A Network Decoupling Transform for Phasor Data Based Voltage Stability Analysis and Monitoring

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Abstract—It is well known that a power network can be represented as a multi-node, multi-branch Thevenin circuit connecting loads to generators. This paper shows that eigen-decomposition can be performed on the Thevenin impedance matrix, creating a set of decoupled single-node, single-branch equivalent circuits. The decoupled circuits can reveal important characteristics of a power system. By applying the transform to calculated or measured voltage phasor data, a technique for tracking the modes of voltage collapse and for identifying areas vulnerable to voltage collapse has been developed. Case studies conducted on multiple power systems have confirmed the effectiveness of the proposed method. In addition to voltage stability applications, the proposed transform presents a new approach for processing and interpreting multi-location phasor data.

Index Terms—Eigen-analysis, Thevenin circuit, Voltage stability.

I. INTRODUCTION

Due to the rapid advancement of measurement and telecommunication technologies, a large amount of data are available nowadays for power system monitoring and control. One example is the synchronized phasor data [1]. Various research works have been conducted to develop applications for the phasor data [2]-[4]. For example, reference [5] has documented the latest attempts to create situation awareness for power system planners and operators using the wide-area phasor data.

The challenges of extracting new and unique information from the phasor data may be partially due to the lack of a support theory for phasor data processing and interpretation. This situation may be understood by examining the use of three-phase voltage phasor data, $V_a, V_b,$ and $V_c$. One can process the data in various ways. However, operations such as $(V_a+V_b+V_c)/3$ $(=V_{zero-sequence})$ or $(V_a+a^2V_b+aV_c)/3$ $(=V_{negative-sequence})$ (where $a=\exp(2\pi i/3)$) have been recognized as the best means to analyze the data. The symmetrical components transform is the support theory for these operations. Because of the theory, converting abc phasors to 012 sequences has become a standard approach to process three-phase phasor data and a number of monitoring and protection schemes have been developed. The wide-area monitoring systems nowadays have made multi-location (positive sequence) voltage phasor data, $V_1, V_2, V_3 \ldots V_n$ available. Inspired by the success of the symmetrical components theory, one would wonder if operations such as $T_1V_1+T_2V_2+\ldots+T_nV_n$ can be rigorously derived for the multi-location (i.e. multi-bus) phasor data, and if such operations can reveal unique characteristics of a power system.

This paper shows that such a transform can be derived. The transform is able to extract important information about a power system from the phasor data. The transform is conceived from the following observation: a power network can be represented as a multi-node, multi-branch Thevenin circuit connecting the loads to the generators. If one applies eigen-decomposition on the Thevenin impedance matrix, the network can be decoupled into a set of single-node, single-branch equivalent circuits. These circuits are much easier to analyze and they carry valuable information of a power system. Similarly, if the variations of the transformed variables can be evaluated, one may be able to predict the complex behaviors of the actual network.

This paper presents the basic theory and characteristics of the proposed transform. The transform is independent of the origin of the phasor data. The data can be either calculated from load flow programs or measured by phasor measurement units. In this paper, the theory is applied to voltage stability analysis and monitoring. Algorithms for voltage instability mode identification are developed. The proposed application is a significant improvement over the Jacobian matrix based voltage stability modal analysis technique [6]-[10].

II. PROPOSED NETWORK DECOUPLING TRANSFORM

A general power system is shown in Fig. 1. This system consists of $n$ loads and $m$ generators. There are many transmission lines and other components such as transformers inside the network. This network (in positive sequence form) can be represented using a generalized multi-node, multi-branch Thevenin equivalent circuit model. The equivalent circuit has the following form:

\[
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n
\end{bmatrix} =
\begin{bmatrix}
k_1 & k_{12} & \cdots & k_{1n} \\
k_{21} & \ddots & \cdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
k_{n1} & k_{n2} & \cdots & k_{nn}
\end{bmatrix}
\begin{bmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n
\end{bmatrix}
\begin{bmatrix}
Z_{11} & Z_{12} & \cdots & Z_{1n} \\
Z_{21} & \ddots & \cdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
Z_{n1} & Z_{n2} & \cdots & Z_{nn}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n
\end{bmatrix}
\]

Or

\[
[V] = [K][E] - [Z][I] = [E'] - [Z][I]
\]

In this model, $[E]$ is the terminal voltages or the internal voltages (if a generator’s $Q_{\text{max}}$ is reached) of the generators and $[V]$ is the nodal voltages at the load buses. Note that...
\[ E^\prime = [T][E] \] is the open circuit voltage vector of the traditional multi-node Thevenin equivalent circuit. A general circuit model for the representation of (1) is shown in Fig. 2.

Eigen-decomposition can be performed on the \([Z]\) matrix of the Thevenin circuit as follows:

\[ [Z] = [T]^\dagger [\Lambda] [T] \]  

(2)

where \([\Lambda]\) and \([T]\) are the eigenvalue and eigenvector matrices of \([Z]\), respectively. Applying the above to (1) yields

\[ [V] = [K][E] - [Z][I] = [K][E] - [T]^\dagger [\Lambda] [T][I] \]

\[ [T][V] = [T][K][E] - [\Lambda][T][I] \]

(3)

Denote \([U] = [T][V]\) as the transformed voltage \([J] = [T][I]\) as the transformed current \([F] = [T][K][E]\) as the transformed voltage source.

This leads to the following decoupled (modal) networks whose circuit representations are shown in Fig. 3.

\[
\begin{bmatrix}
U_1 \\
U_2 \\
\vdots \\
U_n
\end{bmatrix} = \begin{bmatrix}
F_1 & 0 & 0 & 0 \\
0 & F_2 & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & F_n
\end{bmatrix} \begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\vdots \\
\lambda_n
\end{bmatrix} \begin{bmatrix}
J_1 \\
J_2 \\
\vdots \\
J_n
\end{bmatrix}
\]

(4)

The significance of the above transform is the following: a complex network has been transformed into a set of decoupled simple one-source, one-load networks. By analyzing the characteristics of individual decoupled networks, one may extract important information about the actual network.

The above transform is essentially one form of modal (eigen) decomposition that has been widely used in linear system analysis [10]. In the power system field alone, at least three modal transforms have been developed: (1) modal analysis on the state matrix \([A]\) of a linearized dynamic power system [11], (2) modal analysis on the load flow Jacobian matrix [6], and (3) modal analysis on the impedance matrix of multiphase transmission lines [12]. In fact, the symmetrical components transform is also a form of modal analysis. In order to avoid confusion with those transforms, we propose to use “Channel Components Transform (CCT)” to designate the transform of (2). The transformed domain is called the “Channel Domain”. This is based on the consideration that each modal circuit represents a virtual power flow channel or path (i.e. mode). A power network can be viewed as consisting of various virtual power flow channels (paths) defined by \([U]\), \([J]\) and \([F]\). A channel component represents one pattern of currents flowing in a network.

A very simple power system shown in Fig. 4 is used to illustrate the meanings of the channel components. If \(Z_a = Z_b = Z_c\), the system can be transformed to

\[
\begin{bmatrix}
3Z + Z_a & 0 & 0 \\
0 & Z_a & 0 \\
0 & 0 & Z_a
\end{bmatrix}, \text{and } \begin{bmatrix}
F \\
E
\end{bmatrix} = \begin{bmatrix}
\sqrt{3}E \\
0 \\
0
\end{bmatrix}
\]

The results reveal that only channel 1 has a voltage source, i.e. channel 1 is the only channel responsible for power transfer in this case. In fact, the channel impedance \(\lambda_1 = 3(Z + Z_a/3)\) represents the series connection of \(Z\) branch with that of a parallel combination of \(Z_a\) and \(Z_c\) branches. This is exactly what one will do when analyzing the circuit of Fig. 4 “on the back of an envelope”. There is no need to analyze the other channels. So, a four-branch power network has been simplified into a one-channel network.

If \(Z_a, Z_b,\) and \(Z_c\) are not equal, such as \(Z_a = 0.35, Z_b = 0.2, Z_c = 0.1, Z = 0.3, E = 1.0, S_a = 0.3, S_b = 0.25,\) and \(S_c = 0.4,\) the CCT yields the following parameters.

\[
\begin{bmatrix}
1.1165j & 0 & 0 \\
0 & 0.278j & 0 \\
0 & 0 & 0.1055j
\end{bmatrix}, \text{and } \begin{bmatrix}
F \\
E
\end{bmatrix} = \begin{bmatrix}
1.7164 \\
-0.1718 \\
0.1566
\end{bmatrix}
\]

It can be seen that all channels have non-zero source voltages. The channel impedances are also different. Since the power transfer capability of a line is proportional to \(P^2\) and inversely to \(X\), channel 1 is the mode responsible for transferring power to loads. Table I, which shows the channel currents in each branch of the system, reveals more information on how the power is transferred to the loads. Fig. 5 shows the patterns of channel currents in the system. In this figure, the thickness of an arrow on a branch is in proportion to the value of the channel current flowing through that branch. As seen in the table and figure, the Channel 1 currents are all in phase, flowing from the source to the loads. So this is the channel responsible for power transfer. The channel 2 current mostly flows from bus 3 to buses 4 and 5, and the channel 3 current
mostly flows from buses 3 and 4 to bus 5. These are mainly load-to-load power flows that don’t help transferring power from source to loads. In summary, the channel currents can help to identify the main paths (or patterns) of power transfer in a power system.

Each channel transfers different amount of power, defined as \( U_i \times \text{Conj}(J_i) \) at the receiving end. One can calculate the channel power at any given network operating points established by the load flow results. Fig. 6 shows the channel power levels of an actual 2038-bus power system. The results show that only a small number of channels are responsible for transferring the majority of power in a network. One may only need to analyze or monitor a small number of channels to understand the characteristics of a power system.

The computational procedure to form the equivalent circuit matrices is shown below. The standard node equations in the matrix notation are expressed as:

\[
[I] = [V][Y][V]
\]

where, \([V]\) is the bus voltages, \([I]\) is the net injected current, and \([Y]\) is the admittance matrix. All buses can be classified into three types: generator bus (G), load bus (L), and network bus (N) which has no generator or load. As a result, equation (5) can be partitioned as

\[
[I_G] = [Y_{GG} \ Y_{GL} \ Y_{GN} \ V_G]
\]

\[
[I_L] = [Y_{LG} \ Y_{LL} \ Y_{LN} \ V_L]
\]

\[
[I_N] = [Y_{NG} \ Y_{NL} \ Y_{NN} \ V_N]
\]

Since the net injected currents of the network buses are equal to zero, we can write

\[
I_N = Y_{NG}V_G + Y_{NL}V_L + Y_{NN}V_N = 0
\]

Or: \( V_N = -Y^{-1}_{NN}(Y_{NG}V_G + Y_{NL}V_L) \)

Substituting the obtained \( V_N \) in (6) will result in

\[
-I_L = Y_{LG}V_G + Y_{LL}V_L - Y_{LN}Y^{-1}_{NN}(Y_{NG}V_G + Y_{NL}V_L)
\]

Or: \( I_L = (Y_{LG} - Y_{LN}Y^{-1}_{NN}Y_{NG})V_G + (Y_{LL} - Y_{LN}Y^{-1}_{NN}Y_{NL})V_L \)

Rearranging (8) yields

\[
V_L = -(Y_{LL} - Y_{LN}Y^{-1}_{NN}Y_{NL})^{-1}(Y_{LG} - Y_{LN}Y^{-1}_{NN}Y_{NG})V_G
\]

Comparing (9) with (1) reveals that

\[
[K] = -(Y_{LL} - Y_{LN}Y^{-1}_{NN}Y_{NL})^{-1}(Y_{LG} - Y_{LN}Y^{-1}_{NN}Y_{NG})
\]

\[
[Z] = (Y_{LL} - Y_{LN}Y^{-1}_{NN}Y_{NL})^{-1}
\]

The above equation gives the expression of matrices \([K]\) and \([Z]\). Standard eigen-decomposition routines can then be applied to the \([Z]\) matrix.

### III. PV and P\(\delta\) Curves in Channel Domain

This paper presents the application of CCT to voltage stability analysis and monitoring. Maintaining voltage stability is a major objective in power system planning and operation. Although our understanding on the subject has increased significantly in recent years, new findings are still emerging, especially with the emergence of measured phasor data. Many research activities have been conducted on how to utilize the phase information for voltage stability monitoring [14]-[15]. The technique proposed in this paper attempts to help identifying weak areas in a planned or operating power system. Before presenting the complete algorithms, the characteristics of PV and P\(\delta\) curves in the channel domain are explained first.

The process to map PV curves to the channel domain is as follows: A PV curve calculation procedure such as the continuation power flow is applied to the study system. At each PV curve point obtained by this process, the CCT is applied to calculate the channel variables corresponding to that point. The process will produce a set of data points in the channel domain. The channel voltage versus channel power \((P-V)\) or channel power versus channel angle \((P-\delta)\) are plotted. The curves are called channel domain PV or P\(\delta\) curves.

#### A. Channel Domain PV Curve

Since the channel circuit is a very simple one-branch circuit, one can actually derive the entire PV curve in channel domain analytically, as follows:

For a channel shown in Fig. 7, the relationship between the
channel voltage \((U_i)\), and the channel load power \((P_i, Q_i)\) is given as
\[
U_i^4 + U_i^2[2(PR_i + QX_i) - F_i] + \lambda_i^2(P^2 + Q^2) = 0
\] (11)

Using (11), the channel voltage can be plotted for different active powers assuming a constant power factor for the channel load. The resulting curve will be the channel \(PV\) curve.

The channel \(PV\) curves and mapped physical \(PV\) curve points for the conceptual case shown in Fig. 4 when \(Z_a=0.35\), \(Z_b=0.2\), and \(Z_c=0.1\) is illustrated in Fig. 8. There are three groups of channel curves. Each group corresponds to one channel as this is a three-channel system. The stars are the \(PV\) curve points of the actual system mapped to the channel domain. The reason that each channel has a group of curves is due to the following phenomenon. Two variables, the source voltage and the branch impedance, define the shape of a \(PV\) curve for a single-branch network according to (11). In the channel domain, the channel impedance \(\lambda\) is constant but the source voltage \(F\) changes with the physical voltages of the system. Each channel \(PV\) curve therefore corresponds to different source voltage \(F\).

The results shown in Fig. 8 confirmed the observation earlier that each channel carries different amount of power. One can also notice that voltage collapse occurs when one of the channels (channel 1) reaches its maximum power transfer limit, which can be called the critical channel.

B. Channel Domain \(P\delta\) Curve

Voltage stability analysis is usually performed using the \(PV\) curves. But it can also be examined from the \(P\delta\) curves, at least for the single-load system. Since industry is increasingly interested in the angle information provided by the PMUs, it is useful to examine voltage instability from the \(P\delta\) curve. It can be shown that for the system of Fig. 7 where the receiving end voltage is uncontrolled, the \(\delta\) curve points of the actual system mapped to the channel domain. The reason that each channel has a group of curves is due to the following phenomenon. Two variables, the source voltage and the branch impedance, define the shape of a \(PV\) curve for a single-branch network according to (11). In the channel domain, the channel impedance \(\lambda\) is constant but the source voltage \(F\) changes with the physical voltages of the system. Each channel \(PV\) curve therefore corresponds to different source voltage \(F\).

The results shown in Fig. 8 confirmed the observation earlier that each channel carries different amount of power. One can also notice that voltage collapse occurs when one of the channels (channel 1) reaches its maximum power transfer limit, which can be called the critical channel.

Equation (17) is the analytical expression of the \(P\delta\) curve for the \(i^{th}\) channel. Since the channel circuits are decoupled, establishing \(P\delta\) curve for each channel is straightforward. Fig. 9 shows the channel \(P\delta\) curves of the same case study. As seen in this figure, when the actual system reaches the voltage collapse point, the \(P\delta\) curve of channel 1 (critical channel) reaches its maximum point. The maximum angle is about 50°. It can be shown that for the system of Fig. 7 where the receiving end voltage is uncontrolled, the \(\delta_{\text{max}}\) can be determined as
\[
\sin\delta_{\text{max}} = \sqrt{\frac{1 - \sin\theta}{2}}
\] (18)

which is always less than 90° due to the lack of reactive power support at the receiving end.

In summary, the channel \(P\delta\) curves can be used to interpret the voltage stability of a power system. Without the proposed transform, it is impossible to obtain any meaningful \(P\delta\) curves since there are many physical bus angles in an actual power system. Which bus angle differences to examine are difficult to determine.
IV. VOLTAGE STABILITY ANALYSIS USING CCT

The proposed Channel Components Transform is applied to voltage stability analysis in this section. A set of algorithms are proposed for identification of the critical channel and associated load buses called critical buses. The critical buses are those whose load demands cause voltage collapse.

A. Load coupling in channel domain

The advantage of CCT is that it can decouple the supply system into independent channel circuits. When it is applied to the loads (which are physically decoupled), however, the channel loads become coupled. This is because the CCT is a linear transform. A load (Z or S) is a nonlinear variable. This phenomenon can be further understood as follows: The physical loads may be modeled as constant power loads or variable impedance loads that produce the effects of constant power consumptions. Since it is easier to explain the concepts, the impedance load model is used. The physical load at various buses can be expressed as:

\[
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n
\end{bmatrix} = 
\begin{bmatrix}
Z_{i1} & 0 & \cdots & 0 \\
0 & Z_{i2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & Z_{in}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n
\end{bmatrix}
\]

Load:

\[
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n
\end{bmatrix} = 
\begin{bmatrix}
Z_{i1} & 0 & \cdots & 0 \\
0 & Z_{i2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & Z_{in}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n
\end{bmatrix}
\]

(19)

where, \( Z_{ij} \) satisfies load's power demand constraints. In the channel domain, the loads become:

\[
[Y] = [Z_i] \Rightarrow [U] = [T][Z_i][T]^{-1} \Rightarrow [U] = [Z_i][J]
\]

\[
\Rightarrow \text{Channel loads}:
\begin{bmatrix}
U_1 \\
U_2 \\
\vdots \\
U_n
\end{bmatrix} = 
\begin{bmatrix}
Z_{C11} & Z_{C12} & \cdots & Z_{C1n} \\
Z_{C21} & Z_{C22} & \cdots & Z_{C2n} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{Cn1} & Z_{Cn2} & \cdots & Z_{Cnn}
\end{bmatrix}
\begin{bmatrix}
J_1 \\
J_2 \\
\vdots \\
J_n
\end{bmatrix}
\]

(20)

The above equation shows that the loads are coupled in the channel domain. This will create difficulties in deriving useful information from the channel \( PV \) or \( PS \) curves. For example, the mapped channel operating point may reside below the channel \( PV \) curve, which makes it hard to determine the critical channel.

Based on the analysis presented in Appendix A, the coupling term of the \( i \)th channel load consists of the \( F_{ji}, \lambda_j \) components (where \( j \neq i \)). As a result, each channel load is actually a Norton circuit, which is shown in Fig. 10. For any given operating point, the coupling can be represented as a Norton current source. The current source is determined from:

\[
J_{Ei} = J_i - Y_{Cii} U_i
\]

(21)

where \( Y_{Cii} \) is the \( i \)th diagonal element of \( [Y_c] = [Z_c]^{-1} \). In other words, the coupling has been modeled as a current source \( J_{Ei} \). This current source representation is accurate for the operating point from which it is derived. This known current source is then merged into the supply channel circuit as shown in Fig. 11. This results in the change of the channel voltage source from \( F_i \) to \( F_{eq} = F_i - \lambda_i J_{Ei} \).

In summary, the coupling effects have been represented as a modification to the channel voltage source \( F_i \). This is an accurate model for the point from which the parameters are derived. It is an approximation for other points. Extensive case studies presented later have shown that this approximation works very well. One of the outcomes is that whenever a physical system reaches its nose point, one of the channels will approach its nose point as well. This channel becomes the critical channel. In summary, the proposed method to construct the final channel circuit is as follows:

Step 1: Calculate the load impedance for each physical bus, which is \( Z_{li} = V_i/J_i \).

Step 2: Obtain \([Y_c]\) using (22). Note that the computation is minimal as \([T]^{-1}\) is already available.

\[
[Y_c] = [Z_c]^{-1} = ([T][Z_c][T]^{-1})^{-1} = [T][Z_c]^{-1}[T]^{-1}
\]

(22)

Step 3: Calculate the current source \( J_{Ei} \) for each channel according to (21).

Step 4: Calculate the modified channel voltage source \( F_{eq} = F_i - \lambda_i J_{Ei} \) for each channel. The result is the equivalent channel circuit shown in Fig. 11.

With the above treatment, all channels are decoupled from each other. Therefore, it becomes easy to examine the \( PV \) curves from the channel domain. For this purpose, equation (11) can be applied to each of the equivalent channel circuits.

B. Identification of critical channel and critical buses

Among different channels of a system, there is a critical channel responsible for the voltage collapse. This channel can be easily identified based on the channel’s margin, i.e., the critical channel is the one that has the smallest channel margin. Since each channel resembles a two-bus system as shown in Fig. 11, the maximum channel power can be determined analytically as follows:

\[
S_{max} = \frac{\sqrt{\lambda_i^2 - (X_i \sin \theta_i + R_i \cos \theta_i)^2}}{2(X_i \cos \theta_i - R_i \sin \theta_i)}
\]

(23)

where \( \theta_i \) is the power factor angle of the \( i \)th channel load. The channel margin can then be calculated as follows:

\[
\text{Channel margin} = \frac{S_{max} - S_{operating}}{S_{operating}} \times 100
\]

(24)

where \( S_{operating} \) is the channel load power at the current operating condition.
Once the critical channel is known, one can use the information to identify the critical buses or to rank the buses. The method proposed for this task is as follows. Since the low stability margin of the critical channel corresponds to its large channel voltage drop, one can therefore infer that the critical bus is the one that is the most responsible for the voltage drop of the critical channel. Since the channel voltage drop is caused by the channel current, the contribution of bus currents $I$ to the critical channel current $J_i$ can be used to rank the contribution of different buses to the channel voltage drop. The critical channel current $J_i$ is as follows:

$$J_i = T_{ik}I_k + T_{i2}I_2 + \ldots + T_{in}I_n$$  \hspace{1cm} (25)

According to (25), the following contribution index is proposed:

$$\text{Cont}_{ik} = \left| T_{ik} \right| \cos(\alpha) \frac{I_k}{|I|}$$  \hspace{1cm} (26)

where $\text{Cont}_{ik}$ is the contribution of load bus $k$ to the channel current $J_i$, and $\alpha$ is the angle difference between the channel current $J_i$ and the term $T_{ik}I_k$. To determine the critical bus, the contributions of all load bus currents to the critical channel current are calculated using (26) and are ranked. The bus whose current has the highest contribution to the critical channel current is the critical bus.

C. Summary of the proposed algorithms

The CCT-based (off-line) voltage stability analysis technique is summarized in the flowchart of Fig. 12. As seen in this figure, the transformation matrix $[\Lambda]$ and the channel impedances $[\Lambda]$ are first calculated based on the network admittance matrix. These parameters will remain constant unless the network impedance or topology is changed (which includes generators reaching Q limits, see later). The $PV$ curve calculation procedure (which can be the conventional $PV$ curve method or the continuation power flow method) is then applied to the system. At each $PV$ curve point, the CCT is applied to calculate the channel variables and circuits corresponding to that point. The channel margins will be determined as well. The critical channel, and the critical loads are then identified. The generation of channel $PV$ or $P\delta$ curves are optional because channel margins are sufficient for the critical channel identifications. It can be seen that the proposed algorithms can be considered as a diagnostic tool for the $PV$ curves. It uses $PV$ curve results to extract more useful information about the system. This procedure is similar to that used by the Jacobian matrix modal analysis technique of [6].

If one of the generators’ reactive power limits is reached at a $PV$ curve point, that generator shall be represented as a constant excitation voltage behind its synchronous impedance. The terminal of the excitation voltage becomes a new $(PV)$ bus. Thus the network topology is changed. The network $Z$ matrix and CCT transform need to be recalculated accordingly.

The above procedure could be implemented online where each operating point is analyzed to determine the critical buses or areas associated with that operating point. The process is similar to that of Fig. 12 but blocks 3 and 7 are no longer needed. At any given time, the EMS will provide the network topology and voltage phasor data to the online CCT module. If there is a change of network topology, the transform matrix $[\Lambda]$ will be recomputed. Otherwise, the same matrix will be used. The voltage phasor data can come from the PMU directly or from the state-estimator. After performing calculations, the CCT module will output results such as the critical buses, the pattern of critical channel power flow and channel margin etc. This procedure works in theory but more research is needed in at least two areas. The first is to determine the most useful buses to install PMUs for tracking the top channels as it is not practical to measure the phasor voltages of all buses and generators. The second is how to consider generators’ hitting reactive power limits before they actually occur, as the procedure cannot predict their happening. Because of this limitation, the critical buses identified above are valid for the current condition only.

D. Case study results

The CCT-based technique has been applied to several systems including an actual large system. The results for the IEEE 57-bus system are presented first. The analyses are performed according to the procedure shown in Fig. 12. The WECC $PV$ curve methodology [16] is used to perform the $PV$ curve calculations.

Fig. 12. Procedure of the analysis.
Fig. 13 illustrates the channel margins for different load scaling factors obtained for IEEE 57-bus system. According to this figure, channel 1 is identified as the critical channel. Fig. 14 and Fig. 15 show the channel $PV$ and $P\delta$ curves, respectively. Only the top four channels are shown. Both of these figures clearly verify that channel 1 is indeed the critical channel because when the physical system reaches its nose point, the operating point in channel 1 approaches its $PV/P\delta$ curve nose point. It is interesting to note that the power transferred by channel 1 is comparable with those of channel 2 and 5.

The contribution of load bus currents to the critical channel current can be calculated using the proposed contribution index to rank the load buses. The results are shown in Fig. 16. Fig. 16 (a) shows the contributions of buses in the bar chart format. The bubble chart shown in Fig. 16(b) provides the same information in a more visualized way with the topology of the IEEE 57-bus system as the background. The size of the bubble on a load bus is in proportion to the value of the contribution of that bus. This bubble chart clearly shows the locations which are weak with respect to voltage stability. Bus 31 is the bus most far away from the generators relative to its load size. This bus has been ranked as the critical bus correctly.

The results obtained from an actual large system is shown in Figs. 17 to 20. It is interesting to note that both channels 384 and 18 have very small margins. However, channel 18 carries much higher power. So it represents a system wide mode. This is confirmed by the bus ranking indices shown in Figs. 19 and 20. Fig. 19 shows that channel 384 involves very small number of buses while channel 18 has a large number of buses participating. Both channels identify Bus 630 as the critical bus, which is known to industry as problematic.

Several other test systems have also been studied. The results for these systems are summarized in Table II. The results are compared with those obtained from the Jacobian matrix modal analysis (JM) method of [6]. The table confirms that the proposed technique can identify the critical buses for all systems. In a few cases, some of the top buses are different. Sensitivity studies indicate that they are caused by two factors: (1) The index values used to rank buses (by either the JM or the CCT methods) can be very close among top ranked buses in some cases, so the rank order can be affected by the small differences in the index values. (2) The CCT bus rank considers the impact of both active and reactive power loads while the (reduced) Jacobian matrix modal analysis method only considers the impact of reactive power. This subject is further discussed next.
Fig. 17. Channel margins in AIES.

Fig. 18. Channel $PV$ curves of 6 top-ranked channels in AIES.

Fig. 19. Bus ranking obtained from channel 384.

Fig. 20. Bus ranking obtained from channel 18.

Table II: Summary of the Results

<table>
<thead>
<tr>
<th>System</th>
<th>Critical channel</th>
<th>Top ranked critical load buses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Proposed method</td>
</tr>
<tr>
<td>WECC 9-bus</td>
<td>1</td>
<td>9, 5, 7</td>
</tr>
<tr>
<td>30-bus from</td>
<td>4</td>
<td>8, 7, 4, 3</td>
</tr>
<tr>
<td>[17]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IEEE 30-bus</td>
<td>1</td>
<td>30, 21, 24, 26</td>
</tr>
<tr>
<td>IEEE 57-bus</td>
<td>1</td>
<td>31, 25, 33, 30</td>
</tr>
<tr>
<td>AIES 2038-bus</td>
<td>18</td>
<td>630, 393, 277, 388</td>
</tr>
</tbody>
</table>

* Obtained by PSS/E [19] which gives only the first critical bus. The other cases of JM analysis are done using MATPOWER [18].

E. Discussions

For voltage stability analysis, the application procedure of CCT technique is quite similar to that of the Jacobian matrix modal analysis (JM) technique. It is therefore useful to compare the two techniques. The main differences can be summarized below:

1. **Physical meaning:** the channel components and circuits have clear physical meanings and models. It is therefore easy to interpret the results in channel domain and extract useful information. The JM method, on the other hand, is just a numerical technique. There is no circuit model for the modal results. A domain to map and simplify the $PV$ curves does not exist. The $P\delta$ curves are out of reach for the JM method.

2. **Computing effort:** If the network impedance matrix remains the same, the CCT eigen-decomposition only needs to be performed once. The whole $PV$ curves can be mapped into the channel domain with little computing effort. Online implementation is straightforward. The movement of modes can thus be traced easily. The JM method needs to execute eigen-decomposition at every $PV$ curve point if one wants to track the mode changes. Furthermore, a complex, yet-to-be developed mode-trace (i.e. root-locus plotting) technique is needed to relate one eigenvalue obtained at a $PV$ curve point to another eigenvalue obtained at the next $PV$ curve point.

3. **Robustness:** A significant problem that has not been solved for the JM method is which matrix, the reduced Jacobian matrix or the full Jacobian matrix, shall be used for the eigen-analysis. In theory, the full Jacobian matrix shall be used. But case studies have shown that the full Jacobian matrix yields different $\Delta Q$ bus rankings compared to those derived from the reduced Jacobian matrix. Furthermore, the full Jacobian matrix yields two bus ranking lists, one corresponding to $\Delta P$ and one for $\Delta Q$. These two lists are different, which creates additional ambiguity in bus ranking. The JM results presented in this paper are from the reduced Jacobian matrix according to [6]. The CCT method does not have this problem. It includes the impact of both active and reactive power in the form of channel margin. Another practical issue is the
difficulty to identify the critical mode correctly using the JM method since the index to rank modes is the magnitudes of eigenvalues. Because the PV curve technique cannot reach the exact nose point where $\lambda = 0$, the mode ranking can only be done at a PV curve point close to the nose point (the continuation power flow will help to zero in the nose point but numerical differences can still exist). As a result, numerical errors and the existence of multiple small eigenvalues can mask the critical mode.

V. CONCLUSIONS

This paper has proposed and demonstrated a new method to analyze the behavior of complex power systems. The method is in the form of a network-decoupling transform. The main feature of this transform is that it can decouple a complex network into a set of decoupled single-source, single-branch and single-load networks. The transform has been successfully applied to analyze power system voltage stability in terms of identifying the critical buses (or loads) responsible to voltage collapse in a power system. This is done by projecting the PV curves into the channel domain and by evaluating the criticality of each channel in that domain. With the proposed transform, the PV curves can also be examined from the perspective of power-angle relationship in the channel domain. The results help one to gain improved understanding on the role of bus voltage angles in voltage collapse. Finally, the proposed transform can also be considered as a technique for processing or mapping the phasor data ($U = T[V]$). From this perspective, the transform has been applied successfully to process the phasor data calculated from the PV curve algorithms.

REFERENCES


APPENDIX A: CHARACTERISTICS OF CHANNEL LOADS

Theoretical derivation in Section IV.A suggests that all channel loads are coupled to each other. The channel circuits are connected as shown in Fig. A.1. As seen in this figure, the load seen by the channel 1 circuit will contain the $\lambda$ and $\lambda$ components associated with other channels. This indicates that the channel load can be represented by a Thevenin (or Norton) equivalent circuit.

![Equivalent load seen by channel 1 circuit](image)

Fig. A.1. The channel circuits.

The degree of coupling among the channel loads has been investigated. This study is based on a criterion developed in system control field [20]. The system model in (20) can be treated as a MIMO (multiple input multiple output) system in which the channel voltages are the inputs and the channel currents are the outputs. With this approach, the outputs are obtained by multiplying the input matrix in a gain matrix $[Z_c]=\frac{K_c}{Y_c}$. The control theory states that a Relative Gain Array (RGA) can be formed for every MIMO system [20]. RGA will determine how coupled the MIMO system is. It can also determine the optimal input-output variable pairings. RGA can be formed using the gain matrix $[Y_c]$ as follows.

$$RGA = [Y_c] \ast ([Y_c]^{-1})^T \ast (A.1)$$
where (*) indicates the element-wise multiplication. As an example, the RGA for the simple case study shown in Fig. 4 is as follows:

\[
\begin{bmatrix}
1.0309 & 0.0007 & 0.0303 \\
0.0007 & 1.0177 & 0.017 \\
0.0303 & 0.017 & 1.0473
\end{bmatrix}
\]

Since the diagonal elements of the RGA are much larger than the off-diagonal elements, one may conclude that the MIMO system associated with the channels loads is almost decoupled. The RGA has also been computed and examined for several power systems. Fig. A.2 illustrates the RGA computed for several standard test systems when they are at their normal operating conditions. As seen from Fig. A.2, most of the channel loads are decoupled. This is an interesting finding. It supports the approach of approximating the coupling as current sources.

**APPENDIX B: COMPARISON WITH THE SC TRANSFORM**

The CCT shares many common characteristics with the Symmetrical Components transform (SCT). It is therefore useful to compare the two transforms so that more insights can be gained for CCT. The voltage and current relationship of a n-conductor (or n-phase) power line shown in Fig. B.1 is the simple voltage drop relationship in a matrix form:

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} = \begin{bmatrix}
R & L & C \\
L & R & C \\
C & C & R
\end{bmatrix}\begin{bmatrix}
I_a \\
I_b \\
I_c
\end{bmatrix}
\]

where \([Z]\) is the n-phase impedance matrix of the line. The simplest case of the above equation is the 3-phase line where

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix} = \begin{bmatrix}
V_{a0} & V_{a1} & V_{a2} \\
V_{b0} & V_{b1} & V_{b2} \\
V_{c0} & V_{c1} & V_{c2}
\end{bmatrix}
\]

and \([I]\) is \([I_a, I_b, I_c]^{\top}\). If a three-phase line has two shield wires, there are five conductors involved (Fig. B.1), i.e., \([V_a, V_b, V_c, V_{s1}, V_{s2}]^{\top}\). In the EMTP analysis, such a line is called five-phase line [12]. This concept can be generalized to n-phase lines if there are n-conductors in a tower structure.

When conducting EMTP simulations or finding the modes of wave propagation in a line, the \([Z]\) matrix must be diagonalized, i.e., eigen-decomposition shall be performed on \([Z]\). [12] has presented various forms of \([Z]\) decomposition or transform. The symmetrical components transform is the simplest one among them. The SCT yields three single branches in the modal domain. These are the positive-, negative- and zero-sequence branches. The corresponding domain is the well-known sequence domain.

Let’s now compare (B.1) with (1). These two equations have exactly the same form if we set \([E']=[V_d]\) and \([V]=\begin{bmatrix}V_a & V_b & V_c\end{bmatrix}\). The implication is the following: one can treat a multi-generator, multi-load and multi-branch network shown in Fig. 2 as a network that has one generator supplying one load through one transmission line that has multiple “phases”, as shown in Fig. B.2. In other words, each of the real generators can be considered as one “phase” of a “n-phase” generator and each of the loads as one “phase” of a “n-phase” load. The mutual couplings of the “multi-phase transmission line” represent the mutual interaction of various branches inside the actual network.

Once an actual complex network is viewed as a simple “multi-phase” network, the eigen-decomposition techniques well studied in the EMTP theory can be applied to decouple it into n simple “modal” networks. The proposed transform is essentially identical to the transform used by EMTP analysis.

The CCT’s channel domain thus shares many characteristics with the sequence domain. For example, the system shown in

\[
\begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix} \quad \text{“phase coupling”} \quad \begin{bmatrix} P_{1\gamma} & Q_{1\gamma} \\
P_{1\beta} & Q_{1\beta} \\
P_{1\alpha} & Q_{1\alpha}
\end{bmatrix}
\]

Fig. B.2. A Thevenine circuit viewed as a “multi-phase line”. 

Fig. A.2. RGA for different test systems.
Fig. 4 can be decoupled using the SCT when \( Z_a = Z_b = Z_c \). Channel 1 corresponds to the zero sequence component. In fact, \( 3Z + Z_a \) is the zero sequence impedance of an equivalent three-phase line. One may think that the sample system uses “zero sequence mode” to transfer power. Similarly, the observation that a three-phase power system has a much lower negative sequence voltages than its positive sequence voltages also helps one to understand why some channel voltages are way lower than other channel voltages in a (positive sequence) power system.

In summary, if one considers the SCT as a method for processing three-phase phasors (measured or calculated), and the transforms documented in [12] as tools for processing multi-phase phasors, the proposed CCT can be viewed as an operation for processing multi-bus (i.e. multi-location) phasors. It is interesting to note that the PMU was created originally for determining the sequence components needed by a power system protection scheme [21].

Biography

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