Abstract: The single-phase diode-bridge converter-based loads have rapidly proliferated in residential areas. In most harmonic studies, the associated harmonic effect usually does not include the harmonic voltage interaction with the harmonic currents of non-linear loads. This interaction can attenuate or amplify the harmonic current injections of such appliances. This study characterises this effect by showing how prevalent harmonic amplification exists under typical voltage conditions experienced in residential areas, and by understanding how the existence of harmonic attenuation or amplification effect is strongly related to the crest shape of the supply voltage. The analytical model of the single-phase diode-bridge converter-based loads is used to explain why and when harmonic attenuation or amplification happens. The implication is that the impacts of the voltage on the currents at each harmonic order are very different. Failing to include this effect can underestimate or overestimate each order of the harmonic currents.

1 Introduction

The emergence of power electronic-based home appliances have resulted in recent widespread of a new class of harmonic sources. These devices are usually of small rating and are installed throughout the electrical network. Since the supply voltage in actual power systems is typically distorted, the traditional model to represent harmonic loads, namely the constant current injection, cannot accurately represent such devices. This phenomenon has been traditionally referred to as the attenuation effect [1]. Various research works have shown that a non-linear load supplied with distorted voltage will inject less harmonic currents than when supplied by an undistorted voltage [1–6]. It has also been suggested that the attenuation effect has a relationship with supply voltage waveshape [5].

Whereas most studies have ignored the fact that harmonic amplification could happen, it has been found that an increase in current distortion, rather than a decrease, occurred for some voltage conditions. Mansoor et al. [5] extended the work presented in [1] to analyse in more detail the effect of supply voltage harmonics on the load input current waveshape. In this reference, the authors used thousands of random synthetic voltage waveforms in their simulation studies and observed that the input current total harmonic distortion (THD) increased for the single-phase diode-bridge model. The use of random vectors, however, does not necessarily reflect the true performance of a system. In [3], the authors measured two electronic-ballasted fluorescent lamps under distorted voltage supplies and monitored the harmonic current injections, revealing that the THDI increased when the THDV was above 15%. Therefore the measurements presented in this reference also have the limitation of using supply conditions that do not necessarily provide a reliable approach to handle the problem. Although the authors of these papers acknowledged that it is theoretically possible that increased current distortion can occur, this possibility has been neither investigated nor observed under credible voltage conditions.

The objective of this paper is to fully characterise the harmonic attenuation/amplification that occurs under practical conditions. Our research findings suggest that the main impacting factor on whether harmonic attenuation or amplification occurs is the harmonic voltage phase angle and its effect on the supply voltage waveshape. Two indices, namely the peak displacement (PD) and crest factor (CF), are used for describing waveform distortion of supply voltage. This work is based on an equivalent circuit for single-phase diode-bridge converter-based loads. It is supported by extensive measurements that are presented in the corresponding sections in this paper.

2 Impact of input voltage including single harmonic on attenuation/amplification effect

In previous related publications, the harmonic attenuation effect was frequently referred to and traditionally quantified...
by the attenuation factor $AF_h$, defined in [1] as follows

$$AF_h(V_1, V_2 \ldots V_h) = \frac{I_{h-dist}}{I_{h-ideal}}$$ (1)

where $I_{h-dist}$ and $I_{h-ideal}$ are the $h$th harmonic currents under distorted and ideal voltages, respectively. In this case, ideal voltage means the case of applied 60 Hz alternating voltage free of harmonics.

When harmonic attenuation occurs, $AF_h < 1$, whereas $AF_h > 1$ indicates the existence of harmonic amplification. In order to assess the total harmonic attenuation/amplification effect, we propose to extend the definition of $AF$ for individual harmonic to the total harmonic $AF$ by using the definition of THD for reference and considering the weightings of harmonics. The total harmonic attenuation factor, in turn, can be obtained by calculating a weighted average of individual $AF_h$

$$AF_{total}(V_1, V_2 \ldots V_h) = \sqrt{\sum \frac{I_{h-dist}^2}{I_{h-ideal}^2}} = \sqrt{\sum (w_h \times AF_h)^2}$$ (2)

where $w_h = (I_{h-ideal}/\sqrt{\sum I_{h-ideal}^2})$ is the weighting for the individual harmonics.

The equivalent circuit shown in Fig. 1 has been widely accepted for representing the single-phase diode-bridge converter-based loads, and is used in the studies presented in this paper.

The harmonic attenuation and amplification effects are caused by distorted voltage waveforms, especially by the peak characteristics of these waveforms. To establish a correlation between attenuation/amplification effect and the characteristics of voltage waveform for single-phase diode-bridge converter-based loads, indices are developed in this section.

![Fig. 1 Single-phase diode-bridge converter circuit](image)

2.1 Peak indices for distorted voltage

The results of a simulation batch consisting of several thousand cases suggested a likely relationship between AF and voltage waveshape (simulation results are presented later in this paper). Two peak indices describing the peak distortion of the voltage waveform have strong relationship with AF. One is the well-known CF which is defined as the ratio between the peak magnitude and the root mean square (RMS) value of a waveform. The other one is the PD index, which is proposed in this paper and defined as follows

$$PD = \theta_{dist} - \theta_{ideal}$$ (3)

where $\theta_{dist}$ and $\theta_{ideal}$ are the phase angles at the peak of the distorted voltage and of its fundamental voltage, respectively. Therefore negative PD means leading peak and positive PD means lagging peak, as shown in Fig. 2.

2.2 Relationship between AF and PD

The relationship between AF and PD is best visualised through contour plots such as those presented in Fig. 3. Fig. 3 shows the harmonic current AF contours of the 5th, 7th and total harmonic currents generated by circuit in Fig. 1 (ac side) under the voltage (120 V) including only the 5th harmonic voltage. These are shown in the first, second and third figures from the top, respectively. The bottom figure shows the PD contours of the associated voltage. The $x$-axis in each figure is the phase angle of specified harmonic voltage and $y$-axis is the related magnitude. The phase angle, referring to the fundamental phase angle, varies from $-\pi$ to $\pi$. The magnitude in per cent, relative to fundamental magnitude, varies from 0 to 3%. This is shown on the contour plots as lines that have the percentage value of each harmonic shown as contour lines.

The values of AF index greater than 1 indicate the occurrence of current harmonic amplification, which are marked by $AF > 1$. The values which are less than 1 indicate the occurrence of current harmonic attenuation. In the bottom figure, negative tags indicate that the voltage has leading peak waveform (the contour tags are in degrees), and positive tags indicate that the voltage has lagging peak waveform. Therefore for a given point $(x, y)$ in these contour curves, the AF value can be related to PD to show their correlations. The main findings are as follows:

- Whether harmonic attenuation or amplification occurs is solely determined by the harmonic voltage phase angle, and the magnitudes of the harmonic voltages for a given phase

![Fig. 2 PD index for describing the peak leading or lagging of distorted voltage](image)

$a$ Leading peak (PD < 0)

$b$ Lagging peak (PD > 1)
angle only determines the severity of attenuation/amplification effect.

- PD < 0 and AF > 1 (harmonic amplification) occur roughly in the same regions (inside the dashed frames), which implies that the distorted voltage with leading peak always causes harmonic amplification, whereas the distorted voltage with lagging peak always causes harmonic attenuation.
- The phase angle varying from 0 to π leads to PD < 0, whereas from −π to 0 leads to PD > 0.
- The AF and PD increasing/decreasing trends reveal that AF decreases as PD increases, and vice-versa.

The comparisons for other harmonic voltage cases have been made as well, which lead to similar conclusions.

2.3 Relationship between AF and CF

The CF index is also associated with the attenuation and amplification effects. For a sinusoidal waveform, CF = √2. CF > √2 indicates that the waveform is of greater peak magnitude than sinusoidal waveform and is called pulse-topped waveform. Whereas CF < √2 indicates that the waveform is of lower peak magnitude than sinusoidal waveform and is called flat-topped waveform. Similar investigations have been carried out for comparisons of correlation between AF and CF. Fig. 4 shows the main results. Some findings associated with CF are as follows:

- CF > √2 and AF > 1 (amplification effect) mostly occur in the same region (inside the dashed frame), implying that pulse-topped voltages always cause harmonic amplification, whereas flat-topped voltages always cause harmonic attenuation.
The phase angle varying between $-\pi/2 \sim 0 \sim \pi/2$ leads to $\text{CF} > \sqrt{2}$, whereas $\pi/2 \sim \pi \sim -\pi/2$ leads to $\text{CF} < \sqrt{2}$. The magnitude of the harmonic voltages for a given phase angle only determines the severity of pulse/flat-topped peak.

- The AF and CF increasing/decreasing trends reveal that AF decreases as PD decreases, and vice versa.

These conclusions are also confirmed by other harmonic voltage conditions.

Further observations and comparisons suggest that

- It is rare that harmonic attenuation occurs under neither peak lagging nor flat-topped voltage waveforms, and that harmonic amplification occurs under neither peak leading nor pulse-topped voltage waveforms. Overall, the correlations between AF and PD are stronger than those between AF and CF.
- The comparison of the densities of AF contour plots in Figs. 3 and 4 indicates that the higher order harmonic voltage causes more prominent attenuation or amplification effect for currents at each harmonic. Notice that the contour steps of each index are the same, and the magnitude range of the 3rd harmonic voltage in Fig. 4 is 0–5% but the magnitude range of the 5th harmonic voltage in Fig. 3 is 0–3%. Owing to adopted weighting factors at each harmonic order, the $\text{AF}_3$–$\text{AF}_7$ dominate the characteristics (attenuation or amplification) of $\text{AF}_{\text{total}}$.
- The harmonic voltage phase angles that result in the most severe attenuation and amplification have phase angle difference equal to $\pi$.

### 2.4 More results and more findings

Comparing PD $< 0$ region (inside the dashed frames) with the AF $> 0$ region, the percentage of AF $> 1$ in PD $< 0$ is used to quantify the probability of amplification effect occurrence when the peak is leading. Similarly, the percentage of AF $< 1$ in PD $< 0$ is used to quantify the probability of attenuation effect occurrence when peak is lagging. Fig. 5 presents the results including the analysis for the CF index.

The percentages of AF $< 1$ (attenuation) in the cases of PD $> 0$ (lagging peak) in the cases of $\text{CF} < \sqrt{2}$ (flat-topped) and in the cases of either PD $> 0$ or $\text{CF} < \sqrt{2}$ are calculated and shown in the left figures of Figs. 5a–c for voltages including the 3rd, 5th and 7th harmonics, respectively. Similarly, the percentages of AF $> 1$ (amplification) in each case are shown in the right figures of Figs. 5a–c. In these figures, the left bar for each bar...
group refers to the correlation of AF and CF, the middle bar refers to the correlation of AF and PD and the right bar represents the percentage of AF < 1 (AF > 1) cases happen under either lagging peak or flat-topped (either leading peak or pulse-topped) voltages. For the 7th harmonic voltage, AF < 1 is more likely correlated with CF > $\sqrt{2}$, whereas AF > 1 is more likely correlated with CF < $\sqrt{2}$.

Overall, PD has much stronger correlation with AF than with CF, especially for voltages including high-order harmonics.

When the order of harmonic voltage goes higher, the relationship of AF with PD or CF becomes weak. Since the dominant harmonic voltages in actual system are 3rd, 5th and 7th, the strong relationship between AF and PD or CF still exists.

For any given voltage, the characteristics of attenuation/amplification effect are not the same at all harmonic orders. The harmonic attenuation effect can occur at some harmonic orders, whereas the amplification effect can occur at other frequencies. The measurements in Fig. 6 show the 3rd and 5th harmonic currents being attenuated, whereas the 7th and 9th currents are amplified for some THDV. Owing to this difficulty of drawing a general conclusion from these individual indices, it is useful to combine the individual AF$_h$ into the total AF$_{total}$ to quantify the combined effect.

Although the above observations are based on a supply voltage that only contains one harmonic, similar relationships between CF and AF, PD and AF are also expected for cases including more harmonics, as presented in the next section.

3 Impact of input voltage including synthetic harmonics on the attenuation or amplification effect

In reality, the distorted voltage contains more than one harmonic. To identify the correlations of AF, CF and PD, large-scale simulations were conducted. Simulation and measurement results are also presented in this section.

3.1 Simulation results

The AF$_h$ and AF$_{total}$ assessment has been done by using the analytical model of the diode-bridge converter to analyse the behaviour of the attenuation (or amplification) effect. In order to generate considerable amount of synthetic voltage conditions, the following procedure was performed:

1. Generate a 120 V undistorted voltage supply and superimpose all the following harmonic voltage magnitudes (these are bounded by limits such that the ranges represent all practical possible voltages encountered in urban distribution systems).
   - 3rd harmonic varying from 0 to 5%.
   - 5th harmonic varying from 0 to 3%.
   - 7th harmonic varying from 0 to 2%.
   - 9th harmonic varying from 0 to 1%.

2. All the above harmonic voltages are imposed with 1% intermediate steps between the minimum and maximum of the range.

3. Vary all harmonic voltage phase angles from 0 to 360° in increments of 15°.

4. Calculate PD and CF of each synthetic voltage waveform.

5. Apply the synthetic voltage to the diode-bridge circuit model to obtain AF$_h$ and AF$_{total}$ using (1) and (2).

The total number of combinations used to perform simulation is in the order of several million cases.

Final results can be seen in Fig. 7: Fig. 7a shows the amount of cases, among those that result in AF$_{total}$ < 1, that have CF < $\sqrt{2}$ (left bar), PD lagging (centre bar) and either of the cases (right bar). Conversely, Fig. 7b shows the amount of cases, among those that result in AF$_{total}$ > 1, that have CF > $\sqrt{2}$ (left bar), PD leading (centre bar) or either cases (right bar).

It is clear from these results that PD and CF are two indices that can be used to assess the harmonic attenuation (or amplification) effect, and that PD shows stronger correlation. It is also confirmed that it is rare that harmonic attenuation occurs under neither peak lagging nor flatten-top waveform, and that harmonic amplification occurs under neither peak leading nor pulsed-top waveform.

The percentage of the cases that respect the rule leading PD causes AF$_{total}$ > 1 is not 100%, because of the large amount of unrealistic results produced by varying the voltage (e.g. among the synthetic voltage waveforms, there are many cases where 9th harmonic is higher than 3rd, cases where
the 7th harmonic is the dominant, and so on). Next subsection narrows down the amount of possible voltage conditions by using measured conditions in residential areas.

### 3.2 Measurement results

The national instrument NI-6020E 12-bit data acquisition system with a 15.36 kHz sampling rate controlled by a laptop computer was used for the recording. Using this data-acquisition system, we obtained 256 samples per cycle for each waveform. This sampling rate allows obtaining reliable measurements for all harmonic orders presented in this paper.

Measurements were taken in five residential locations in Canada (more than 600 measurement snapshots were acquired). Using those measurements, we were able to do further verification of the relationship between AF_total and PD. A scatter plot of PD against AF is shown in Fig. 8a. This figure shows that AF_total has a roughly linear relationship with PD. Measurements show that lagging peak cause AF_total < 1 for all 3rd and 5th harmonics, and for part of 7th harmonics. The index AF_total is always observed to be less than 1. Similar observation is seen between CF and AF in Fig. 8b.

### 4 Explanation of the relationship between AF and distorted voltage waveform

In [7], a diode-bridge capacitor filtered converter model was developed. This model is an analytical derivation based on the single-phase diode-bridge converter circuit shown in Fig. 1. Therefore this model can be treated as frequency domain model for a generic single-phase diode-bridge converter. This model is expressed as follows

\[
I_k = Y_{k,1} \dot{V}_k + \sum_{h=1}^{N} Y_{k,h} \dot{V}_h
\]  

(4)

where \(I_k\), \(\dot{V}_k\) and \(\dot{V}_h\) are phasors of \(k\)th harmonic current, fundamental and \(h\)th harmonic voltages, respectively, \(k = 1, 3, 5, \ldots, K\), \(h = 3, 5, \ldots, H\) are the harmonic orders of current and voltage, \(K\) and \(H\) are the highest concerned harmonic orders, respectively, \(\dot{V}_1 = V_1 \angle 0\) and \(\dot{V}_h = V_h \angle \phi_h\), \(\phi_h\) is the phase angle referring to fundamental voltage.

The admittances in (4) are

\[
Y_{1,1} = \frac{\sqrt{1 + (\omega a R C)^2}}{\pi R} (\delta - \alpha) e^{\text{arc tan}(\omega a R C)}
\]

\[
+ 2 \frac{\sqrt{1 + (\omega R C)^2}}{\pi R(1 + k)} \sin \left(1 + k \frac{\delta - \alpha}{2}\right) e^{-j\left((1 + k)(\delta - \alpha)/2 + \text{arc tan}(\omega a R C)\right)}
\]

\[
Y_{k,1} = \frac{2 \sqrt{1 + (\omega a R C)^2}}{\pi R(h - k)} \sin \left(h - k \frac{\delta - \alpha}{2}\right) e^{-j\left(((h - k)(\delta - \alpha)/2 + \text{arc tan}(\omega a R C)\right)}
\]

\[
+ 2 \frac{\sqrt{1 + (\omega R C)^2}}{\pi R(1 + k)} \sin \left(1 + k \frac{\delta - \alpha}{2}\right) e^{-j\left((1 + k)(\delta - \alpha)/2 + \text{arc tan}(\omega a R C)\right)}
\]

\[
Y_{k,h} = \frac{\sqrt{1 + (\omega a R C)^2}}{\pi R} (\delta - \alpha) e^{\text{arc tan}(\omega a R C)} (h = k \neq 1)
\]

\[
Y_{k,h} = \frac{2 \sqrt{1 + (\omega a R C)^2}}{\pi R(h - k)} \sin \left((h - k) \frac{\delta - \alpha}{2}\right) e^{-j\left(((h - k)(\delta - \alpha)/2 + \text{arc tan}(\omega a R C)\right)} (h \neq k \neq 1)
\]

where \(\alpha, \delta\) are the firing and extinction angles of the diode-bridge, \(R\) and \(C\) are the resistance and capacitance in Fig. 1, \(\omega = 2 \pi f\) is the fundamental angular frequency, \(f\) is the fundamental frequency.

This model shows that for a given circuit, the output current varies with the firing and extinction angles and with the phase angles of the harmonic voltages. However, the firing and extinction angles are essentially determined by waveform of the imposed voltage. Therefore the most determining factor of attenuation or amplification effect is indeed the supply voltage. In fact, this finding was verified in [7].

The model presented in [7] is employed to explain how the peak indices impact AF. Equations (4)–(7) suggest that three

![Fig. 8 Relationship PD against AF and CF against AF](image-url)
and (d) distorted voltage causes (a) angle at the middle point of the charging period. If the voltages, Fig. 9 shows the contour plots for (a) cause the harmonic current to change with the harmonic admittances in the model will change, what can in turn decreases, whereas (d) becomes lower that is attenuation effect. Otherwise, the harmonic components of the current shall become higher and amplification effect shall occur. The relationship between the CF alone and the attenuation effect has been addressed in [5]. In this paper, previous findings are extended to completely describe the relationships of the voltage waveshape and the harmonic attenuation/ amplification, which is that attenuation always occurs when either PD > 0 or CF < √2, and amplification always occurs when either PD < 0 or CF > √2.

For the distorted voltage including synthetic harmonics, the cause of amplification effect also can be understood as follows: if the peak of distorted voltage waveform leads the peak of ideal voltage, the converter circuit is fired earlier, which results in longer diode conducting period. The resulting harmonic currents is larger than that under ideal voltage, which leads to AF > 1.

5 Predicting AF from PD and CF

As explained throughout this paper, AF has a strong relationship with PD, but it also depends on CF in a lesser extent. Each harmonic voltage contributes to AF. As shown in Fig. 8, in many cases the AFh is related to PD, CF or to both of them. Further detailing on PD and CF can suggest it is possible to predict AFh. In this case, we need to separate these parameters into more detailed entities as follows:

- PDh is the angle displacement caused by harmonic voltage component h only.
- CFh is the voltage crest factor caused by harmonic voltage component h only.

For a short range of voltage conditions, it is initially assumed that AFh has linear relationship with PDh and CFh as per the results shown in Figs. 3 and 4. We can use the snapshots recorded in residential areas to do this study. The proposed linear approximate structure is given by

\[
AF_h = \sum_{h=3}^{H+1} k_h CF_h + c_h PD_h
\]  

where \(k_h\) and \(c_h\) are the gains for CF and PD, respectively, and \(H\) is the highest concerned harmonic order.

To increase the confidence on the results presented in this section, the AFh is initially (steps 1 and 2) obtained from simulated results but during the validation (step 3) this AFh is the actual ratio obtained from the current measurements.

An approach is proposed to obtain the coefficients \(k\) and \(c\). It is based on least-squares estimation. The following procedure is used:

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**Fig. 9 (a)\((\delta - \alpha)\), \((\delta + \alpha)/2\) and PD contours**

- (a) \((\delta - \alpha)\) contours
- (b) \((\delta + \alpha)/2\) contours
- (c) PD contours

Factors affect the output current for a given circuit. These factors are \((\delta - \alpha)\), \((\delta + \alpha)/2\) and the phase angle of the harmonic voltage \(\varphi_h\). Actually, \((\delta - \alpha)\) is the charging period of the circuit, whereas \((\delta + \alpha)/2\) is the phase angle at the middle point of the charging period. If the distorted voltage causes \((\delta - \alpha)\) and \((\delta + \alpha)/2\) to change, the admittances in the model will change, what can in turn cause the harmonic current to change with the harmonic voltages. Fig. 9 shows the contour plots for \((\delta - \alpha)\), \((\delta + \alpha)/2\) and PD when the circuit is supplied by a distorted voltage that includes the 3rd harmonic. The comparison reveals that PD indeed has a strong correlation with \((\delta - \alpha)\) and \((\delta + \alpha)/2\) (more significantly with \((\delta + \alpha)/2\)). The contours show that when PD decreases, \((\delta + \alpha)/2\) also decreases, whereas \((\delta - \alpha)\) generally increases and vice versa. The contour plots for the other harmonic voltage also show similar characteristics, but are not presented here due to space limitations.

Figs. 10a and b are presented to further explain how PD impacts \((\delta - \alpha)\) and \((\delta + \alpha)/2\). In Fig. 10, the dashed lines and curves denote the distorted voltage and related current, whereas the solid lines and curves denote the ideal voltage and the related current. If the peak of the distorted voltage waveform becomes lagging (leading) the ideal one, \((\delta + \alpha)/2\) will move along it since the firing instant will be delayed (brought ahead) as shown in Fig. 10a (Fig. 10b) and the peak of current becomes lower (higher). In addition, lagging peak also makes the charging duration \((\delta - \alpha)\) of circuit become longer, since the rising slope becomes slow. Conversely, leading peak results in shorter \((\delta - \alpha)\). As the waveform of lagging peak voltage intends to decrease the current peak and extend the charge duration, the harmonic components of current shall become lower that is attenuation effect. Otherwise, the harmonic components of the current shall become higher and amplification effect shall occur. The relationship between the CF alone and the attenuation effect has been addressed in [5]. In this paper, previous findings are extended to completely describe the relationships of the voltage waveshape and the harmonic attenuation/amplification, which is that attenuation always occurs when either PD > 0 or CF < √2, and amplification always occurs when either PD < 0 or CF > √2.
1. Use a number of measurements to obtain $k$ and $c$. In this case, the measurements are about 600 snapshots taken at five residential locations as explained in Section 3.2.

2. Obtain the indices PD and CF from the measured voltage data.

3. Test a number of measurements that have not been used to obtain $k$ and $c$. Thirty acquired snapshots were used. In this case, the attenuation factors are not obtained from analytical model, but the actual measured attenuation factors obtained from measurements performed in that location.

4. Evaluate the error using histograms. The histogram is drawn taken the relationship $AF_h$ estimated against $AF_h$ measured, and evaluating the approximation to the unity slope. One histogram per harmonic order is used.

The results for the test diagnosis (step 2) are shown in Fig. 11a. Not surprisingly, the estimated $AF_h$’s are in very good agreement with the simulated one, especially for harmonics 7th and 9th. Fig. 11b shows the results from step 3, which also represent good agreement with the gains obtained from step 1.

The obtained gains are presented in Table 1. Notice that for the 3rd harmonic, the only adjacent harmonic order used is the 5th. It is interesting that usually certain $AF_h$ will strongly depend on $CF_{h-1}$, $PD_{h-1}$, $CF_{h-k}$ and $PD_{h-k}$, and to a less extent on $CF_{h+1}$ and $PD_{h+1}$. This implies on higher coupling on the orders inferior and on itself. It depends little on the superior orders.

Finally, the error was evaluated (step 4) and found to be always smaller than 1% for $h = 3$, 2% for $h = 5$, and smaller than 4% for $h = 7$ and $h = 9$. Results are omitted because of space limitations.

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6 Conclusions

This paper has shown that the harmonic attenuation/amplification effect can be significant for single-phase power electronic loads. Previous research has recognised the fact that harmonic amplification can occur, but has not investigated or analysed the phenomenon under credible voltage conditions. In this paper, we have proposed describing the voltage waveshape by using crest indices, which are used to predict the harmonic current amplification or reduction. The main findings are

- Whether harmonic amplification or attenuation occurs depends on the phase angle displacement PD and on the CF. Higher harmonic currents are more sensitive attenuation or amplification effects.
- The dependence on PD is clearly stronger than that on CF, as it could be shown in the results presented in this paper. This finding greatly expands the development presented in [5], which attempted to characterise the current distortion by using CF only.
- The voltage angle displacement PD is caused by the type of load. In residential areas, where predominantly single-phase power electronic loads are the dominant non-linear loads, harmonic attenuation or amplification can be experienced. But most of the voltage conditions result in harmonic attenuation, and few cases result in voltage amplification.
- For narrow ranges of voltages (voltages measured in residential areas), it is possible to predict the individual harmonic AF$_h$ of a diode-bridge single-phase load using the coupled PD$_h$ and CF$_h$. The associated errors are small and therefore acceptable.

Other indices, such as zero crossing shifting caused by interharmonics, may also impact the harmonic amplification/attenuation effect of non-linear loads. This analysis is currently under research.

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8 References