A Study on the Harmonic Contributions of Residential Houses (V1.0)

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Abstract—This paper presents a measurement technique to determine the harmonic sources and impedances of residential houses at the utility metering point. The results are then applied to quantify a residential house's harmonic voltage and current contributions. Five residential houses are investigated using the proposed method. Characteristics of the load-side harmonic impedances and sources are studied and their harmonic contributions are determined. The results show that voltage distortions are mainly affected by background harmonic sources inside the supply system. The current harmonics are affected by both the residential loads and the supply system.

Index Terms—Harmonic impedance, harmonic source, harmonics, power quality, residential load

I. INTRODUCTION

Single-phase commercial and residential loads are one of the main sources of harmonic distortions in low voltage distribution systems. Domestic appliances such as TV sets, computers and fluorescent lamps can generate significant harmonics in comparison to their sizes. Although the harmonic currents produced by individual appliances are negligible, the cumulative effect produced by a large number of small harmonic sources can be substantial. For example reference [1] showed that when many people watch sport events on TVs the harmonic distortion level could increase by as high as 100%. With this proliferation of harmonic producing appliances, utility companies have started to contemplate methods to evaluate and manage the ever-increasing harmonic distortion levels in modern power distribution systems.

One of the first steps to address this concern is to determine the actual harmonic contributions of residential houses through field measurements. The results will help utilities to understand the relative significance of an individual house on the harmonic distortion level at the utility metering point. For this purpose, a practical and easy to implement harmonic source measurement technique has been developed and showed in this paper. The harmonic contributions of the sources are then determined. The technique has been applied to five residential houses and a lot of interesting results have been obtained. This paper presents the findings. The paper is organized as follows: Section II presents the problem description and proposed harmonic customer impedance measurement model and technique. Implementation issues are also discussed in this section. Section III proposes a new model and approach for quantifying the harmonic contributions of both customer and utility side. Simulation study is used to demonstrate the effectiveness of the proposed technique in determining customer side impedances in sections IV. Finally section V presents case studies. In this section, the proposed techniques and models are applied over measured data. Finally, conclusions are drawn in section VI.

II. THE PROBLEM DESCRIPTION AND PROPOSED METHOD

In North America, residential and commercial customers are usually supplied through three-wire single-phase distribution transformers [8]. Loads are supplied with a neutral and two hot phases carrying 120V with respect to the neutral and 240V with respect to themselves as shown in Figure 1. To determine the harmonic contributions of the customer and the supply at the revenue meter point, the harmonic impedance and sources contained in both customer and utility systems should be determined. In the previous paper [7], authors proposed a method to determine the harmonic impedance and sources of the supply side. The focus of current paper is to determine the model parameters for the customer side.

![Figure 1: Power distribution system for three-wire single-phase feeding systems](image)

The proposed measurement method is to switch a portable capacitor at the utility meter point as shown in Figure 2. The resulting disturbances can be used to estimate both the load side and the system side parameters. To determine the load...
side parameters, the current \( I_{\text{load-side}} \) downstream from the capacitor was used, as shown in Figure 2. In a general sense, load impedance can be calculated by the ratio

\[
Z_{\text{load}} = \frac{\Delta V}{\Delta I_{\text{load side}}},
\]

(1)

where \( \Delta V \) and \( \Delta I_{\text{load-side}} \) represent the subtraction of the voltage and the currents, respectively, before and after the capacitor switching. In practical cases, only the system and capacitor currents can be measured; as a result, the load side current is calculated from the measured system and capacitor currents:

\[
I_{\text{load side}} = I_{\text{system side}} - I_{\text{cap}}.
\]

(2)

In order to overcome this difficulty, a new equivalent model for the two-branch load has been developed and is presented in Figure 4(a). \( Z_3 \) and \( I_{c3} \) can be separated into two parts, in which

\[
Z_4 = \frac{V_a \text{pre}}{V_a \text{pre} + V_b \text{pre}} Z_b \quad \text{and} \quad Z_5 = \frac{V_b \text{pre}}{V_a \text{pre} + V_b \text{pre}} Z_c.
\]

The load impedances for this system can be represented by two equivalent phase-to-neutral impedances, \( Z_a \) and \( Z_b \), which can be calculated using (3).

\[
\begin{align*}
Z_a &= Z_4 \parallel Z_4, \\
Z_b &= Z_2 \parallel Z_5
\end{align*}
\]

(3)

In this way, the impedance estimation is performed for the two branches simultaneously. The results are two equivalent impedances \( (Z_a, Z_b) \) and two equivalent current sources \( (I_{c_a}, I_{c_b}) \). Equivalent impedances can be determined by using the \( \Delta V/\Delta I \) ratio, which is obtained from a capacitor switching, as is shown in equation (4):

\[
Z_{a-h} = \frac{V_a \text{pre}_h - V_a \text{pre}_p}{I_a \text{pre}_h - I_a \text{pre}_p} = \Delta \frac{V_h}{I_h},
\]

(4)

where \( V_{\text{pre}} \) and \( I_{\text{pre}} \) represent the voltage and current phasors before the capacitor is switched; conversely, \( V_{\text{post}} \) and \( I_{\text{post}} \) are obtained after the switching. The subscript \( h \) represents the harmonic order. The current source equivalent \( I_{c_a} \) can be calculated as follows:

\[
I_{c_a-h} = \frac{V_{a \text{pre}_h}}{Z_{a-h}} - I_{a \text{pre}_h},
\]

(5)

where \( V_{a \text{pre}_h} \) and \( I_{a \text{pre}_h} \) are the voltage and current calculated at the interface point. The branch \( b \) impedance and current source, \( Z_b \) and \( I_{c_b} \), are similarly calculated.


### B. Implementation Issues

The branch currents used in load-side impedance calculations must be obtained by subtracting the capacitor current from the actual measured current at the utility side. This subtraction can propagate measurement errors and eventually affect the impedance calculation. The set of data used in this paper was based on multi-channel measurements; therefore, all the channels had to be sampled synchronously. Due to hardware costs and constraints, however, a common data-acquisition practice is to sample the channels sequentially. Since the acquired signals (currents and voltages) are recorded sequentially, a time delay occurs that produces what is called a skewing error. Load impedance results are significantly sensitive to this time delay. Several ways to correct this error are discussed in the literature [9-11]. For instance, one way is to increase the number of points per cycle and then align the signals to be subtracted [12]. Depending on the number of points increased, this approach may be computationally inefficient. The method for skewing-error correction used here is based on the principle of linear interpolation. Assume that the first channel is the reference channel to which the values of all the other channels will be aligned; then, the corrected value of the $4^{th}$ channel can be calculated according to Figure 5 and Equation (6): 

\[
x_{nc}(k+1) = \frac{n - 1}{N} x_n(k) + \frac{N - (n - 1)}{N} x_r(k + 1)
\]

where subscript $c$ indicates the corrected value. Equation (6) shows that the corrected value is the weighted summation of two adjacent values. Since $x_r(k)$ is closer to the reference time, it has a weighting factor of 3/4. $x_c(k+1)$ is 3ΔT away from the reference time. Its weighting factor is 1/4. The general correction formula for the $N$ channel sequential interval sampling scheme is

\[
x_{nc}(k+1) = \frac{n - 1}{N} x_n(k) + \frac{N - (n - 1)}{N} x_r(k + 1), \quad (7)
\]

where $N$ is the total number of channels, and $n$ is the specific channel number. For the $1^{st}$ channel, $n=1$ and $x_c(k+1)=x_r(k+1)$, the corrected value equals the sampled value. Figure 6 shows how this error was corrected for the branch currents at each side of the interface point and the error generated by the subtraction of the capacitor current.

### III. Harmonic Contributions

Harmonic contribution of supply or load is defined as the portion of the PCC harmonic currents or voltages that are caused by the supply or load harmonic sources. For example, if a PCC’s $5^{th}$ harmonic current is 5A, the harmonic source in
the supply system may contribute to 1A and the customer load may contribute to 4A. So the supply system contributes 20% and the load contributes to 80%. In some cases, the harmonic contributions of the supply and the load may cancel each other that cause negative contribution of one of them. A rigorous method to determine the harmonic contribution has been established in [2]. This method is mainly designed for single-phase two-wire systems. To handle this situation, this section first proposes a model to convert single-phase three-wire systems to an equivalent single-phase two-wire system. Thereafter, the procedure of determining the harmonic contributions is presented.

A. Equivalent Single-Phase Voltages and Currents

Utility companies supply three-wire single-phase loads by using three-winding single-phase distribution transformers as shown in Figure 7. Harmonic voltages and currents in secondary side of the transformer for both phase ‘a’ and ‘b’ are not exactly equal especially in high harmonic orders. The unbalanced voltage between phases is caused by the unbalanced currents of the loads, while the no-load voltages in both phases are equal.

![Figure 7: Three-winding single-phase distribution transformer (shown in per-unit)](image)

Neglecting the impedances of wires connecting load to the transformer, equation (8) can be derived.

\[
\begin{align*}
E &= V_{a_{pcc}} + Z_{1s} \cdot I_{a_{pcc}} \\
E &= V_{b_{pcc}} + Z_{1s} \cdot I_{b_{pcc}}
\end{align*}
\]  

(8)

Using (8), the leakage impedance in secondary side is achieved as shown in (9).

\[
Z_{1s} = \frac{V_{a_{pcc}} - V_{b_{pcc}}}{I_{a_{pcc}} - I_{b_{pcc}}}
\]

(9)

The transformer coupling voltage and currents can be simply calculated as in (10).

\[
\begin{align*}
E &= \frac{V_{a_{pcc}} + V_{b_{pcc}}}{2} + \frac{Z_{1s}}{2} \cdot (I_{a_{pcc}} + I_{b_{pcc}}) \\
I'_{p} &= I_{a_{pcc}} + I_{b_{pcc}}
\end{align*}
\]

(10)

The main drawback of observing the customer from the transformer coupling point is that the impedance of the secondary side of the transformer is considered as a part of the customer-side impedance. However, this impedance should be considered as a part of the utility-side impedance for harmonic contribution studies. Referring to Figure 8, we want to calculate equivalent transformer impedance and remove it from the equivalent single phase load.

![Figure 8: Equivalent single-phase model of the two-phase system](image)

The idea is to use the power loss balance to calculate the equivalent impedance of the transformer. The power dissipation in \(Z_{eq-T}\) should equalize the total power dissipations in the secondary side of the transformer. So we have:

\[
Z_{1s} \{ |I_{a_{pcc}}|^2 + |I_{b_{pcc}}|^2 \} = Z_{eq-T} |I'_{p}|^2
\]

(11)

For the circuit shown in Fig. 2 we can write:

\[
\begin{align*}
V_{eq} &= E - Z_{eq-T} \times I'_{p} \\
I_{eq} &= I'_{p} = I_{a_{pcc}} + I_{b_{pcc}}
\end{align*}
\]

(12)

Substituting \(E\) from (10) and \(Z_{eq-T}\) from (11) into (12), we can achieve the equivalent voltage on the customer side.

\[
V_{eq} = \frac{V_{a_{pcc}} + V_{b_{pcc}} + Z_{1s} \cdot I'_{p} \times |U_{t}|^2}{2 - \frac{Z_{eq-T} \cdot |U_{t}|^2}{4}}
\]

(13)

where \(U_{t}\) is the current unbalance. The unbalance is defined in (14) where \(X\) can be replaced with \(V\) or \(I\).

\[
U_{s} = \frac{X_{a} - X_{b}}{(X_{a} + X_{b})/2}
\]

(14)

Replacing \(Z_{1s}\) from (9) into (13), we can get the following notable equations.

\[
\begin{align*}
V_{eq} &= \frac{V_{a_{pcc}} + V_{b_{pcc}}}{2} \left( 1 + \frac{U_{V}}{U_{I}} \times \frac{|U_{t}|^2}{4} \right) \\
I_{eq} &= I_{a_{pcc}} + I_{b_{pcc}}
\end{align*}
\]

(15)

Equation (15) is a two-phase to single-phase transformation that converts voltages and currents of a two-phase system into their equivalent single-phase voltage and current. It is seen that if the unbalance in voltage or current is zero, the equivalent voltage will be the average of voltages for the two phases, which is expected for a balanced load.

B. Determination of Harmonic Contributions

Using the equivalent circuit model of the supply and load shown in Figure 9, the harmonic contributions can be determined using the following procedure:

1. The contribution of the utility side to the PCC currents and voltages can be determined according to Figure 9(a).
The results are phasor currents \((I_{a_{pc}}\text{ and } I_{b_{pc}})\) and voltages \((V_{a_{pc}}\text{ and } V_{b_{pc}})\).

2. Similarly the contribution of the customer side to the PCC currents and voltages can be determined according to Figure 9(b). The result are phasor currents \((I_{c_{pc}}\text{ and } I_{d_{pc}})\) and voltages \((V_{c_{pc}}\text{ and } V_{d_{pc}})\).

3. Using (15), converts currents and voltages of phase \(a\) and \(b\) into their single-phase equivalents \((I_{eq_{pc}}\text{ and } V_{eq_{pc}})\).

4. Convert utility side contribution currents and voltages to their single-phase equivalents \((I_{eq_{u}}\text{ and } V_{eq_{u}})\).

5. Convert load side contribution currents and voltages to their single-phase equivalent \((I_{eq_{c}}\text{ and } V_{eq_{c}})\).

6. Using the scalar contribution index propose in [2], the harmonic contribution of each side can be determined.

IV. SIMULATION RESULTS

The circuit presented in Figure 10 was simulated. The supply system was represented by two voltage sources of 120V per branch, which were connected to the load side through branch impedances of \((0.062+j0.03)\Omega\). The load side was composed of the impedances \(Z_a = (1.18+j0.754)\Omega\), \(Z_b = (0.73+j1.508)\Omega\) and \(Z_1 = (5.0+j1.885)\Omega\). Fifth-order harmonic sources were intentionally added at each load in order to have some harmonic background. 5.5 A were injected into each branch and another source of 10A was connected in parallel with \(Z_1\).

A 1200 \(\mu\text{F}\) was switched after the fifth cycle of the simulation. In Figure 11 and Figure 12, the recorded waveform and the harmonic spectra of the voltage and currents of each branch are presented. As expected, only the fundamental and fifth harmonic components are present. Table 1 shows the equivalent impedance results obtained for these frequency components. These results agree closely with the expected values.

V. FIELD MEASUREMENT RESULTS

The proposed harmonic impedance and source measurement techniques have been applied to four residential houses. (change five to four in the previous sections) The impedances and harmonic sources obtained from the have been examined. These measurements consist of several capacitor switching events. Each test site involves 20 to 60 capacitor switching events, called snapshots. Table 2 summarizes the characteristics of the measured residential sites. Old and new houses with different sizes are included in this study.

<table>
<thead>
<tr>
<th>House ID</th>
<th>Year Built</th>
<th>Square Footage (ft²)</th>
<th>No. of Inhabitants</th>
<th>THD Voltage [%]</th>
<th>THD Current [%]</th>
<th>Load [A] during test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE 1: SIMULATION LOAD IMPEDANCE RESULTS

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>Fundamental</th>
<th>5th harmonic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Z_a_{eq}) [(\Omega)]</td>
<td>(Z_b_{eq}) [(\Omega)]</td>
</tr>
<tr>
<td>Expected</td>
<td>0.8115+j0.4403</td>
<td>0.7439+j1.885</td>
</tr>
<tr>
<td>PSCAD® simulation</td>
<td>0.8156+j0.4343</td>
<td>0.7473+j1.8217</td>
</tr>
</tbody>
</table>

TABLE 2: CHARACTERISTICS OF THE MEASURED HOUSES
A. Measuring Load Harmonic Impedances

Site #4 is selected to evaluate load-side harmonic impedances. 30 snapshots were taken (15 switching events for each branch). The fundamental impedance was calculated by using \( \frac{\Delta V}{\Delta I} \). Figure 13 shows the results obtained for this particular site. Since each measured field case includes multiple snapshots, multiple estimated impedance values were obtained. The \( \Delta V/\Delta I \) results agree for the snapshots where the capacitor was switched on, because the energy level on the switched branch was much higher than on the non-switched one. The results show consistent results for the fundamental component for the analyzed site.

B. Characteristics of Load Harmonic Impedances

Figure 15 shows the average resistance and (absolute) reactance values of measured sites as a function of harmonic frequency. This figure reveals that the resistive component dominates the load-side impedance. Both \( R \) and |\( X | \) increase slightly with the frequency. At the fundamental frequency, the load impedances are much higher and, so one cannot use the fundamental frequency impedance to predict the harmonic impedance of the loads. In Figure 15, the load impedances are also compared with the average system impedance. The results show that the load impedances are higher than the system impedances, at least to the 13\(^{th} \) harmonic.

Figure 14: Sample measured load impedances.

The harmonic characteristic of the measured residential sites are presented in the following figures. Figure 16 presents the impedance results (\( R \) and \( X \)) for phase \( a \) of houses. The resistance component shows a fairly constant trend for the harmonic orders. On the other hand, the reactance increases with the frequency. Harmonic impedances mainly present a capacitive characteristic. This result is not surprising since the capacitors in the power electronic-based loads have been previously found to create such a characteristic [13]. Figure 17 presents the impedance results for phase \( b \) of measured houses. Our results show that for most of the cases the harmonic impedances present a capacitive characteristic and harmonic resistance is fairly constant. In addition, for most of sites, harmonic resistances are bigger than harmonic reactances.
C. Characteristics of the Harmonic Sources

Based on the impedances of the system and load sites, we can estimate the harmonic sources inside the system and the load. The results will help to identify the causes of harmonic distortions, voltage and current, at the system-load interface points. System and customer side harmonic sources are determined for measurement sites from available impedance information. The harmonic source in supply side is modeled by a harmonic voltage source, $E_s$, and harmonic source in load side is modeled by a harmonic current source, $I_c$.

Table 3 presents the harmonic spectra of $V_{pcc}$ and $E_s$ for the studied cases. The results show that the patterns of $E_s$ and $V_{pcc}$ spectra are similar. It also confirms that $E_s$ has a major influence on $V_{pcc}$. The PCC voltage distortion is mainly caused by the harmonic voltage sources inside the system.

<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>3rd</th>
<th>5th</th>
<th>7th</th>
<th>9th</th>
<th>11th</th>
<th>13th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site #1</td>
<td>$V_{pcc}$</td>
<td>1.96</td>
<td>2.21</td>
<td>0.96</td>
<td>0.67</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>$E_s$</td>
<td>1.95</td>
<td>2.30</td>
<td>0.95</td>
<td>0.67</td>
<td>0.25</td>
</tr>
<tr>
<td>Site #2</td>
<td>$V_{pcc}$</td>
<td>2.25</td>
<td>2.32</td>
<td>0.66</td>
<td>0.72</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>$E_s$</td>
<td>2.21</td>
<td>2.27</td>
<td>0.63</td>
<td>0.71</td>
<td>0.21</td>
</tr>
<tr>
<td>Site #3</td>
<td>$V_{pcc}$</td>
<td>0.97</td>
<td>1.04</td>
<td>0.46</td>
<td>0.42</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>$E_s$</td>
<td>0.95</td>
<td>1.02</td>
<td>0.45</td>
<td>0.41</td>
<td>0.44</td>
</tr>
<tr>
<td>Site #4</td>
<td>$V_{pcc}$</td>
<td>1.47</td>
<td>2.07</td>
<td>0.32</td>
<td>0.81</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>$E_s$</td>
<td>1.39</td>
<td>1.96</td>
<td>0.26</td>
<td>0.8</td>
<td>0.27</td>
</tr>
</tbody>
</table>

The customer side sources are represented as current sources $I_c$. Table 4 presents the harmonic spectra of $I_{pcc}$ and $I_c$ for the studied cases. Again, the similarities between $I_{pcc}$ and $I_c$ are noticeable. The results reveal that the patterns of $I_c$ and $I_{pcc}$ are similar. However, this similarity cannot confirm that $I_{pcc}$ is mainly affected by $I_c$ since they represent the most common spectra patterns seen in a power distribution system; however, the harmonic currents recorded at the revenue meter point are not entirely caused by the nonlinear loads inside the customer’s facility.

<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>3rd</th>
<th>5th</th>
<th>7th</th>
<th>9th</th>
<th>11th</th>
<th>13th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site #1</td>
<td>$I_{pcc}$</td>
<td>1.12</td>
<td>0.8</td>
<td>0.39</td>
<td>0.14</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>$I_c$</td>
<td>8.06</td>
<td>7.7</td>
<td>4.01</td>
<td>2.45</td>
<td>0.89</td>
</tr>
<tr>
<td>Site #2</td>
<td>$I_{pcc}$</td>
<td>1.2</td>
<td>0.81</td>
<td>0.53</td>
<td>0.4</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>$I_c$</td>
<td>3.44</td>
<td>3.01</td>
<td>1.11</td>
<td>0.65</td>
<td>0.39</td>
</tr>
<tr>
<td>Site #3</td>
<td>$I_{pcc}$</td>
<td>2.01</td>
<td>1.33</td>
<td>0.57</td>
<td>0.05</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>$I_c$</td>
<td>1.92</td>
<td>1.41</td>
<td>0.88</td>
<td>0.5</td>
<td>0.32</td>
</tr>
<tr>
<td>Site #4</td>
<td>$I_{pcc}$</td>
<td>1.7</td>
<td>1.12</td>
<td>0.42</td>
<td>0.03</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>$I_c$</td>
<td>3.22</td>
<td>1.6</td>
<td>0.63</td>
<td>0.77</td>
<td>0.24</td>
</tr>
</tbody>
</table>

D. Harmonic Contribution

The results of harmonic sources have revealed some interesting characteristics of the harmonic distortions at the point of common coupling. This section will investigate the characteristics further by quantifying the harmonic contributions of the system and customer side sources. Error! Reference source not found. shows the average harmonic contribution results obtained for studied cases. The voltage contribution chart reveals that the utility side is responsible for almost 100% of the fundamental frequency voltage. This agrees well with reality that utility side supplies the load. For the 3rd harmonic voltage, the utility side contributes about 91% while the customer side contributes about 9%. For the other harmonics, the customer side contribution is always less than 17%. This fact indicates that utility side is the main contributor for the harmonic voltage. The current contribution chart can be understood similarly. For the fundamental frequency, the customer side has negative contribution. It implies that there are probably motors in customer side. At 3rd and 13th harmonic...
order, the harmonic contribution of customer side is almost 17% and 41%, respectively. The remaining portions are contributed by the utility side. However for the other harmonic orders such as 5th, 7th and etc, the utility side has negative contributions. This shows that utility side cancels out some portion of harmonic currents that are injected by customer resulting in less harmonic currents in the PCC.

![Harmonic Contribution Chart](image)

**VI. CONCLUSION**

This paper proposed a model and a method to estimate the load-side harmonic impedances and sources for single-phase two wire residential houses. Combined with the system side parameters, harmonic contributions of four residential houses have been determined. The studied cases show that the load impedances are higher than the system impedances, at least to the 13th harmonic. Some of the houses present a capacitive impedance characteristic. The harmonic voltage at the metering point is mainly caused by background harmonic sources inside the system. The utility side sources help to cancel out the harmonic currents injected by residential houses. (confirm if this conclusion is correct and revise accordingly if not)

![Figure 18: Average Harmonic Contribution of studied cases](image)

**VII. REFERENCES:**


A Study on the Harmonic Contributions of Residential Houses (Final Version)

Hooman E. Mazin, Student Member, IEEE, Edwin E. Nino, Member, IEEE, Wilsun Xu, Fellow, IEEE, and Jing Yong, Member, IEEE

Abstract—This paper presents a measurement technique to determine the harmonic sources and impedances of residential houses at the utility metering point. The results are then applied to quantify the harmonic and current contributions of residential premises. Four residential houses are investigated by using the proposed method. The characteristics of the load-side harmonic impedances and sources are studied, and their harmonic contributions are determined. The results show that voltage distortion is affected mainly by background harmonic sources that exist within the supply system. The current harmonics are affected by both the residential loads and the supply system.

Index Terms—Harmonic impedance, harmonic source, harmonics, power quality, residential load

I. INTRODUCTION

Single-phase commercial and residential loads are one of the main sources of harmonic distortion in low-voltage distribution systems. Despite their small sizes, domestic appliances such as TV sets, computers and fluorescent lamps can generate significant harmonic currents. Although the harmonic currents produced by individual appliances are negligible, the cumulative effect produced by a large number of small harmonic sources can be substantial. For example, reference [1] found that when many people watched sports events on TVs, the harmonic distortion level increased by as much as 100%. In response to the proliferation of harmonic-producing appliances, utility companies have started to contemplate methods to evaluate and manage the ever-increasing harmonic distortion levels in modern power distribution systems.

One of the first steps to address this concern is to determine the actual harmonic contributions of residential houses through field measurements. The results will help utilities to understand the relative significance of an individual house on the harmonic distortion level at the utility metering point. For this purpose, a practical and easy-to-implement harmonic source measurement technique has been developed and is presented in this paper. The harmonic contributions of the sources are determined. The technique is applied to four dwellings, and their harmonic impedances are determined, and the loads’ harmonic impedances are characterized. The characteristics of the harmonic sources and the harmonic contribution of both the utility and customer sides are also studied. The paper is organized as follows: Section II presents the problem description and proposed harmonic customer impedance measurement model and technique. Implementation issues are also discussed in this section. Section III proposes a new model and approach for quantifying the voltage and current harmonic contributions for three-wire single-phase systems. In Section IV, a simulation study is used to demonstrate the effectiveness of the proposed technique in determining customer-side impedances. The proposed techniques and models are applied to some case studies in Section V. Finally, conclusions are drawn in Section VI.

II. THE PROBLEM DESCRIPTION AND PROPOSED METHOD

In North America, residential and commercial customers are usually supplied through three-wire single-phase distribution transformers [2]. Loads are supplied with a neutral and two hot phases carrying 120V with respect to the neutral and 240V with respect to themselves, as shown in Fig. 1. To determine the harmonic contributions of the customer and the supply at the revenue meter point, the harmonic impedance and sources contained in both the customer and utility systems should be determined. In a previous paper [3], the authors proposed a method to determine the harmonic impedance and sources of the supply side. The focus of the current paper is to determine the model parameters for the customer side.

Fig. 1: Power distribution system for three-wire single-phase feeding systems
The proposed measurement method is to switch a portable capacitor at the utility meter point, as shown in Fig. 2. The resulting disturbances can be used to estimate both the load-side and the system-side parameters. The capacitor size should be carefully selected to produce a sufficient disturbance for both the utility and customer sides. The criterion for capacitor selection is explained in Section II.A of [3]. To determine the load-side parameters, the current (I_{load-side}) downstream from the capacitor can be used, as shown in Fig. 2. In a general sense, the load impedance can be calculated by using the ratio shown in (1):

\[
Z_{load} = \frac{\Delta V}{\Delta I_{load-side}} \quad (1)
\]

where \(\Delta V\) and \(\Delta I_{load-side}\) respectively, represent the difference between the voltages and currents before and after the capacitor switching. In practical cases, only the system and capacitor currents can be measured; as a result, the load-side current is calculated from the measured system and capacitor currents, as shown in (2).

\[
I_{load-side} = I_{system-side} - I_{cap} \quad (2)
\]

The load impedances for this system can be represented by two equivalent phase-to-neutral impedances, \(Z_a\) and \(Z_b\), which can be calculated by using following equations.

\[
Z_a = Z_1 PZ_4 \quad (5)
\]

\[
Z_b = Z_2 PZ_5 \quad (6)
\]

### A. Equivalent Circuit for Three-Wire Single-Phase Load

Although the above impedance measurement technique is simple in theory, its application to the three-wire single-phase loads is a challenging task, since not all measurements of the current are available to directly estimate the impedances in question. To determine the harmonic contributions of the customer and the supply at the revenue meter point, the equivalent circuit model shown in Fig. 3 is proposed. In this model, there are harmonic sources on both sides. On the load side, appliances are supplied with phase-to-neutral 120V (modeled by \(Z_6, Ic_1\) and \(Z_2, Ic_2\)) and phase-to-phase 240V (modeled by \(Z_5, Ic_3\)). The above circuit does not show the impedance of the neutral conductor. It can be shown that the effect of neutral impedance can be included in the above circuit without any need to change the structure of the proposed circuit. In practical cases, voltage and current measurements are available only at the utility-interface point, so that only the phase currents \(I_a\) and \(I_b\) can be accessed, but not the currents flowing through the impedances \(Z_1, Z_2\) and \(Z_3\). This problem increases the complexity of estimating the customer-side load impedances. In order to overcome this difficulty, a new equivalent model for the two-branch load has been developed and is presented in Fig. 4(a). \(Z_3\) and \(Ic_3\) can be separated into two parts, as shown in Fig. 4.

\[
Z_4 = \frac{V_a}{V_a + V_b} Z_3 \quad (3)
\]

\[
Z_5 = \frac{V_b}{V_a + V_b} Z_3 \quad (4)
\]
switching, as is shown in (7):

$$Z_{a-h} = \frac{V_{a_{pcc-h-post}} - V_{a_{pcc-h-pre}}}{I_{a_{pcc-h-post}} - I_{a_{pcc-h-pre}}} = \frac{\Delta V_h}{\Delta I_h},$$

(7)

where the subscripts ‘pre’ and ‘post’ relate to the phasors of the harmonic voltage and current before and after the harmonic switching. The subscript $h$ represents the harmonic order. The current source equivalent $I_{c_{a}}$ can be calculated as follows:

$$I_{c_{a-h}} = \frac{V_{a_{pcc-h}} - I_{a_{pcc-h}}}{Z_{a-h}},$$

(8)

where $V_{a_{pcc-h}}$ and $I_{a_{pcc-h}}$ are the voltage and current calculated at the interface point. The branch $b$ impedance and current source, $Z_{b}$ and $I_{c_{b}}$, are similarly calculated.

B. Implementation Issues

The branch currents used in load-side impedance calculations must be obtained by subtracting the capacitor current from the actual measured current on the utility side. This subtraction can propagate measurement errors and eventually affect the impedance calculation. The set of data used in this paper was based on multi-channel measurements; therefore, all the channels had to be sampled synchronously. Due to hardware costs and constraints, however, a common data-acquisition practice is to sample the channels sequentially. Since the acquired signals (currents and voltages) are recorded sequentially, a time delay occurs that produces what is called a skewing error. The load impedance results are significantly sensitive to this time delay. Several ways to correct this error are discussed in the literature [4-5]. For instance, one way is to increase the number of points per cycle and then align the signals to be subtracted [6]. Depending on the number of points increased, this approach may be computationally inefficient. The method for skewing-error correction used here is based on the principle of linear interpolation. Assume that the first channel is the reference channel to which the values of all the other channels will be aligned; then, the corrected value of the $5^{th}$ channel can be calculated according to Eq. 5 and Equation (9):

$$x_{c_{5}}(k+1) = \frac{x_{5}(k+1) - x_{5}(k)}{5T} (5\Delta T - 4\Delta T) + x_{5}(k),$$

(9)

where the subscript $c$ indicates the corrected value. Equation (10) shows that the corrected value is the weighted summation of the two adjacent values. Since $x_{5}(k)$ is closer to the reference time, it has a weighting factor of 4/5. $x_{5}(k+1)$ is $4\Delta T$ away from the reference time. Its weighting factor is 1/5. The general correction formula for the $N$ channel sequential interval sampling scheme is

$$x_{c_{n}}(k+1) = \frac{n-1}{N} x_{n}(k) + \frac{N-(n-1)}{N} x_{n}(k+1),$$

(10)

where $N$ is the total number of channels, and $n$ is the specific channel number. For the 1$^{st}$ channel, $n=1$ and $x_{1}(k+1) = x_{1}(k+1)$, and the corrected value equals the sampled value.

Fig. 5: Correction of skewing error for the sequential interval sampling scheme.

Fig. 6 shows how this error was corrected for the load current in phase $b$ before and after applying skewing-error correction method. $I_{b_{dir}}$ is the load current (in phase $b$) directly measured at load side and $I_{b_{sub}}$ is the load current calculated by subtracting the capacitor current from the utility current. Ideally the difference ($I_{b_{error}}$) should be zero, but in practice because of skewering error it is not zero. Fig. 6(b) shows how the proposed correction method can reduce the aforementioned error.

III. HARMONIC CONTRIBUTIONS

The harmonic contribution of the supply or load is defined as the portion of the PCC harmonic currents or voltages that are caused by the supply or load harmonic sources. For example, if a PCC’s 5$^{th}$ harmonic current is 5A, the harmonic source in the supply system may contribute to 1A, and the customer load may contribute to 4A, so the supply system
contributes 20% and the load contributes 80%. In some cases, the harmonic contributions of the supply and the load may cancel out each other and cause one of them to have a negative contribution. A rigorous method to determine the harmonic contribution was established in [7]. This method is designed mainly for single-phase two-wire systems. To handle this situation, this section first proposes a model to convert a single-phase three-wire load to an equivalent single-phase two-wire load, and then, the procedure for determining the harmonic contributions is presented.

A. Equivalent Single-Phase Voltages and Currents

Utility companies supply three-wire single-phase loads by using three-winding single-phase distribution transformers, as shown in Fig. 7. The harmonic voltages and currents on the secondary side of the transformer for both phase ‘a’ and ‘b’ are not exactly equal, especially for high harmonic orders. The unbalanced voltage between the phases is caused by the unbalanced currents of the loads, while the no-load voltages in both phases are equal.

![Fig. 7: Three-winding single-phase distribution transformer (per-unit representation)](image)

By neglecting the impedances of the wires connecting the load to the transformer, equations (11) and (12) can be derived.

\[ E = V_a p_c + Z_s I_a p_c \]  
(11)

\[ E = V_b p_c + Z_s I_b p_c \]  
(12)

By using these equations, the leakage impedance of the secondary side is achieved.

\[ Z_{ls} = \frac{V_a p_c - V_b p_c}{I_a p_c - I_b p_c} \]  
(13)

The transformer coupling voltage and current can be simply calculated as in (14) and (15).

\[ E = \frac{V_a p_c + V_b p_c}{2} + Z_{ls} \left( \frac{I_a p_c + I_b p_c}{2} \right) \]  
(14)

\[ I_p' = I_a p_c + I_b p_c \]  
(15)

The main drawback of observing the customer side from the transformer coupling point is that the impedance of the secondary side of the transformer is considered as a part of the customer-side impedance. However for harmonic contribution studies, this impedance should be considered as a part of the utility-side impedance. Referring to Fig. 8, it is required to calculate equivalent transformer impedance and remove it from the equivalent single-phase load.

The idea is to use the power-loss balance to calculate the equivalent impedance of the transformer. The active and reactive power dissipation in \( Z_{eq-T} \) should equalize the active and reactive power dissipations in the secondary side of the transformer, so we have

\[ Z_{ls} \left( \left| I_a p_c \right|^2 + \left| I_b p_c \right|^2 \right) = Z_{eq-T} \left| I_p' \right|^2. \]  
(16)

For the circuit shown in Fig. 8, it can be written:

\[ V_{eq} = E - Z_{eq-T} \times I_p' \]  
(17)

\[ I_{eq} = I_p' = I_a p_c + I_b p_c \]  
(18)

By substituting \( E \) from (14) and \( Z_{eq-T} \) from (16) into (17), the equivalent voltage on the customer side can be achieved.

\[ V_{eq} = \frac{V_a p_c + V_b p_c - Z_s I_p'}{2} \times \frac{\left| U_I \right|^2}{4} \]  
(19)

where \( U_I \) is the current unbalance. The unbalance is defined in (20), where \( X \) can be replaced with \( V \) or \( I \).

\[ U_I = \frac{X_a - X_b}{(X_a + X_b)/2} \]  
(20)

![Fig. 8: Equivalent single-phase model of the two-phase system](image)

By putting \( Z_{ls} \) from (13) into (19), the following notable equations are derived.

\[ V_{eq} = \frac{V_a p_c + V_b p_c}{2} \times \left( 1 + \frac{U_V}{U_I} \times \frac{\left| U_I \right|^2}{4} \right) \]  
(21)

\[ I_{eq} = I_a p_c + I_b p_c \]  
(22)

Equations (21) and (22) represent a two-phase to single-phase transformation that converts the voltages and currents of a two-phase system into their equivalent single-phase voltage and current. If the unbalance in the voltage or current is zero, the equivalent voltage will be the average of the voltages for the two phases. This result is expected for a balanced load. However, the proposed equivalent load model was originally developed for the cases that the distribution transformer supplies one load. Its application is not only limited to this case. Replacing \( U_I \) and \( U_V \) in (21), this equation can be
rewritten as follows:

\[ V_{eq} = \frac{V_{a_{pcc}} I_{a_{pcc}} + V_{b_{pcc}} I_{b_{pcc}}}{I_{eq}} \]  

It is interesting to note that by using (23), it can simply be verified that \( V_{eq} I_{eq} = (V_{a_{pcc}} I_{a_{pcc}} + V_{b_{pcc}} I_{b_{pcc}}) \). In another words, the input power (active and reactive) of the three-wire single-phase load equals to the input power of its single-phase equivalent counterpart. This fact can broaden the application of the proposed equivalent load. For example, in a case that a transformer feeds two or more three-wire single-phase residential loads each load can be modeled to its single-phase equivalent model by using (21) and (22) with guaranteeing that the input power of the original and equivalent load are the same.

B. Determination of Harmonic Contributions

By using the equivalent circuit model of the supply and load shown in Fig. 9, the harmonic contribution of both utility and customer can be determined using following procedure:

1. To find contribution of the utility side to the PCC currents and voltages, remove harmonic current sources in the customer side (\( I_{c_u} \) and \( I_{c_b} \)), as shown in Fig. 9(a). The obtained phasor currents (\( I_{a_{pcc}} \) and \( I_{b_{pcc}} \)) and the voltages (\( V_{a_{pcc}} \) and \( V_{b_{pcc}} \)) are only generated by harmonic sources inside the utility side (\( E_s_a \) and \( E_s_b \)).

2. Similarly, the contribution of the customer side to the PCC currents and voltages can be determined by removing (shout circuiting) the harmonic voltage sources (\( E_s_a \) and \( E_s_b \)) in the utility side, as shown in Fig. 9(b). The obtained phasor currents (\( I_{a_{pcc}} \) and \( I_{b_{pcc}} \)) and the voltages (\( V_{a_{pcc}} \) and \( V_{b_{pcc}} \)) are only generated by harmonic current sources inside the customer side (\( I_{c_u} \) and \( I_{c_b} \)).

3. Using (18) and (19), convert currents and voltages of phase \( a \) and \( b \) into their single-phase equivalents (\( I_{eq_{pcc}} \) and \( V_{eq_{pcc}} \)).

4. Convert the utility-side contribution currents and voltages to their single-phase equivalents (\( I_{eq_{u_{pcc}}} \) and \( V_{eq_{u_{pcc}}} \)).

5. Convert the load-side contribution currents and voltages to their single-phase equivalent (\( I_{eq_{c_{pcc}}} \) and \( V_{eq_{c_{pcc}}} \)).

6. \( I_{eq_{u_{pcc}}} \) and \( I_{eq_{c_{pcc}}} \) are phasors and \( I_{eq_{pcc}} \) is also a phasor, which make it difficult to use simple percentage numbers to quantify the contribution level. To solve this problem, [7] proposed a scalar contribution index. Referring to Fig. 10, \( I_{eq_{pcc}} \) is the phasor addition of \( I_{eq_{u_{pcc}}} \) and \( I_{eq_{c_{pcc}}} \). The projections of \( I_{eq_{u_{pcc}}} \) and \( I_{eq_{c_{pcc}}} \) onto the phasor \( I_{eq_{pcc}} \), \( I_{eq_{u_{pcc}}} \) and \( I_{eq_{c_{pcc}}} \), are the components of interest. Based on the projected quantities, the contribution of each party can be defined using the following equations:

\[ \text{Current Contribution}_{utility} = \frac{I_{eq_{u_{pcc}}}}{I_{eq_{pcc}}} \times 100\% \]  

\[ \text{Current Contribution}_{customer} = \frac{I_{eq_{c_{pcc}}}}{I_{eq_{pcc}}} \times 100\% \]  

7. Similar approach should be done to find voltage contributions.

The circuit presented in Fig. 11 was simulated by using PSCAD/EMTDC®. The supply system was represented by two voltage sources of 120V per branch, which were connected to the load side through branch impedances of \( (0.062+j0.03)\Omega \). The load side was comprised of the impedances \( Z_1 = (1.18+j0.754)\Omega \), \( Z_2 = (0.73+j1.508)\Omega \) and \( Z_3 = (5.0+j1.885)\Omega \). Fifth-order harmonic sources were intentionally added at each load in order to have some harmonic background. 5.5A were injected into each branch, and another source of 10A was connected in parallel with \( Z_3 \).

A 1200 \( \mu \)F was switched after the second cycle of the simulation. The recorded waveforms are shown in Fig. 12. Before capacitor switching, the PCC harmonic voltages in both phase \( a \) and \( b \) are measured, as presented in Table 1. By using these voltages and (3) and (4), Z4 and Z5 are calculated for each harmonic order.
multiple snapshots, multiple estimated impedance values were obtained. The \( \Delta V/\Delta I \) results agree for the snapshots where the capacitor was switched on, because the energy level on the switched branch was much higher than on the non-switched one. The results show consistent results for the fundamental component for the analyzed site.

![Fig. 12: Branch currents and voltage waveforms](image)

**Table 1: The PCC Voltages Before Capacitor Switching**

<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>( V_{a_{pcc}} ) [V]</th>
<th>( V_{b_{pcc}} ) [V]</th>
<th>( Z_a ) [Ω]</th>
<th>( Z_b ) [Ω]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>112.3+j1.732</td>
<td>112.3+j1.732</td>
<td>2.50+j0.94</td>
<td>2.49+j0.94</td>
</tr>
<tr>
<td>5</td>
<td>-0.2759+j1.737</td>
<td>-0.276+j1.737</td>
<td>2.50+j4.712</td>
<td>2.49+j4.712</td>
</tr>
</tbody>
</table>

By using (5) and (6), \( Z_a \) and \( Z_b \) are calculated. These theoretically expected values are presented in Table 2. By using (7) and the harmonic voltages and currents before and after capacitor switching, the harmonic impedances are estimated. These results agree closely with the expected values.

**Table 2: Simulation Load Impedance Results**

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>( Z_a ) [Ω]</th>
<th>( Z_b ) [Ω]</th>
<th>( Z_{a_{eff}} ) [Ω]</th>
<th>( Z_{b_{eff}} ) [Ω]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental</td>
<td>Expected 0.8156+0.4343</td>
<td>0.7473+0.8217</td>
<td>0.8115+j0.4403</td>
<td>2.50+j0.94</td>
</tr>
<tr>
<td></td>
<td>Estimated 0.8156+j0.4343</td>
<td>0.7473+j0.8217</td>
<td>0.8115+j0.4403</td>
<td>2.50+j0.94</td>
</tr>
<tr>
<td>5th Harmonic</td>
<td>Expected 0.7439+0.816</td>
<td>2.50+4.712</td>
<td>2.50+j4.712</td>
<td>2.50+j4.712</td>
</tr>
<tr>
<td></td>
<td>Estimated 0.7439+0.816</td>
<td>2.50+4.712</td>
<td>2.50+j4.712</td>
<td>2.50+j4.712</td>
</tr>
</tbody>
</table>

**V. Field Measurement Results**

The proposed harmonic impedance and source measurement techniques were applied to four residential houses. The impedances and harmonic sources obtained from them were examined. These measurements consisted of several capacitor switching events. Each test site involved 20 to 60 capacitor-switching events, called snapshots. Table 3 summarizes the characteristics of the measured residential sites. Old and new houses of different sizes are included in this study.

**Table 3: Characteristics of the Measured Houses**

<table>
<thead>
<tr>
<th>House ID</th>
<th>Year Built</th>
<th>Square Footage [ft²]</th>
<th>No. of Inhabitants</th>
<th>THD Voltage [%]</th>
<th>THD Current [%]</th>
<th>Load [A] during test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1976</td>
<td>1000</td>
<td>2</td>
<td>3.2</td>
<td>21.2</td>
<td>7.0</td>
</tr>
<tr>
<td>2</td>
<td>1997</td>
<td>2000</td>
<td>4</td>
<td>1.6</td>
<td>27.4</td>
<td>6.1</td>
</tr>
<tr>
<td>3</td>
<td>2003</td>
<td>1800</td>
<td>2</td>
<td>3.4</td>
<td>29.2</td>
<td>8.9</td>
</tr>
<tr>
<td>4</td>
<td>1985</td>
<td>1200</td>
<td>4</td>
<td>2.7</td>
<td>19.6</td>
<td>10.8</td>
</tr>
</tbody>
</table>

**A. Measuring Load Harmonic Impedances**

Site #4 was selected to evaluate the load-side harmonic impedances. 30 snapshots were taken (15 switching events for each branch). The fundamental impedance was calculated by using \( \Delta V/\Delta I \). Fig. 13 shows the results obtained for this particular site. Since each measured field case included multiple snapshots, multiple estimated impedance values were obtained. The \( \Delta V/\Delta I \) results agree for the snapshots where the capacitor was switched on, because the energy level on the switched branch was much higher than on the non-switched one. The results show consistent results for the fundamental component for the analyzed site.

![Fig. 13: Load side fundamental impedance, comparison results using V/I and \( \Delta V/\Delta I \) (a) Branch a (b) Branch b](image)

Fig. 14 presents the average harmonic impedance data (average of phase a and b) obtained for the measured site #4. All harmonic orders have consistent results. In comparison to system-side impedance determination, the estimated load impedances are much more scattered, due mainly to the energy level available for load-side impedance determination. Because the system side had much smaller impedances, most of the capacitor-switching transients traveled to the system side instead of the load side. Thus, the determination of load impedance was much more challenging, and few snapshots yielded acceptable results. Therefore, the estimated load impedances are less reliable than the estimated system impedances.

![Fig. 14: Harmonic load-side impedance of site #4](image)
B. Characteristics of Load Harmonic Impedances

Fig. 15 shows the frequency-dependent impedance (average of phase $a$ and $b$) of site #4 for both the system and load sides. In the system side, the reactance increases (almost linearly) by the harmonic order, and the resistance is fairly constant. In contrast, the fundamental frequency of the load-side impedance is much higher than that of the harmonic impedances, so one cannot use the fundamental frequency impedance to predict the harmonic impedance of the loads. A comparison of the load-side harmonic impedances with the system-side harmonic impedances reveals that the load impedances are larger than the system impedances, at least up to the 13th harmonic.

![Figure 15: The frequency-dependent impedance of site #4](image)

(a) System side (b) Load side

The harmonic characteristic of the measured residential sites are presented in the following figures.

Fig. 16 presents the impedance results ($R$ and $X$) for phase $a$ of the houses. The resistance component shows a fairly constant trend for the harmonic orders. On the other hand, the reactance increases with the frequency. The harmonic impedances present mainly a capacitive characteristic. This result is not surprising since the capacitors in the power electronic-based loads have been previously found to create such a characteristic [8]. Fig. 17 presents the impedance results for phase $b$ of the measured houses. The results show that for most of the cases, the harmonic impedances present a capacitive characteristic, and the harmonic resistance is fairly constant. In addition, for most of the sites, the harmonic resistances are greater than the harmonic reactances.

![Figure 16: Load impedance results for phase a (a) Resistance, (b) Reactance](image)

(b) Site #1

(a) Site #2

R (ohms)

Fig. 17: Load impedance results for phase $b$ (a) Resistance, (b) Reactance

C. Characteristics of the Harmonic Sources

Using the impedances of the system and load sites, the harmonic sources inside the system and the load were estimated. The results helped to identify the causes of the harmonic distortion at the system-load interface points. The system- and customer-side harmonic sources were determined for measurement sites from the available impedance information. The harmonic source on the supply side is modeled by using a harmonic voltage source, $E_s$, and the harmonic source on the load side is modeled by using a harmonic current source, $I_c$. Table 4 presents the harmonic spectra of the average $V_{pcc}$ and $E_s$ for the studied cases. The results show that the patterns of $E_s$ and $V_{pcc}$ spectra are similar and also confirm that $E_s$ has a major influence on $V_{pcc}$. The PCC voltage distortion is caused mainly by the harmonic voltage sources inside the system.

<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>$3^{rd}$</th>
<th>$5^{th}$</th>
<th>$7^{th}$</th>
<th>$9^{th}$</th>
<th>$11^{th}$</th>
<th>$13^{th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site #1 $V_{pcc}$</td>
<td>1.96</td>
<td>2.21</td>
<td>0.96</td>
<td>0.67</td>
<td>0.67</td>
<td>0.25</td>
</tr>
<tr>
<td>$E_s$</td>
<td>1.95</td>
<td>2.2</td>
<td>0.95</td>
<td>0.67</td>
<td>0.67</td>
<td>0.25</td>
</tr>
<tr>
<td>Site #2 $V_{pcc}$</td>
<td>2.25</td>
<td>2.32</td>
<td>0.66</td>
<td>0.72</td>
<td>0.72</td>
<td>0.33</td>
</tr>
<tr>
<td>$E_s$</td>
<td>2.21</td>
<td>2.27</td>
<td>0.63</td>
<td>0.71</td>
<td>0.71</td>
<td>0.32</td>
</tr>
<tr>
<td>Site #3 $V_{pcc}$</td>
<td>0.97</td>
<td>1.04</td>
<td>0.46</td>
<td>0.42</td>
<td>0.42</td>
<td>0.15</td>
</tr>
<tr>
<td>$E_s$</td>
<td>0.95</td>
<td>1.02</td>
<td>0.45</td>
<td>0.41</td>
<td>0.41</td>
<td>0.15</td>
</tr>
<tr>
<td>Site #4 $V_{pcc}$</td>
<td>1.43</td>
<td>2.07</td>
<td>0.32</td>
<td>0.81</td>
<td>0.81</td>
<td>0.24</td>
</tr>
<tr>
<td>$E_s$</td>
<td>1.39</td>
<td>1.96</td>
<td>0.26</td>
<td>0.8</td>
<td>0.27</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Table 4: Harmonic Spectra of Average $V_{pcc}$ and $E_s$ for Studied Cases
The customer-side sources are represented as the current sources $I_c$. Table 5 presents the harmonic spectra of the average $I_{pcc}$ and $I_c$ for the studied cases. As Table 5 reveals, the harmonic current at the point of common coupling ($I_{pcc}$) is usually smaller than the harmonic current generated by the load harmonic source ($I_c$). One reason for this result is that a portion of $I_c$ is absorbed by the load impedance $Z_l$. Another reason is that the harmonic currents generated by the load harmonic sources are partially cancelled out by the utility harmonic sources. This cancellation is explained in the next section.

<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>$3^{rd}$</th>
<th>$5^{th}$</th>
<th>$7^{th}$</th>
<th>$9^{th}$</th>
<th>$11^{th}$</th>
<th>$13^{th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site #1 $I_{pcc}$</td>
<td>1.12</td>
<td>0.8</td>
<td>0.39</td>
<td>0.14</td>
<td>0.12</td>
<td>0.18</td>
</tr>
<tr>
<td>Site #1 $I_c$</td>
<td>8.06</td>
<td>7.7</td>
<td>4.01</td>
<td>2.45</td>
<td>0.89</td>
<td>1.07</td>
</tr>
<tr>
<td>Site #2 $I_{pcc}$</td>
<td>1.2</td>
<td>0.81</td>
<td>0.53</td>
<td>0.4</td>
<td>0.24</td>
<td>0.09</td>
</tr>
<tr>
<td>Site #2 $I_c$</td>
<td>3.44</td>
<td>3.01</td>
<td>1.11</td>
<td>0.65</td>
<td>0.39</td>
<td>0.47</td>
</tr>
<tr>
<td>Site #3 $I_{pcc}$</td>
<td>2.01</td>
<td>1.33</td>
<td>0.57</td>
<td>0.05</td>
<td>0.19</td>
<td>0.24</td>
</tr>
<tr>
<td>Site #3 $I_c$</td>
<td>1.92</td>
<td>1.41</td>
<td>0.88</td>
<td>0.5</td>
<td>0.32</td>
<td>0.41</td>
</tr>
<tr>
<td>Site #4 $I_{pcc}$</td>
<td>1.7</td>
<td>1.12</td>
<td>0.42</td>
<td>0.03</td>
<td>0.17</td>
<td>0.1</td>
</tr>
<tr>
<td>Site #4 $I_c$</td>
<td>3.22</td>
<td>1.6</td>
<td>0.63</td>
<td>0.77</td>
<td>0.24</td>
<td>0.26</td>
</tr>
</tbody>
</table>

**D. Harmonic Contribution**

The results for the harmonic sources have revealed some interesting characteristics of the harmonic distortion at the point of common coupling. This section will investigate the characteristics further by quantifying the harmonic contributions of the system- and customer-side sources. Figure 18 shows the average harmonic contribution results obtained for the studied cases.

<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>Customer</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^{rd}$</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>$5^{th}$</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>$7^{th}$</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$9^{th}$</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$11^{th}$</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>$13^{th}$</td>
<td>0.003</td>
<td>0.003</td>
</tr>
</tbody>
</table>

**Fig. 18: Average Harmonic Contribution of studied cases**
(a) Voltage contribution (b) Current contribution

The voltage-contribution chart reveals that the utility side is responsible for almost 100% of the fundamental frequency voltage. This result agrees well with the reality that the utility side supplies the load. For the $3^{rd}$ harmonic voltage, the utility side contributes about 91%, while the customer side contributes about 9%. For the other harmonics, the customer-side contribution is always less than 17%. This fact indicates that utility side is the main contributor of the harmonic voltage. The current-contribution chart can be understood similarly. For the fundamental frequency, the customer side has a negative contribution. This result implies that there are probably motors on the customer side. At the $3^{rd}$ and $13^{th}$ harmonic orders, the harmonic contribution of the customer side is almost 17% and 41%, respectively. The remaining portions are contributed by the utility side. However, for the other harmonic orders such as the $5^{th}$ and $7^{th}$, the utility side has negative contributions. This finding shows that the utility side cancels out some portion of the harmonic currents that are injected by the customer and results in less harmonic currents in the PCC.

**VI. CONCLUSION**

This paper proposed a model and a method to estimate the load-side harmonic impedances and sources for single-phase two-wire residential houses. By using the system-side parameters, the harmonic contributions of four residential houses were determined. The studied cases show that the load impedances are higher than the system impedances, at least up to the $13^{th}$ harmonic order. Some of the houses presented a capacitive-impedance characteristic. The harmonic voltage at the metering point is caused mainly by the background harmonic sources within the supply system. In most of the cases, the utility-side sources helped to cancel out the harmonic currents injected by the residential houses.

**VII. REFERENCES**


