(1-\rho)+1)^{r_1+m_1}(p_1+1)^{s_1}}\sum_{t_1=0}^{s_1+m_1-1}\binom{s_1+m_1-1}{t_1} \\
& \times \sum_{p_2=0}^{m_2-1}\sum_{t_2=0}^{\min(r_2,|t_1-p_2+1|-1)}\frac{(-1)^{q_1+r_2+1}(|t_1-p_2+1|-t_2-1)!}{p_2!(r_2-t_2)!t_2!}\left(\frac{m_2C}{\eta_2}\right)^{\min(p_2,t_1+1)+t_2-m_1} \\
& + \frac{k\binom{N_r}{k}}{((m_1-1)!)^2}\sum_{q_1=0}^{k-1}\binom{k-1}{q_1}\sum_{r_1=0}^{p_1(m_1-1)}\frac{\phi_{p_1(r_1)}^{m_1-1}(r_1+m_1-1)!(1-\rho)^{r_1}}{(p_1(1-\rho)+1)^{r_1+m_1}} \\
& \times \sum_{t_1=0}^{m_1-1}\binom{m_1-1}{t_1}\sum_{p_2=0}^{\min(t_1+1,m_2-1)}\frac{(-1)^{t_1-p_2+q_1+1}}{(t_1-p_2+1)!p_2!}\left(\frac{m_2C}{\eta_2}\right)^{t_1+1-m_1} \\
& \times \left(\ln\left(\frac{(p_1+1)}{(p_1(1-\rho)+1)}\right)-\psi(1)-\psi(t_1-p_2+2)\right),
\end{aligned} \tag{3.23}$$

$$\begin{aligned}
\tau_2 = {} & \frac{k\binom{N_r}{k}}{((m_1-1)!)^2}\sum_{q_1=0}^{k-1}\binom{k-1}{q_1}\sum_{r_1=0}^{p_1(m_1-1)}\frac{\phi_{p_1(r_1)}^{m_1-1}(r_1+m_1-1)!(1-\rho)^{r_1}}{(p_1(1-\rho)+1)^{r_1+m_1}} \\
& \times \sum_{t_1=0}^{m_1-1}\binom{m_1-1}{t_1}\left(\frac{m_2C}{\eta_2}\right)^{t_1+1-m_1}\sum_{p_2=0}^{\min(t_1+1,m_2-1)}\frac{(-1)^{t_1-p_2+1+q_1}}{(t_1-p_2+1)!p_2!},
\end{aligned} \tag{3.24}$$

and

$$\begin{aligned}
\tau_3 = {} & \frac{k\binom{N_r}{k}}{(m_1-1)!}\sum_{q_1=0}^{k-1}\binom{k-1}{q_1}\sum_{r_1=0}^{p_1(m_1-1)}\phi_{p_1(r_1)}^{m_1-1}\sum_{s_1=0}^{r_1}\frac{(r_1+m_1-1)!}{(s_1+m_1-1)!} \\
& \times \frac{\binom{r_1}{s_1}\rho^{s_1}(1-\rho)^{r_1-s_1}}{(p_1(1-\rho)+1)^{r_1+m_2}(p_1+1)^{s_1+m_1-m_2}}\sum_{t_1=0}^{s_1+m_1-1}\binom{s_1+m_1-1}{t_1} \\
& \times \sum_{p_2=0}^{\min(t_1+1,m_2-1)}\sum_{t_2=0}^{\min(s_2,t_1-p_2)}\frac{(-1)^{q_1+s_2+1}(t_1-p_2-t_2)!}{p_2!(s_2-t_2)!t_2!}\left(\frac{m_2C}{\eta_2}\right)^{p_2+t_2-m_2}
\end{aligned} \tag{3.25}$$

where $r_2 = \max(0, t_1-p_2+1)-s_1$, $s_2 = t_1-p_2+1+m_2-m_1-s_1$ and $\psi(x)$ is the digamma function [7, Eq. (6.3.1)].

Remark 1: It is noted that when $\rho < 1$ (outdated CSI), the diversity order of the system is given by $\min(m_1, m_2)$. The diversity order, if S had perfect CSI is $\min(m_1 k, m_2)$. If $m_2 < m_1$, the impact of outdated CSI is not very significant, as the $R \to D$ link acts as the performance bottleneck. However, if $m_1 > m_2$, and for large N_r and k, the impact of outdated CSI is large, particularly so in the high SNR region. This is due to the potential loss of the system's diversity order, compared to the perfect CSI case.

In the special case of Rayleigh fading, by substituting $m_1 = m_2 = 1$ in (3.22) yields

$$F_{\gamma_{eq1}}(\gamma_T) = k\binom{N_r}{k}\sum_{q_1=0}^{k-1}\frac{(-1)^{q_1}\binom{k-1}{q_1}}{(p_1(1-\rho)+1)}\left(1-\frac{C}{\eta_2}\right. \tag{3.26}$$
$$\left.\times\left(\ln\left(\frac{(p_1+1)C\gamma_T}{(p_1(1-\rho)+1)\eta_1\eta_2}\right)-\psi(1)-\psi(2)\right)\right)\left(\frac{\gamma_T}{\eta_1}\right).$$

3.3.2 Probability Density Function

Since the CDF of the end-to-end SNR of the system had been already derived, its derivative is taken to obtain the PDF ($f_{\gamma_{eq1}}$):

$$f_{\gamma_{eq1}}(x) = 2k\binom{N_r}{k}\sum_{q_1=0}^{k-1}\frac{(-1)^{q_1}\binom{k-1}{q_1}}{(m_1-1)!}e^{-\frac{m_1(p_1+1)x}{(p_1(1-\rho)+1)\eta_1}}\sum_{r_1=0}^{p_1(m_1-1)}\phi_{p_1(r_1)}^{m_1-1} \tag{3.27}$$
$$\times\sum_{s_1=0}^{r_1}\frac{(r_1+m_1-1)!\left(\frac{m_1}{\eta_1}\right)^{s_1+m_1}\binom{r_1}{s_1}\rho^{s_1}(1-\rho)^{r_1-s_1}}{(p_1(1-\rho)+1)^{r_1+s_1+m_1}(s_1+m_1-1)!}\sum_{t_1=0}^{s_1+m_1-1}\binom{s_1+m_1-1}{t_1}$$
$$\times\sum_{p_2=0}^{m_2-1}\frac{m_2^{p_2}C^{p_2}x^{s_1+m_1+\frac{p_2-t_1-3}{2}}}{p_2!\eta_2^{p_2}}\left(\frac{(p_1(1-\rho)+1)m_2C\eta_1}{m_1(p_1+1)\eta_2}\right)^{\frac{t_1-p_2+1}{2}}$$
$$\times\left(\left(\frac{m_1(p_1+1)x}{(p_1(1-\rho)+1)\eta_1}-s_1-m_1-p_2+t_1+1\right)K_{t_1-p_2+1}\left(2\sqrt{\frac{(p_1+1)m_1m_2Cx}{(p_1(1-\rho)+1)\eta_1\eta_2}}\right)\right.$$
$$\left.+\sqrt{\frac{(p_1+1)m_1m_2Cx}{(p_1(1-\rho)+1)\eta_1\eta_2}}K_{t_1-p_2}\left(2\sqrt{\frac{(p_1+1)m_1m_2Cx}{(p_1(1-\rho)+1)\eta_1\eta_2}}\right)\right).$$

The effect of the correlation coefficient ρ on the PDF of γ_{eq1} can be investigated using this result.

In Fig 3.1, the PDF of γ_{eq1} at $\rho = 0.1, 0.5$ and 0.9 is plotted. It can be seen that the peak of the PDF of shifts towards the left as ρ increases. Hence, a reduction in the expected value of γ_{eq1} is expected as ρ reduces. Additionally, it is seen that, as

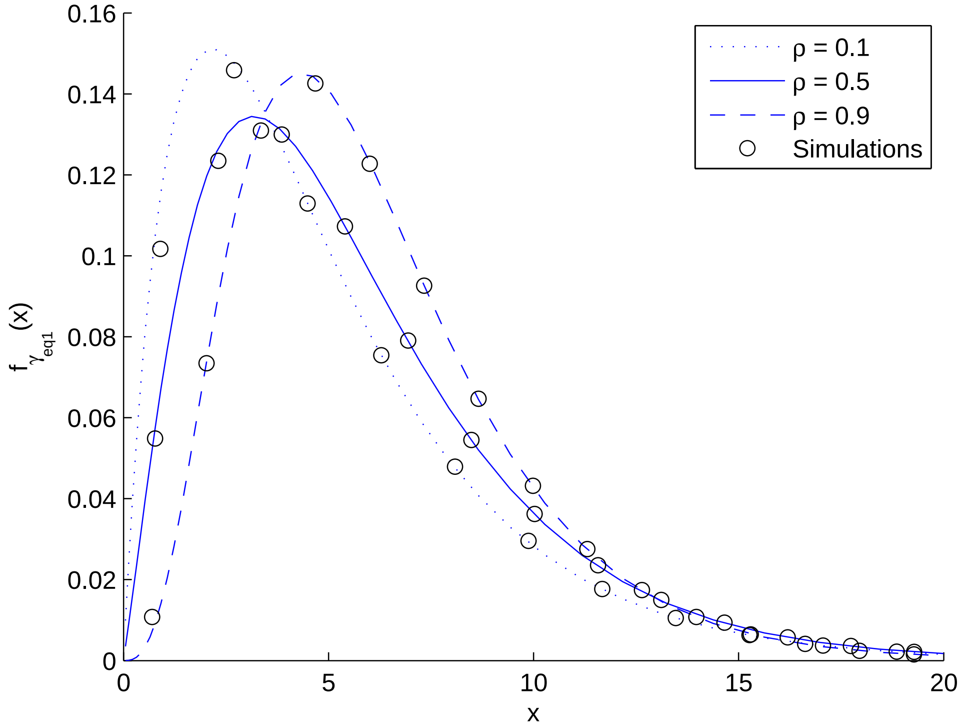

Figure 3.1: The probability density function of γ_{eq1}, for different ρ. ($N_r = 5, k = 5, m_1 = 2, m_2 = 5, \eta_1 = \eta_2 = 10$dB).

ρ reduces, the portion of the area under the PDF close to zero is increasing. This means that the system outage probability will increase as ρ goes down.

3.3.3 Average BER

The analysis then proceeds to the system's average BER. For many modulation formats used in wireless applications, the average BER can be expressed as

$$P_b = \alpha\, E[Q(\sqrt{\beta \gamma_{eq1}})] = \frac{\alpha}{\sqrt{2\pi}} \int_0^\infty F_{\gamma_{eq1}}\left(\frac{t^2}{\beta}\right) e^{-\frac{t^2}{2}} dt, \tag{3.28}$$

where $\alpha, \beta > 0$ are constants depending on the modulation scheme, and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{y^2}{2}} dy$ is the Gaussian Q-function. Using the result of (3.20) and [51, Eq. (6.631.3)] the following expression for the BER given by (3.29) is arrived at, where $\mathcal{W}_{\mu,\nu}(\cdot)$ is the Whittaker hypergeometric function W [7, Eq. (13.1.33)].

$$\begin{aligned} P_b = \frac{\alpha}{2} - \frac{\alpha k}{\sqrt{2\pi}} \binom{N_r}{k} \sum_{q_1=0}^{k-1} \frac{(-1)^{q_1} \binom{k-1}{q_1}}{(m_1-1)!} \sum_{r_1=0}^{p_1(m_1-1)} \phi_{p_1(r_1)}^{m_1-1} \\ \times \sum_{s_1=0}^{r_1} \frac{(r_1+m_1-1)! \left(\frac{m_1}{\eta_1}\right)^{s_1+m_1} \binom{r_1}{s_1} \rho^{s_1} (1-\rho)^{r_1-s_1}}{(p_1(1-\rho)+1)^{r_1+s_1+m_1} (s_1+m_1-1)!} \sum_{t_1=0}^{s_1+m_1-1} \binom{s_1+m_1-1}{t_1} \\ \times \sum_{p_2=0}^{m_2-1} \frac{m_2^{p_2} C^{p_2}}{p_2! \eta_2^{p_2}} \left(\frac{(p_1(1-\rho)+1) m_2 C \eta_1}{m_1 (p_1+1) \eta_2}\right)^{\frac{t_1-p_2+1}{2}} \frac{\omega_1^{t_1-p_2+1-s_1+m_1} e^{\frac{\omega_2^2}{8\omega_1}}}{\omega_2} \\ \times \Gamma\left(\frac{1}{2}+s_1+m_1\right) \Gamma\left(\frac{1}{2}+m_2-s_2\right) \mathcal{W}_{\frac{t_1+1-p_2-2s_1-2m_1}{2}, \frac{t_1-p_2+1}{2}}\left(\frac{\omega_2^2}{4\omega_1}\right). \end{aligned} \tag{3.29}$$

$\omega_1 = \frac{m_1(p_1+1)\gamma_T}{(p_1(1-\rho)+1)\beta\eta_1} + \frac{1}{2}$ and $\omega_2 = 2\sqrt{\frac{(p_1+1)m_1 m_2 C \gamma_T}{(p_1(1-\rho)+1)\beta\eta_1\eta_2}}$.

Substituting (3.22) in (3.28), the following high SNR approximation is obtained for the average BER.

As $\eta_1, \eta_2 \to \infty$,

$$P_b = \begin{cases} 2^{m_1-1} \frac{\alpha \Gamma(m_1+\frac{1}{2})}{\sqrt{\pi}} \left(\tau_1 + \tau_2 \left(\ln(\frac{2m_1 m_2 C}{\beta\eta_1\eta_2}) + \psi(m_1+\frac{1}{2})\right)\right) \left(\frac{m_1 m_2 C}{\beta\eta_1\eta_2}\right)^{m_1} & \text{(a)} \\ 2^{m_2-1} \tau_3 \Gamma(m_2+\frac{1}{2}) \frac{\alpha}{\sqrt{\pi}} \left(\frac{m_1 m_2 C}{\beta\eta_1\eta_2}\right)^{m_2} & \text{(b)} \end{cases} \tag{3.30}$$

where (a) $= m_1 \leq m_2$ and (b) $= m_1 > m_2$.

3.4 Analysis of Variable Gain Relaying

In the case of variable gain relaying, the instantaneous end-to-end SNR of the selected relay (γ_{eq2}) is given by

$$\gamma_{eq2} = \frac{\widetilde{\gamma}_{1(k)}\gamma_2}{\widetilde{\gamma}_{1(k)} + \gamma_2 + 1}. \tag{3.31}$$

Therefore, for further analysis, it is desirable to define a new RV of the form: $Y = \frac{\widetilde{\gamma}_{1(k)}\gamma_2}{\widetilde{\gamma}_{1(k)}+\gamma_2+c}$, where $c \geq 0$ is a constant. Note that $c = 1$ gives the exact form of γ_{eq2} in (3.31), while $c = 0$ gives a mathematically more tractable tight approximation for γ_{eq2} in the medium-to-high SNR region.

3.4.1 Outage Probability

The CDF of Y given by Pr $\left(\frac{\widetilde{\gamma}_{1(k)}\gamma_2}{\widetilde{\gamma}_{1(k)}+\gamma_2+c} < \gamma_T\right)$ can be evaluated as

$$\begin{aligned} F_Y(\gamma_T) &= \int_0^{\gamma_T} f_{\tilde{\gamma}_{1(k)}}(x)dx + \int_{\gamma_T}^{\infty} \Pr\left(\gamma_2 < \frac{x(\gamma_T + c)}{x - \gamma_T}\right) f_{\tilde{\gamma}_{1(k)}}(x)dx \\ &= 1 - \int_0^{\infty} \frac{\Gamma\left(m_2, \frac{m_2\gamma_T(x+\gamma_T+c)}{\eta_2 x}\right)}{\Gamma(m_2)} f_{\tilde{\gamma}_{1(k)}}(x+\gamma_T)dx. \end{aligned} \tag{3.32}$$

Use of the series expansion for $\Gamma\left(m_2, \frac{m_2\gamma_T(x+\gamma_T+c)}{\eta_2 x}\right)$ [7, Sec. (6.5)],and subsequent binomial expansions and substituting (3.11) and $c = 1$, results in the expression (3.33) for $F_{\gamma_{eq2}}(\gamma_T)$.

$$\begin{aligned} F_{\gamma_{eq2}}(\gamma_T) &= 1 - \frac{2k\binom{N_r}{k}}{(m_1-1)!} \sum_{p_1=0}^{m_2-1} \frac{m_2^{p_1}}{p_1!\eta_2^{p_1}} \sum_{q_1=0}^{p_1} \binom{p_1}{q_1} \sum_{q_2=0}^{k-1} (-1)^{q_2} \binom{k-1}{q_2} \sum_{r_2=0}^{p_2(m_1-1)} \phi_{p_2(r_2)}^{m_1-1} \\ &\times (r_2+m_1-1)! \sum_{s_2=0}^{r_2} \frac{\binom{r_2}{s_2}\left(\frac{m_1}{\eta_1}\right)^{s_2+m_1} \rho^{s_2}(1-\rho)^{r_2-s_2} e^{-\gamma_T\left(\frac{m_2}{\eta_1}+\frac{m_1(p_2+1)}{(p_2(1-\rho)+1)\eta_2}\right)}}{(p_2(1-\rho)+1)^{r_2+s_2+m_1}(s_2+m_1-1)!} \\ &\times \sum_{q_2=0}^{s_2+m_1-1} \binom{s_2+m_1-1}{q_2} \left(\frac{(p_2(1-\rho)+1)m_2\eta_1}{(p_2+1)m_1\eta_2}\right)^{\frac{q_2-q_1+1}{2}} \gamma_T^{m_1+p_1+s_2-\frac{(q_2+q_1+1)}{2}} \\ &\times (1+\gamma_T)^{\frac{q_2+q_1+1}{2}} K_{q_2-q_1+1}\left(2\sqrt{\frac{(p_2+1)m_1m_2(\gamma_T+1)\gamma_T}{(p_2(1-\rho)+1)\eta_1\eta_2}}\right). \end{aligned} \tag{3.33}$$

Although (3.33) gives the exact outage probability, asymptotic results are also of interest due to the insights they offer on high SNR behavior of the systems.

It has been shown that in high SNR, the system performance is governed by the weakest link [55]. Since $\gamma_2 \sim \mathcal{G}\left(m_2, \frac{\eta_2}{m_2}\right)$,

$$\begin{aligned} f_{\gamma_2}(x) &= \frac{m_2^{m_2} x^{m_2-1}}{\eta^{m_2}\Gamma(m_2)} e^{-\frac{m_2 x}{\eta_2}} \\ &= \frac{m_2^{m_2}}{\eta_2^{m_2}\Gamma(m_2)} x^{m_2-1} + o(x^{m_2}). \end{aligned} \tag{3.34}$$

From (3.11) substituting the Maclaurin series expansion for the exponential function and selecting only the lowest power of γ_T with a non-zero coefficient, yields

$$f_{\tilde{\gamma}_{1(k)}}(x) = \frac{\kappa}{\eta_1^{m_1}} x^{m_1-1} + o(x^{m_1}), \tag{3.35}$$

where

$$\begin{aligned} \kappa = & \frac{k\binom{N_r}{k} m_1{}^{m_1}}{((m_1-1)!)^2} \sum_{q_2=0}^{k-1} (-1)^{q_2} \binom{k-1}{q_2} \\ & \times \sum_{r_2=0}^{p_2(m_1-1)} \frac{\phi_{p_2(r_2)}^{m_1-1}(r_2+m_1-1)!(1-\rho)^{r_2}}{(p_2(1-\rho)+1)^{r_2+m_1}}. \end{aligned} \tag{3.36}$$

Using the results of [55], the following asymptotic result is derived for the outage probability. As $\eta_1, \eta_2 \to \infty$, for $\rho < 1$

$$F_{\gamma_{eq2}}(\gamma_T) = \begin{cases} \frac{\kappa}{m_1 \eta_1^{m_1}} \gamma_T^{m_1} & m_1 < m_2, \\ \left(\frac{m_2^{m_2}}{\eta_2^{m_2} m_2!} + \frac{\kappa}{m_2 \eta_2^{m_2}} \right) \gamma_T^{m_2} & m_1 = m_2, \\ \frac{m_2^{m_2}}{\eta_2^{m_2} m_2!} \gamma_T^{m_2} & m_1 > m_2. \end{cases} \tag{3.37}$$

Remark 2: Similar to the fixed gain case, it is observed that the diversity order of the system is $\min(m_1, m_2)$. As discussed in Section III, the performance loss due to outdated CSI will be most significant in the case $m_1 < m_2$, as imperfect CSI would cause a reduction in the achievable diversity order for the considered dual hop relay network.

An asymptotic approximation for outage probability in Rayleigh fading can be found by substituting $m_1 = m_2 = 1$ in (3.37). Under Rayleigh fading, as $\eta_1, \eta_2 \to$

∞, $F_{\gamma_{eq2}}(\gamma_T)$ can be approximated as

$$F_{\gamma_{eq2}}(\gamma_T) = \frac{\gamma_T}{\eta_2} + \sum_{p=0}^{k-1} \frac{k\binom{N_r}{k}(-1)^p\binom{k-1}{p}\gamma_T}{((N_r-k+p)(1-\rho)+1)\eta_1}. \tag{3.38}$$

3.4.2 Probability Density Function

As in the analysis of the FG relaying, the derivative of 3.33 with respect to γ_T can be taken to obtain the PDF of γ_{eq2}.

$$\begin{aligned}
f_{\gamma_{eq2}}(x) = {} & \frac{2k\binom{N_r}{k}}{(m_1-1)!} \sum_{p_1=0}^{m_2-1} \frac{m_2^{p_1}}{p_1!\eta_2^{p_1}} \sum_{q_1=0}^{p_1} \binom{p_1}{q_1} \sum_{q_2=0}^{k-1} (-1)^{q_2} \binom{k-1}{q_2} \sum_{r_2=0}^{p_2(m_1-1)} \phi_{p_2(r_2)}^{m_1-1} \\
& \times (r_2+m_1-1)! \sum_{s_2=0}^{r_2} \frac{\binom{r_2}{s_2}\left(\frac{m_1}{\eta_1}\right)^{s_2+m_1} \rho^{s_2}(1-\rho)^{r_2-s_2} e^{-x\left(\frac{m_2}{\eta_1}+\frac{m_1(p_2+1)}{(p_2(1-\rho)+1)\eta_2}\right)}}{(p_2(1-\rho)+1)^{r_2+s_2+m_1}(s_2+m_1-1)!} \\
& \times \sum_{q_2=0}^{s_2+m_1-1} \binom{s_2+m_1-1}{q_2} \left(\frac{(p_2(1-\rho)+1)m_2\eta_1}{(p_2+1)m_1\eta_2}\right)^{\frac{q_2-q_1+1}{2}} x^{m_1+p_1+s_2-\frac{(q_2+q_1+1)}{2}} \\
& \times (1+x)^{\frac{q_2+q_1+1}{2}} \left(K_{q_2-q_1+1}\left(2\sqrt{\frac{(p_2+1)m_1m_2(x+1)x}{(p_2(1-\rho)+1)\eta_1\eta_2}}\right)\right. \\
& \times \left((1+x)\left(\frac{m_2x}{\eta_1} + \frac{m_1(p_2+1)x}{(p_2(1-\rho)+1)\eta_2} + q_2 + 1 - m_1 - p_1 - s_2\right) - q_1x\right) \\
& \left. + (2x+1)\sqrt{\frac{(p_2+1)m_1m_2(x+1)x}{(p_2(1-\rho)+1)\eta_1\eta_2}} K_{q_2-q_1}\left(2\sqrt{\frac{(p_2+1)m_1m_2(x+1)x}{(p_2(1-\rho)+1)\eta_1\eta_2}}\right)\right).
\end{aligned} \tag{3.39}$$

Using the result above the effect of ρ on the distribution of γ_{eq2} can be analyzed. In Fig 3.2, the PDF of γ_{eq2} is plotted at $\rho = 0.1, 0.5$ and 0.9 at 10 dB unfaded hop SNR η_1, η_2.

It is seen that,as ρ reduces, γ_{eq2} tend to be more concentrated on lower values. The peak of the curve shifts to the left. i.e. the mode of γ_{eq2} reduces with ρ. From fig 3.2 it can be seen that the expected value of γ_{eq2} also seem to reduce as $\rho \to 0$. As the distribution of γ_{eq2} gets concentrated near lower values as ρ reduces, and hence higher system outage probability is expected as ρ reduces.

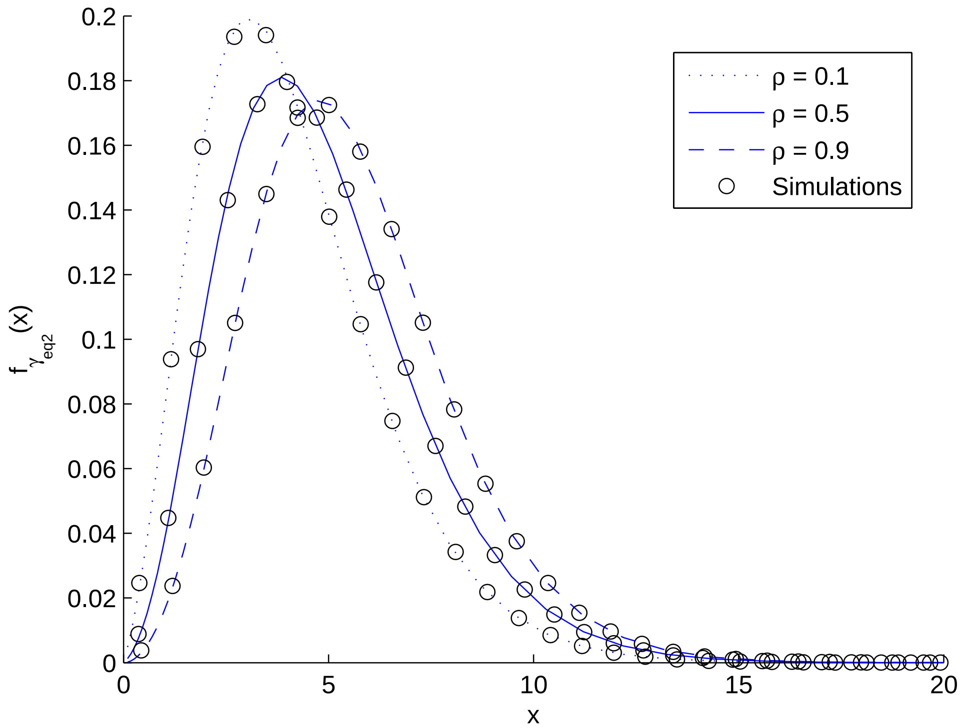

Figure 3.2: The probability density function of γ_{eq2}, for different ρ. ($N_r = 5, k = 5, m_1 = 2, m_2 = 5, \eta_1 = \eta_2 = 10$dB).

3.4.3 Average BER

(3.33) is substituted into (3.8) to derive the average BER. However, since there is no closed-form solution, $c = 0$ is substituted in (3.32). Hence, with the help of [51, Eq.(6.621.3)], a tight lower bound for the average BER can be obtained as (3.40),

$$P_b \geq \frac{\alpha}{2} - \frac{\alpha\sqrt{\beta}k\binom{N_r}{k}}{(m_1-1)!}\sum_{p_1=0}^{m_2-1}\frac{m_2^{p_1}}{p_1!\eta_2^{p_1}}\sum_{q_1=0}^{p_1}\binom{p_1}{q_1}\sum_{q_2=0}^{k-1}(-1)^{q_2}\binom{k-1}{q_2}\sum_{r_2=0}^{p_2(m_1-1)}\phi_{p_2(r_2)}^{m_1-1} \tag{3.40}$$

$$\times (r_2+m_1-1)!\sum_{s_2=0}^{r_2}\frac{\binom{r_2}{s_2}\left(\frac{m_1}{\eta_1}\right)^{s_2+m_1}\rho^{s_2}(1-\rho)^{r_2-s_2}}{(p_2(1-\rho)+1)^{r_2+s_2+m_1}(s_2+m_1-1)!}\sum_{q_2=0}^{s_2+m_1-1}\binom{s_2+m_1-1}{q_2}$$

$$\times\left(\frac{(p_2(1-\rho)+1)m_2^3\eta_1}{(p_2+1)m_1\eta_2^3}\right)^{\frac{q_2-q_1+1}{2}}\frac{2^{2q_2-2q_1+\frac{3}{2}}\Gamma\left(r_1+q_2-q_1+\frac{3}{2}\right)\Gamma\left(r_1-q_2+q_1+\frac{1}{2}\right)}{\left(\frac{m_2}{\eta_2}+\frac{m_1(p_2+1)}{(p_2(1-\rho)+1)\eta_1}+2\sqrt{\frac{(p_2+1)m_1m_2}{(p_2(1-\rho)+1)\eta_1\eta_2}}+\frac{1}{2}\right)(r_1)!}$$

$$\times\ {}_2F_1\left(r_1+q_2-q_1+\frac{3}{2}, q_2-q_1+\frac{3}{2}; r_1+1; \frac{\frac{m_2}{\beta\eta_2}+\frac{m_1(p_2+1)}{(p_2(1-\rho)+1)\beta\eta_1}-\frac{2}{\beta}\sqrt{\frac{(p_2+1)m_1m_2}{(p_2(1-\rho)+1)\eta_1\eta_2}}+\frac{1}{2}}{\frac{m_2}{\beta\eta_2}+\frac{m_1(p_2+1)}{(p_2(1-\rho)+1)\beta\eta_1}+\frac{2}{\beta}\sqrt{\frac{(p_2+1)m_1m_2}{(p_2(1-\rho)+1)\eta_1\eta_2}}+\frac{1}{2}}\right),$$

where $r_1 = p_1+s_2+m_1$ and ${}_2F_1(a,b;c;x)$ is the Gauss hypergeometric function [7, Eq. (15.1.1)].

Substituting (3.37) in (3.28), the following high SNR approximation is derived for the average BER (P_b^∞).

As $\eta_1, \eta_2 \to \infty$,

$$P_b^\infty = \begin{cases} \frac{2^{m_1-1}\alpha\kappa\Gamma(m_1+\frac{1}{2})}{\sqrt{\pi}m_1\beta^{m_1}\eta_1^{m_1}} & m_1 < m_2, \\ \left(\frac{m_2^{m_2}}{\eta_2^{m_2}m_2!}+\frac{\kappa}{m_2\eta_2^{m_2}}\right)\frac{2^{m_2-1}\alpha\Gamma(m_2+\frac{1}{2})}{\sqrt{\pi}\beta^{m_2}} & m_1 = m_2, \\ \frac{2^{m_2-1}m_2^{m_2}\alpha\Gamma(m_2+\frac{1}{2})}{\sqrt{\pi}\beta^{m_2}\eta_2^{m_2}m_2!} & m_1 > m_2. \end{cases} \tag{3.41}$$

3.5 Numerical and Simulation Results

Figures 3.3,3.4 and 3.5 show the performance of the fixed gain (FG) and variable gain(VG) relaying schemes discussed earlier.

Effect of relay rank on outage probability: Figure 3.3 shows the outage prob-

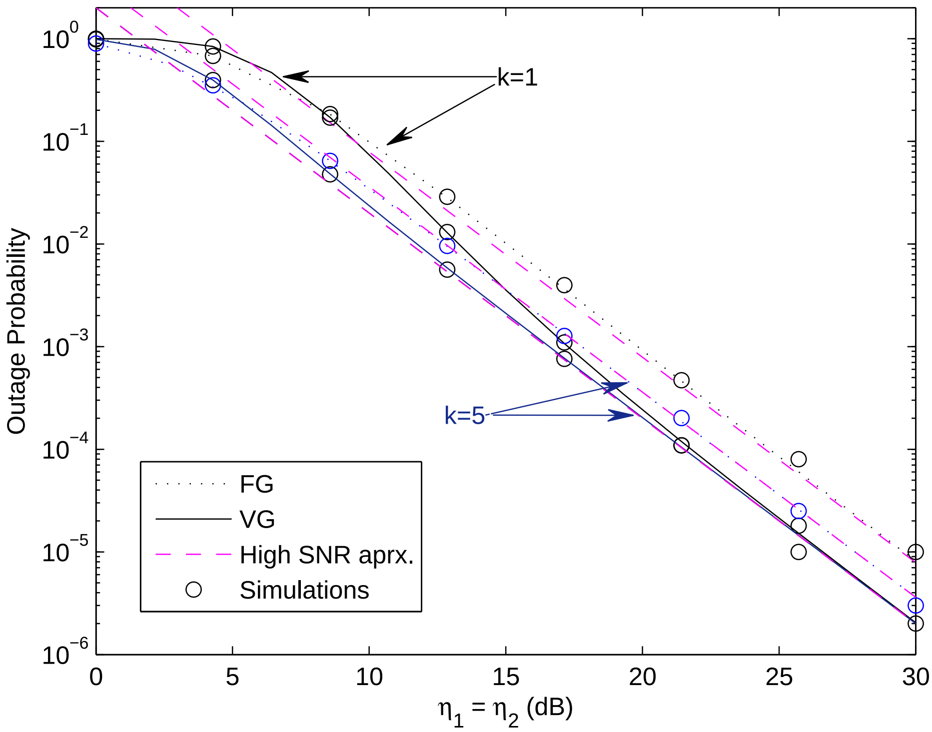

Figure 3.3: The outage probability, for best ($k = 5$) and worst ($k = 1$) relay selection. ($N_r = 5, m_1 = 3, m_2 = 2, \rho = 0.8, \gamma_T = 1$).

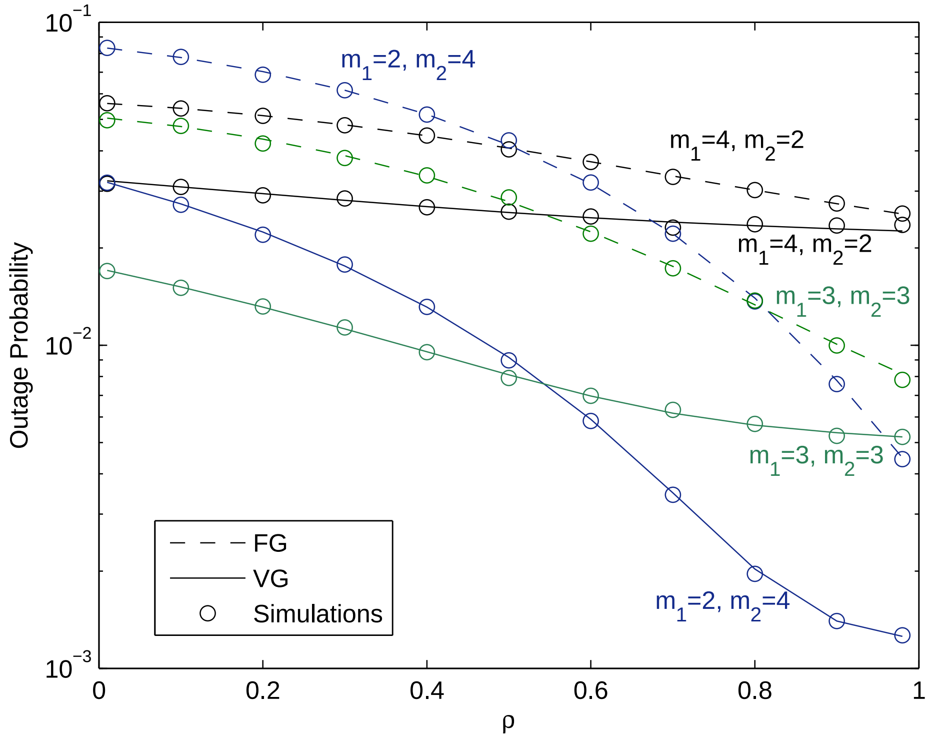

Figure 3.4: The effect of correlation on the outage probability, for different fading parameters m_1, m_2. ($N_r = 5, k = 5, \gamma_T = 1, \eta_1 = \eta_2 = 10$ dB).

ability of the systems for the best ($k = 5$) and worst ($k = 1$) relay selection, with $N_r = 5, \rho = 0.8, \gamma_T = 1, m_1 = 3$ and $m_2 = 2$. The VG scheme outperforms the FG scheme with respect to the outage probability. It is seen that at high SNR, for VG, the performance is the same for both best and worst relay selection. This is so since the system has $m_1 > m_2$ and the high SNR performance is dependent only on the $R -$ D link which has identical fading for all relays. The simulations and asymptotic results shows excellent agreement with the analytical results. It is observed that the outage diversity order is equal to m_1 in all cases.

Effect of correlation on outage probability: Figure 3.4 shows the outage probability of FG, VG systems with varying correlation ρ, with $N_r = k = 5$ and $\eta_1 = \eta_2 = 10$ dB. Under all conditions, the VG relaying outperforms FG relaying. A more important observation to notice is that, the highest variation with ρ is shown for $m_1 < m_2$. With $m_1 > m_2$, the performance loss with reducing ρ is small, par-

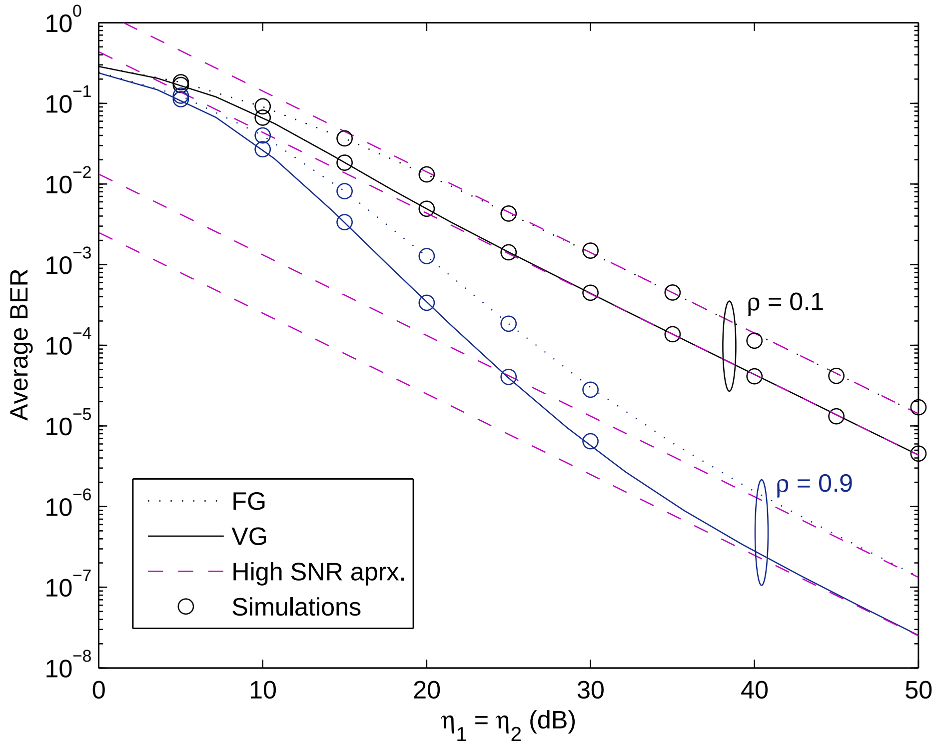

Figure 3.5: The average BER for QPSK ($\alpha = \beta = 1$), under high ($\rho = 0.9$) and low($\rho = 0.1$) correlation.

ticularly for the VG scheme. The performance bottleneck in the case of $m_1 > m_2$ is the $R \to D$ link. As such the impact of outdated CSI is not so significant. However, in the case of $m_1 < m_2$, there is a reduction in the diversity order due to outdated CSI, and hence has a highly negative impact on the system's performance.

Effect of correlation on BER: Figure 3.5 shows the average BER of the two schemes under QPSK modulation with $N_r = k = 5$, $m_1 = 1$ and $m_2 = 2$. It can be seen that the VG scheme outperforms the FG scheme, exhibiting a performance gain of approximately 3 dB in the high SNR region. It is further observed that there is an increase in the coding gain at high correlation ($\rho = 0.9$) over low correlation ($\rho = 0.1$). As expected, the diversity gain with imperfect CSI is equal to m_1.

3.6 Conclusion

This chapter presents new expressions for the outage probability and the average BER of an AF based PRS system over Nakagami-m fading channels, when relay selection is performed using outdated CSI. The derived expressions quantify the performance degradation in the presence of outdated CSI for both fixed gain and variable gain AF relaying protocols. Additionally, high SNR approximations for the outage probability and the average BER were presented. By doing so, this work quantifies the diversity order and the coding gain of the considered relay systems. All the results obtained in this chapter are verified using extensive Monte-Carlo simulations.

Chapter 4

Conclusions

This thesis has investigated impact of outdated channel information at the relays for dual hop relay systems. Outdated or stale CSI occurs because of the time taken for the relay selection process and the variations of the channel during that time.

In Chapter 2, the system model was introduced. Performance of the partial relay selection and opportunistic relay selection schemes under Rayleigh fading was derived. Both fixed gain and variable gain relaying systems were analyzed and compared. The diversity gain in the opportunistic relay selection scheme decreases to unity under imperfect CSI, whilst with perfect CSI it is equal to k, the rank of the relay selected. Fixed gain systems tend to perform better than variable gain systems when the actual channel SNR is less correlated with the channel estimate at the relay, and vice versa.

In Chapter 3, the system performance was analyzed under more general Nakagami-m fading. The performance of the partial relay selection scheme is presented, and expressions for the outage probability and the average BER of the system were derived. With perfect channel information, the diversity gain of the system would be $\min(m_1, m_2 k)$. But there is a diversity gain reduction to $\min(m_1, m_2)$ in both variable gain and fixed gain systems under imperfect CSI.

4.1 Future Research Directions

This work investigated the impact of outdated channel state information on relay system performance.

This study assumed the perfect CSI is available at the destination. In future, the effect of imperfect CSI at the destination can be investigated. As well as the performance impact of these effects, coding schemes, and CSI estimation method and feedback schemes to successfully mitigate the performance degradations can be studied.

In this work we have only considered dual-hop single antenna SRS systems. The analysis can be extended to multihop systems, MRS systems and multiple antenna systems.

Appendix I : High SNR Approximation - Fixed Gain Relaying

Let $\gamma_u = \min\left\{\tilde{\gamma}_{1(k)}, \frac{\tilde{\gamma}_{1(k)}\gamma_{2(k)}}{C}\right\}$. It is claimed that γ_u is an upper bound for γ_{eq1} in (2.3) [49].

$$\begin{aligned}\overline{F}_{\gamma_u}(\gamma_T) &= \Pr\{\gamma_u > \gamma_T\} = \Pr\left\{\frac{\gamma_2\tilde{\gamma}_{1(k)}}{C} > \gamma_T \cap \tilde{\gamma}_{1(k)} > \gamma_T\right\} \\ &= \int_{\gamma_T}^{\infty} f_{\tilde{\gamma}_{1(k)}}(x)\overline{F}_{\gamma_2}(C\gamma_T/x)dx, \end{aligned} \tag{4.1}$$

where $\overline{F}_X(\cdot)$ denotes the CCDF of the RV, X. Since the above distributions are known, $\overline{F}_{\gamma_u}(\gamma_T)$ can be expressed as

$$\overline{F}_{\gamma_u}(\gamma_T) = k\binom{N_r}{k}\sum_{m=0}^{k-1}\frac{a}{\eta_1}I, \tag{4.2}$$

where $I = \int_{\gamma_T}^{\infty} e^{-\left(\frac{C\gamma_T}{\eta_2 x}+\frac{bx}{\eta_1}\right)}dx, \quad a = \frac{(-1)^m\binom{k-1}{m}}{(N_r-k+m)(1-\rho)+1}$ and $b = \frac{N_r-k+m+1}{(N_r-k+m)(1-\rho)+1}$. Then I simplifies to

$$I = \sum_{i=0}^{\infty}\frac{(-1)^i C^i\gamma_T^i}{i!\eta_2^i}\int_{\gamma_T}^{\infty}\frac{e^{-\frac{bx}{\eta_1}}}{x^i}\,dx = \sum_{i=0}^{\infty}\frac{(-1)^i C^i}{i!\eta_2^i}e^{-\frac{b\gamma_T}{\eta_1}}\int_{0}^{\infty}\frac{e^{-\frac{bx}{\eta_1}}}{\left(\frac{x}{\gamma_T}+1\right)^i}\,dx. \tag{4.3}$$

For large η_1 and η_2, I can be approximated using [49, Eq. (11)] as

$$I \approx e^{-\frac{b\gamma_t}{\eta_1}}\left(\sum_{i=2}^{\infty}\frac{(-1)^i C^i\gamma_T}{(i-1)i!\eta_2^i} + \frac{\eta_1}{b} + \frac{C\gamma_T}{\eta_2}\ln\left(\frac{b}{\eta_1}\right)\right). \tag{4.4}$$

Substituting (4.4) into (4.2), using CDF of γ_u, $F_{\gamma_u}(\gamma_T) = 1 - \overline{F}_{\gamma_u}(\gamma_T)$, and after further simplifications, the following can be obtained.

$$\begin{aligned} F_{\gamma_u}(\gamma_T) &\approx k\binom{N_r}{k}\frac{\gamma_T}{\eta_1}\sum_{m=0}^{k-1}\frac{(-1)^m\binom{k-1}{m}}{((N_r-k+m)(1-\rho)+1)} \\ &\times\left(1-\sum_{r=2}^{\infty}\frac{(-1)^r C^r}{{\eta_2}^r r!(r-1)} - \frac{C}{\eta_2}\ln\left(\frac{(N_r-k+m+1)}{((N_r-k+m)(1-\rho)+1)\eta_1}\right)\right). \end{aligned} \tag{4.5}$$

With further simplifications, the approximation given in (2.14) can be obtained.

Appendix II : High SNR Approximation - Variable Gain Relaying

Let $\gamma_{u2} = \min\left\{\tilde{\gamma}_{1(k)}, \frac{\tilde{\gamma}_{1(k)}\gamma_{2(k)}}{\gamma_{1(k)}}\right\}$. It can be shown that $\gamma_{u2} \approx \gamma_{eq2}$ given by (2.7) in high SNR region. Following a similar approach as in Appendix I, the following expression is arrived at.

$$\begin{aligned}\overline{F}_{\gamma_{u2}}(\gamma_T) &= \int_{\gamma_T}^{\infty}\int_{0}^{\infty} f_{\tilde{\gamma}_{1(k)},\gamma_{1(k)}}(x,y)e^{-\frac{\gamma_T y}{\eta_2 x}}dydx \\ &= \frac{k\binom{N_r}{k}}{(1-\rho)\eta_1^2}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m}\int_{\gamma_T}^{\infty}e^{-\frac{x}{(1-\rho)\eta_1}} \\ &\times\int_{0}^{\infty}e^{-y\left(\frac{\omega}{\eta_1}+\frac{\gamma_T}{\eta_2 x}\right)}I_0\left(\frac{2\sqrt{\rho x y}}{(1-\rho)\eta_1}\right)\,dy\,dx\end{aligned} \tag{4.6}$$

where $\omega = \frac{((N_r-k+m)(1-\rho)+1)}{(1-\rho)}$.

Using [50, Eq 4.16.14], substituting $\eta_2 = \mu\eta_1$, and further simplification results in,

$$\begin{aligned}\overline{F}_{\gamma_{u2}}(\gamma_T) &= \frac{k\binom{N_r}{k}}{(1-\rho)\eta_1}\sum_{m=0}^{k-1}(-1)^m\binom{k-1}{m} \\ &\times\int_{\gamma_T}^{\infty}\frac{\mu x}{\omega\mu x+\gamma_T}\exp\left(-\frac{x}{(1-\rho)\eta_1}\left(1-\frac{\rho\mu x}{(1-\rho)(\omega\mu x+\gamma_T)}\right)\right)dx.\end{aligned} \tag{4.7}$$

Using partial fractions, the argument of the exponential function can be re-expressed as

$$\frac{x}{(1-\rho)\eta_1}\left(1-\frac{\rho\mu x}{(1-\rho)(\omega\mu x+\gamma_T)}\right) = \frac{p_1}{\eta_1}x+\frac{p_2\gamma_T}{\eta_1}+\frac{p_3}{\eta_1(\gamma_T+\omega\mu x)}, \tag{4.8}$$

where $p_1 = \frac{N_r-k+m+1}{(N_r-k+m)(1-\rho)+1}$, $p_2 = \frac{\rho}{\mu((N_r-k+m)(1-\rho)+1)^2}$ and $p_3 = \frac{-\gamma_T^2\rho}{\mu((N_r-k+m)(1-\rho)+1)^2}$.

Substituting (4.8) into (4.7) yields

$$\overline{F}_{\gamma_{u2}}(\gamma_T) = \sum_{m=0}^{k-1}\frac{(-1)^m k\binom{N_r}{k}\binom{k-1}{m}}{(1-\rho)\eta_1}e^{-\frac{p_2\gamma_T}{\eta_1}}\int_{\gamma_T}^{\infty}\frac{\mu x}{\omega\mu x+\gamma_T}e^{\left(-\frac{p_1 x}{\eta_1}-\frac{p_3}{\eta_1(\gamma_T+\omega\mu x)}\right)}dx. \tag{4.9}$$

Using a variable transformation to modify the range of the above integral to $(0, \infty)$, and the Maclaurin series expansion of $\exp\left(\frac{p_3}{\eta_1(\gamma_T+\omega\mu x)}\right)$ yields

$$\overline{F}_{\gamma_{u2}}(\gamma_T) = \frac{k\binom{N_r}{k}}{(1-\rho)} \sum_{m=0}^{k-1} (-1)^m \binom{k-1}{m} e^{-\frac{(p_1+p_2)\gamma_T}{\eta_1}} \sum_{i=0}^{\infty} \frac{(-1)^i p_3^i \mu}{i! \eta_1^{i+1}} \times \int_0^{\infty} \frac{x+\gamma_T}{(\omega\mu x + \gamma_T(1+\omega\mu))^{i+1}} e^{-\frac{p_1}{\eta_1}x}\, dx. \tag{4.10}$$

With further simplifications, using approximation from [49, Eq. (11)] and ignoring higher order terms of η_1^{-1}, one can obtain the following high SNR approximation for the CCDF. As $\eta_1 \to \infty$,

$$\overline{F}_{\gamma_{u2}}(\gamma_T) \approx \frac{k\binom{N_r}{k}}{(1-\rho)} \sum_{m=0}^{k-1} \frac{(-1)^m \binom{k-1}{m}}{\omega} e^{-\frac{(p_1+p_2)\gamma_T}{\eta_1}} \left(\frac{1}{p_1} - \frac{\gamma_T \ln(\eta_1/p_1)}{\mu\omega\eta_1}\right). \tag{4.11}$$

Using the above result, the following approximation is obtained for the CDF as $\eta_1 \to \infty$

$$F_{\gamma_{u2}}(\gamma_T) \approx \frac{k\binom{N_r}{k}\gamma_T}{\eta_1} \sum_{m=0}^{k-1} (-1)^m \binom{k-1}{m} \left(\frac{p_1+p_2}{N_r-k+m+1} + \frac{\ln(\eta_1/p_1)}{\mu\omega^2(1-\rho)}\right). \tag{4.12}$$

Appendix III - Outage Probability - Variable gain relaying II

To derive the CDF of γ_{eq3}, a new random variable (RV), Z, is defined as

$$Z = \frac{\tilde{\gamma}_{1(k)}\tilde{\gamma}_{2(k)}}{\tilde{\gamma}_{1(k)} + \tilde{\gamma}_{2(k)} + c}, \tag{4.13}$$

where $c \geq 0$ is a constant.

The CDF of Z can be written as

$$F_Z(z) = \int_0^{\infty} \Pr\left(\frac{x\tilde{\gamma}_{2(k)}}{x+\tilde{\gamma}_{2(k)}+c} < \gamma_T\right) f_{\tilde{\gamma}_{1(k)}}(x) dx \tag{4.14}$$

where $\Pr(\cdot)$ denotes the probability and $f_{\tilde{\gamma}_{1(k)}}(x)$ is the probability density function of $\tilde{\gamma}_{1(k)}$. After applying some algebraic manipulations to (4.14), it can be shown that

$$F_Z(\gamma_T) = 1 - \int_0^{\infty} f_{\tilde{\gamma}_{1(k)}}(\gamma_T + x)\left(1 - F_{\tilde{\gamma}_{2(k)}}\left(\gamma_T + \frac{\gamma_T^2 + c\gamma_T}{x}\right)\right) dx, \tag{4.15}$$

where $F_{\tilde{\gamma}_{2(k)}}(x) = 1 - e^{-\frac{x}{\eta_2}}$ is the CDF of $\tilde{\gamma}_{2(k)}$. In order to simplify (4.15) further, an expression for $f_{\tilde{\gamma}_{1(k)}}(x)$ is required.

According to the principles of concomitants or induced order statistics, the PDF of $\tilde{\gamma}_{1(k)}$, denoted by $f_{\tilde{\gamma}_{1(k)}}(x)$, is given by

$$f_{\tilde{\gamma}_{1(k)}}(x) = \int_0^\infty f_{\tilde{\gamma}_{1(k)}|\gamma_{1(k)}}(x|y) f_{\gamma_{1(k)}}(y) dy \tag{4.16}$$

where $f_{\tilde{\gamma}_{1(k)}|\gamma_{1(k)}}(x|y) = \frac{f_{\tilde{\gamma}_{1(\ell)},\gamma_{1(\ell)}}(x,y)}{f_{\gamma_{1(\ell)}}(y)}$ is the PDF of $\tilde{\gamma}_{1(k)}$ conditioned on $\gamma_{1(k)}$. Since $\tilde{\gamma}_{1(\ell)}$ and $\gamma_{1(\ell)}$ are two correlated exponentially distributed RVs, their joint PDF is given by

$$f_{\tilde{\gamma}_{1(\ell)},\gamma_{1(\ell)}}(x,y) = \frac{1}{(1-\rho)\eta_1^2} e^{-\frac{x+y}{(1-\rho)\eta_1}} I_0\left(\frac{2\sqrt{\rho x y}}{(1-\rho)\eta_1}\right), \tag{4.17}$$

where $I_0(x)$ is the modified Bessel function of the first kind.

The PDF $f_{\gamma_{1(k)}}(y)$ is given by

$$f_{\gamma_{1(k)}}(y) = \frac{N_r!}{(k-1)!(N_r-k)!} [F_{\gamma_{1(\ell)}}(y)]^{k-1} [1 - F_{\gamma_{1(\ell)}}(y)]^{N_r-k} f_{\gamma_{1(\ell)}}(y) \tag{4.18}$$

where $f_{\gamma_{1(\ell)}}(y) = \frac{1}{\eta_1} e^{-\frac{y}{\eta_1}}$ and $F_{\gamma_{1(\ell)}}(y) = 1 - e^{-\frac{y}{\eta_1}}$. Following the approach in [56] and simplifying yields

$$f_{\tilde{\gamma}_{1(k)}}(x) = k\binom{N_r}{k} \sum_{m=0}^{k-1} \frac{(-1)^m}{\eta_1} \binom{k-1}{m} \frac{1}{(N_r-k+m)(1-\rho)+1} e^{-\frac{(N_r-k+m+1)x}{((N_r-k+m)(1-\rho)+1)\eta_1}} \tag{4.19}$$

Now substituting (4.19) into (4.15), the integral can be solved in closed-form using [51, Eq. (3.478.4)]. Therefore, $F_Z(\gamma_T)$ is given by (2.34) where $K_1(x)$ is the first order modified Bessel function of the second kind. Finally, the exact outage probability follows by substituting $c = 1$ in (2.34).

Bibliography

[1] A. Goldsmith, *Wireless Communications*. New York, NY, USA: Cambridge University Press, 2005.

[2] *IEEE Standard for Information technology Telecommunications and information exchange between systems Local and metropolitan area networks Specific requirements Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications*, IEEE Computer Society Std., Nov. 2007.

[3] *IEEE Standard for Information technology - Telecommunications and information exchange between systems - Local and metropolitan area networks - Specific requirements. Part 15.1: Wireless Medium Access Control (MAC) and Physical Layer (PHY) Specifications for Wireless Personal Area Networks (WPANs)*, IEEE Computer Society Std., Jun. 2005.

[4] Global mobile cellular subscriptions - World Telecommunication/ICT Indicators Database. International Telecommunication Union. [Online]. Available: http://www.itu.int/ITU-D/ict/statistics/material/excel/2010/Global_mobile_cellular_00-10.xls

[5] D. Tse and P. Viswanath, *Fundamentals of wireless communication*. New York, NY, USA: Cambridge University Press, 2005.

[6] W. C. Jakes, Jr., *Microwave Mobile Communications*. New York: Wiley, 1974.

[7] M. Abramowitz and I. Stegun, *Handbook of mathematical functions*. New York: Dover Publications Inc., 1970.

[8] M. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. Wiley-IEEE Press, Nov. 2004.

[9] K. Stewart, G. Labedz, and K. Sohrabi, "Wideband channel measurements at 900 MHz," in *IEEE 45th Vehicular Technology Conference*, vol. 1, Jul. 1995, pp. 236 –240.

[10] M. Nakagami, "The m-distribution, a general formula for intensity distribution of rapid fading," *Statistical Methods in RadioWave Propagation*, pp. 3–36, 1960.

[11] M. Yacoub, "The κ- μ distribution and the η-μ distribution," *IEEE Antennas Propag. Mag.*, vol. 49, no. 1, pp. 68 –81, Feb. 2007.

[12] M. D. Yacoub, "The α- μ distribution: A physical fading model for the stacy distribution," *IEEE Trans. Veh. Technol.*, vol. 56, no. 1, pp. 27 –34, Jan. 2007.

[13] C. Tarhini and T. Chahed, "On capacity of OFDMA-based IEEE802.16 WiMAX including Adaptive Modulation and Coding (AMC) and inter-cell interference," in *15th IEEE Workshop on Local Metropolitan Area Networks, 2007.*, Jun. 2007, pp. 139 –144.

[14] M. Alasti, B. Neekzad, J. Hui, and R. Vannithamby, "Quality of service in WiMAX and LTE networks," *IEEE Commun. Mag.*, vol. 48, no. 5, pp. 104 –111, May 2010.

[15] J. G. Proakis, *Digital Communications*, 4th ed. McGraw-Hill, 2001.

[16] L. Zheng and D. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 5, pp. 1073 – 1096, May 2003.

[17] Z. Wang and G. Giannakis, "A simple and general parameterization quantifying performance in fading channels," *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1389 – 1398, Aug. 2003.

[18] J. Laneman, G. Wornell, and D. Tse, "An efficient protocol for realizing cooperative diversity in wireless networks," in *Proc. IEEE International Symposium on Information Theory*, 2001, p. 294.

[19] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity. part I. system description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927 – 1938, Nov. 2003.

[20] ——, "User cooperation diversity. part II. implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939 – 1948, Nov. 2003.

[21] A. Nosratinia, T. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 74 – 80, Oct. 2004.

[22] T. Cover and A. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. 25, no. 5, pp. 572 – 584, Sep. 1979.

[23] P. Anghel and M. Kaveh, "Exact symbol error probability of a cooperative network in a Rayleigh-fading environment," *IEEE Trans. Wireless Commun.*, vol. 3, no. 5, pp. 1416 – 1421, Sep. 2004.

[24] M. Hasna and M.-S. Alouini, "A performance study of dual-hop transmissions with fixed gain relays," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1963 – 1968, nov. 2004.

[25] J. Laneman, "Cooperative diversity in wireless networks:algorithms and architectures," Ph.D. dissertation, MASSACHUSETTS INSTITUTE OF TECHNOLOGY, Cambridge, MA., Sep. 2002.

[26] P. Herhold, E. Zimmermann, and G. Fettweis, "A simple cooperative extension to wireless relaying," in *International Zurich Seminar on Communications*, 2004, pp. 36 – 39.

[27] D. Chen and J. Laneman, "Modulation and demodulation for cooperative diversity in wireless systems," *IEEE Trans. Wireless Commun.*, vol. 5, no. 7, pp. 1785 –1794, Jul. 2006.

[28] Z. Yi and I.-M. Kim, "Diversity order analysis of the decode-and-forward cooperative networks with relay selection," *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1792 –1799, May 2008.

[29] J. Laneman and G. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415 – 2425, Oct. 2003.

[30] A. Bletsas, A. Khisti, D. Reed, and A. Lippman, "A simple cooperative diversity method based on network path selection," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 659 – 672, Mar. 2006.

[31] J. Hu and N. Beaulieui, "Performance analysis of Decode-and-Forward relaying with selection combining," *IEEE Commun. Lett.*, vol. 11, no. 6, pp. 489 –491, Jun. 2007.

[32] Y. Zhao, R. Adve, and T. Lim, "Improving amplify-and-forward relay networks: optimal power allocation versus selection," *IEEE Trans. Commun.*, vol. 6, no. 8, pp. 3114 –3123, Aug. 2007.

[33] Y. Jing and H. Jafarkhani, "Single and multiple relay selection schemes and their achievable diversity orders," *IEEE Trans. Commun.*, vol. 8, no. 3, pp. 1414 –1423, Mar. 2009.

[34] I. Krikidis, J. Thompson, S. McLaughlin, and N. Goertz, "Amplify-and-forward with partial relay selection," *IEEE Commun. Lett.*, vol. 12, no. 4, pp. 235 –237, Apr. 2008.

[35] D. Michalopoulos, G. Karagiannidis, T. Tsiftsis, and R. Mallild, "Wlc41-1: An optimized user selection method for cooperative diversity systems," in *IEEE Global Telecommunications Conference (GLOBECOM), 2006*, Dec. 2006, pp. 1 –6.

[36] S. Ikki and M. Ahmed, "Performance analysis of generalized selection combining for amplify-and-forward cooperative-diversity networks," in *IEEE International Conference on Communications (ICC), 2009*, Jun. 2009, pp. 1 –6.

[37] K. Loa, C.-C. Wu, S.-T. Sheu, Y. Yuan, M. Chion, D. Huo, and L. Xu, "IMT-advanced relay standards [WiMAX/LTE Update]," *IEEE Commun. Mag.*, vol. 48, no. 8, pp. 40 –48, Aug. 2010.

[38] A. Bletsas, H. Shin, and M. Z. Win, "Outage optimality of opportunistic amplify-and-forward relaying," *IEEE Commun. Lett.*, vol. vol.11, pp. 261–263, Mar. 2007.

[39] Y. Zhao, R. Adve, and T. J. Lim, "Symbol error rate of selection amplify-and-forward relay systems," *IEEE Commun. Lett.*, vol. 10, no. 11, pp. 757 –759, Nov. 2006.

[40] B. Maham and A. Hjørungnes, "Performance analysis of amplify-and-forward opportunistic relaying in Rician fading," *IEEE Signal Process. Lett.*, vol. 16, pp. 643–646, Aug. 2009.

[41] S. Ikki and M. H. Ahmed, "Performance analysis of cooperative diversity wireless networks over Nakagami-m fading channel," *IEEE Commun. Lett.*, vol. 11, pp. 334–336, Apr. 2007.

[42] M. Torabi, W. Ajib, and D. Haccoun, "Performance analysis of amplify-and-forward cooperative networks with relay selection over Rayleigh fading channels," in *Proc. IEEE Vehicular Technology Conference (VTC) Spring, 2009*, Barcelona, Spain, Apr. 2009, pp. 1–5.

[43] D. S. Suraweera, A. Michalopoulos and G. K. Karagiannidis, "Semi-blind amplify-and-forward with partial relay selection," *Electron. Lett.*, vol. 45, pp. 317–318, Mar. 2009.

[44] D. da Costa and S. Aissa, "End-to-end performance of dual-hop semi-blind relaying systems with partial relay selection," *IEEE Trans. Wireless Commun.*, vol. 8, pp. 4306–4315, Aug. 2009.

[45] H. Ding, J. Ge, D. B. da Costa, and Z. Jiang, "Diversity and coding gains of fixed-gain amplify-and-forward with partial relay selection in Nakagami-m fading," *IEEE Commun. Lett.*, vol. 14, pp. 734–736, Aug. 2010.

[46] H. Suraweera, M. Soysa, T. Tellambura, and H. Garg, "Performance analysis of partial relay selection with feedback delay," *IEEE Signal Process. Lett.*, vol. 17, pp. 531–534, Jun. 2010.

[47] D. S. Michalapoulos, H. A. Suraweera, G. K. Karagiannidis, and R. Schober, "Relay selection with outdated channel estimates," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM), 2010*, Dec. 2010, pp. 1936 –1941.

[48] J. Vicario, A. Bel, J. A. Lopez-Salcedo, and G. Seco, "Opportunistic relay selection with outdated CSI: Outage probability and diversity analysis," *IEEE Trans. Wirless Commun.*, vol. vol. 8, pp. 2872–2876, Jun. 2009.

[49] F. Xu, F. Lau, and D.-W. Yue, "Diversity order for amplify-and-forward dual-hop systems with fixed-gain relay under Nakagami fading channels," *IEEE Trans. Commun.*, vol. 9, no. 1, pp. 92 –98, Jan. 2010.

[50] A. Erdelyi, W. Magnus, F. Oberhettinger, and F. Tricomi, *Tables of Integral Transforms*, ser. Bateman Manuscript Project, Ed. California Institute of Technology. McGraw-Hill Book Company, Inc., 1954, vol. 1.

[51] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series and Products*, 6th ed. Academic Press, Inc., 2000.

[52] A. H. Nuttall, "Some integrals involving the Q_M function," *IEEE Trans. Inf. Theory*, pp. 95–96, Jan. 1975.

[53] S. Ikki and M. H. Ahmed, "On the analysis of the nth best-relay cooperative-diversity networks," *IEEE Trans. Commun.*, Accepted.

[54] J. Tang and X. Zhang, "Transmit selection diversity with maximal-ratio combining for multicarrier DS-CDMA wireless networks over Nakagami-m fading channels," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 1, pp. 104 – 112, Jan. 2006.

[55] W. Mo and Z. Wang, "Average symbol error probability and outage probability analysis for general cooperative diversity system at high signal to noise ratio," in *Proc. Conference on Information Sciences and Systems*, Princeton, NJ, Mar. 2004.

[56] R. Radaydeh, "Impact of delayed arbitrary transmit antenna selection on the performance of rectangular QAM with receive MRC in fading channels," *IEEE Commun. Lett.*, vol. 13, no. 6, pp. 390 –392, Jun. 2009.