Energy Efficient Network Beamforming Design Using Power-Normalized SNR

Yichen Hao, Yindi Jing, and Shahram ShahbazPanahi

Abstract

In this paper, we adopt a novel efficiency measure, namely the received signal to noise ratio (SNR) per unit power, in amplify-and-forward (AF) based relay networks. The measure is addressed as the power-normalized SNR (PN-SNR). For several relay network scenarios, we solve the PN-SNR maximization problems and analyze the network performance. First, for single-relay networks, we find the optimal relay power control scheme that maximizes the PN-SNR for a given transmitter power. Then, for multi-relay networks with a sum relay power constraint, we prove that the PN-SNR optimization problem has a unique maximum, thus the globally optimal solution can be found using a gradient-ascent algorithm. Finally, for multi-relay networks with an individual power constraint on each relay, we propose an algorithm to obtain the globally optimal solution and a low complexity suboptimal solution. Our results show that with the same average relay transmit power, the PN-SNR maximizing scheme is superior to the fixed relay power scheme not only in the PN-SNR but also in the outage probability for both single and multi-relay networks. Compared with SNR-maximizing scheme, it is significantly superior in PN-SNR with moderate degradation in outage probability. Our results show the potential of using PN-SNR as efficiency measure in network design.

Index Terms

Relay network, power-normalized SNR, efficiency, power control, outage probability.

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I. Introduction

Wireless communication networks are in consistently increasing demands for higher date rates without sacrificing reliability. One way to improve the date rate and/or the reliability of a wireless link is to use cooperative schemes in the network [1]–[8]. Power control and network performance optimization are among hot topics in cooperative relay network design in recent years. There have been numerous results on the global performance optimization such as signal-ro-noise ratio (SNR) maximization, throughput maximization, and error rate minimization for fixed transmit power [9]–[14]. Specifically, [9] and [10] dealt with the received SNR optimization in single-user multi-relay networks under sum relay power constraint and separate relay power constraints, respectively. In [11] and [12], the same model in [9] and [10] was considered, while the goal was to maximize the capacity of the network. The authors of [13] and [14] focused on two-way relay networks where either minimum SNR or sum-rate was optimized. In [15], the authors proposed relay selection schemes that maximize the received SNR and used block error rate as performance measure.

As the popularity of wireless users and wireless traffic rapidly multiply, the drastic increase in energy consumption in wireless infrastructure leads to the increase of greenhouse gas emission, causing severe environmental depredation. As a result, green communication design has attracted significant attention in recent years [16], [17].

Popular efficiency measures include spectral efficiency (or capacity if the bandwidth is fixed) and energy efficiency. There has been a significant volume of literature addressing these two efficiency measures for various network configurations, e.g., [11], [12], [18]–[21]. Spectral efficiency is defined as the achievable transmission bit-rate and its maximization guarantees the highest amount of information flow for a fixed transmit power. But it does not consider how efficient the power is used in achieving the maximum. Energy efficiency is defined as the number of transmitted bits per unit energy or power. It is thus a natural efficiency measure. However, for most communication systems, energy efficiency is maximized when the transmit power approaches 0, i.e., when the system works in the low SNR regime. To see this, we consider the simple point-to-point single-antenna system with transmit power P, unit-variance noise, and channel gain λ. The energy efficiency of the system is given as $[\log(1+\lambda P)]/P$, which takes its maximum λ when $P \to 0$. Hence, an energy-efficiency-optimal scheme will trap the system in the low SNR regime, where the communication bit-rate and reliability can be low.

In this work, the aforementioned limitations of spectral efficiency and energy efficiency measures has inspired us to study new efficiency metric, namely SNR-per-unit-power or power-normalized SNR (PN-SNR), to design network beamforming algorithms and to evaluate the network efficiency. For a single-user network, the PN-SNR is defined as

$$\eta \triangleq \frac{\mathrm{SNR}}{P_{\mathrm{total}}} \tag{1}$$

where SNR is the end-to-end received SNR and P_{total} is the total power consumed in the network. The parameter η represents the achievable received SNR per unit transmit power. If the received noise has a unit variance, P_{total} can also be seen as the transmit SNR. In this sense, η represents the received SNR the system provides per unit transmit SNR.

Compared with the spectral efficiency metric, the PN-SNR is a more natural efficiency measure as it shows the performance per unit power. Compared with the energy efficiency metric, the PN-SNR does not trap the network in the low power regime. To see this, we revisit the same point-to-point single-antenna system with transmit power P_{total}, unit-variance noise, and channel gain λ. The PN-SNR of the system is $\eta = (\lambda P_{\mathrm{total}})/P_{\mathrm{total}} = \lambda$, which is independent of the transmit power. For a point-to-point direct communication system without relaying (e.g., multi-antenna system), the PN-SNR is equal to the array gain of the system, independent of the transmit power. Thus, the maximization of PN-SNR in such systems is trivial. For cooperative relay networks, however, the maximization can be involved, as will be seen later in this paper.

The PN-SNR was first proposed as an efficiency measure in [15], where it was called power efficiency. It was later used in [22], [23], and [24], where the term PN-SNR was introduced. While the PN-SNR was employed for numerical performance evaluation in [15], [22], [23], its properties and optimal designs using this measure have yet to be investigated. For asynchronous two-way multi-relay networks, [24] investigated the joint subcarrier transceiver power loading and relay beamforming optimization that maximizes the minimum SNR of the two users across all subcarriers, where, on each subcarrier, the subproblem of the SNR-maximization for given subcarrier power vectors was proved to result in PN-SNR optimization. But this work focused on SNR optimization and PN-SNR was not considered as an energy efficiency measure.

In this paper, adopting the PN-SNR as an efficiency criterion for designing power-efficient relay networks, we propose a PN-SNR-optimal relay power control scheme. We analyze the properties of the proposed scheme and then compare these properties with those of the SNR-maximizing scheme and the fixed relay power scheme. For single-relay networks, we find the optimal relay power that maximizes the PN-SNR for arbitrarily given transmitter power in close-form. We also study the average received SNR, the outage probability, and the average relay power offered by the proposed design; and compare them with those of existing schemes. Then, for multi-relay networks with a total relay power constraint, we prove that the PN-SNR maximization problem has a unique maximum, thus the globally optimal solution can be found with gradient-ascent algorithm. Finally, for multi-relay networks with separate power constraints on relays, we propose a numerical algorithm for the globally optimal solution and a low complexity suboptimal solution. Our simulation results show that compared with fixed relay power scheme with the same average relay power, the proposed scheme is superior in both the PN-SNR and the outage probability. Also, compared with the SNR-maximizing scheme, the proposed scheme has considerably higher PN-SNR with moderate degradation in the outage probability.

In this introduction section, we have motivated the use of PN-SNR as an efficiency measure and summarized the main results of our work. The remaining of this paper is organized as follows. In Section II, we introduce the system model and the underlying communication protocol. Section III considers a single-relay network, where a closed-form solution to the PN-SNR maximization problem is obtained and the network performance is analyzed. In Sections IV and V, we study the

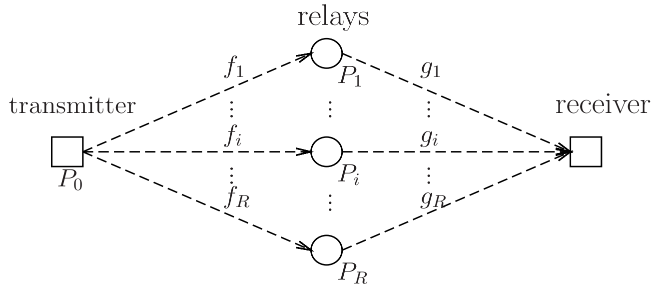

Fig. 1. Multi-relay network.

PN-SNR maximization problems for multi-relay networks under a sum relay power constraint and under separate relay power constraints, respectively. Section VI provides our simulation results. Section VII concludes the paper. Involved proofs are included in the appendices.

Notation: For a matrix $\mathbf{A}$, $\mathbf{A}^T$ denotes the transpose of $\mathbf{A}$. For a complex scalar α, $|\alpha|$ and $\angle\alpha$ represent the amplitude and phase of α, respectively. For a vector $\mathbf{a}$, $\|\mathbf{a}\|$ stands for its Euclidean norm. For two vectors $\mathbf{a}$ and $\mathbf{b}$ of the same dimension, $\mathbf{a} \circ \mathbf{b}$ is the Schur-Hadamard product of the two vectors. $\mathrm{erf}(\cdot)$ is the error function, $\tan^{-1}(\cdot)$ is the inverse tangent function, and $K_1(\cdot)$ is the first order modified Bessel function of second kind. $\ln(\cdot)$ and $\log(\cdot)$ denote the natural logarithm function and common logarithm function, respectively. $\mathbb{P}(\cdot)$ stands for the probability.

II. System Model

We consider a general distributed network with one transmitter, one receiver and R relays, as depicted in Fig. 1. Each relay has only one single antenna which can be used for both transmission and reception. We denote the channel from the transmitter to the ith relay as f_i and the channel from the ith relay to the receiver as g_i. We assume that there is no direct link between the transmitter and the receiver. We assume that f_i and g_i are i.i.d. complex Gaussian with zero-mean and unit-variance, so the channel magnitudes follow Rayleigh distribution. All channels are assumed to be flat-fading channels. We also assume that each relay knows its own channels, i.e., the ith relay knows f_i and g_i, and the receiver knows all channels. The required channel state information at the receiver can be obtained via channel estimation and feedback [25]–[28]. The ith relay can obtain f_i by training and g_i by feedback. Let $\mathbf{f} \triangleq [f_1 \; f_2 \; \ldots \; f_R]^T$ and $\mathbf{g} \triangleq [g_1 \; g_2 \; \ldots \; g_R]^T$, which are the transmitter-relay and relay-receiver channel vectors. We define the effective end-to-end channel vector between the transmitter and receiver as $\mathbf{h} \triangleq \mathbf{f} \circ \mathbf{g}$.

We herein consider a two-step amplify-and-forward (AF) relaying protocol with relay beamforming, where the relays adjust the amplitudes and the phases of their received signals before forwarding them. During the first step, the transmitter sends $\alpha_0\sqrt{P_0}s$, where the information symbol s is randomly selected from the codebook $\mathcal{S}$. We assume that s in the codebook are normalized as $\mathbb{E}\{|s|^2\} = 1$. Thus, the average power used at the transmitter is $\alpha_0^2 P_0$, where P_0 is the maximum power of the transmitter and the coefficient $0 \leq \alpha_0 \leq 1$ is introduced to adjust the power of the transmitter. The signals received at the relays can be represented as

$$\mathbf{x} = \alpha_0\sqrt{P_0}\mathbf{f}s + \mathbf{z}, \tag{2}$$

where $\mathbf{x}$ is the $R \times 1$ complex vector of the signals received by relays and $\mathbf{z}$ is the $R \times 1$ complex vector of the relay noises. We assume that all noises are i.i.d. complex Gaussian random variables with zero-mean and unit-variance.

In the second step, the ith relay multiplies its received signal by a complex weight w_i to adjust the phase and magnitude of the signal and transmits the adjusted signal. All relays share the same channel and are assumed to be perfectly synchronized. The $R \times 1$ complex vector $\mathbf{t}$ of the transmitted signals of all relays can then be expressed as

$$\mathbf{t} = \mathbf{w} \circ \mathbf{x}, \tag{3}$$

where $\mathbf{w} \triangleq [w_1 \; w_2 \; \ldots \; w_R]^T$ is referred to as the relay beamforming vector. Denoting the ith entry of $\mathbf{t}$ as t_i, the power consumed on the ith relay, denoted as P_i, can be calculated, using (3), as

$$P_i = \mathbb{E}\{|t_i|^2\} = (1 + \alpha_0^2 P_0 |f_i|^2)|w_i|^2, \tag{4}$$

The signal received at the receiver, denoted as y, can be written as

$$y = \alpha_0 \sqrt{P_0}\mathbf{w}^T\mathbf{h}s + \mathbf{w}^T(\mathbf{g} \circ \mathbf{z}) + n\,.$$

where the noise n at the receiver is assumed to be independent of $z_1, \ldots, z_R$ and is Gaussian distributed with zero mean and unit variance. With straightforward calculation, the end-to-end received SNR can be expressed as

$$\text{SNR} = \frac{\alpha_0^2 P_0 (\mathbf{w}^T\mathbf{h})^2}{1 + \|\mathbf{w} \circ \mathbf{g}\|^2}. \tag{5}$$

Recalling the power consumed by the ith relay is given as in (4). The total transmit power consumed on all relays is $\sum_{i=1}^{R}(1 + \alpha_0^2 P_0 |f_i|^2)|w_i|^2 = P_0\|\mathbf{w} \circ \mathbf{a}\|^2$, where

$$\mathbf{a} \triangleq \left[\sqrt{\frac{1}{P_0} + \alpha_0^2|f_1|^2} \quad \cdots \quad \sqrt{\frac{1}{P_0} + \alpha_0^2|f_R|^2}\right].$$

The total transmit power consumed in the whole network is thus $P_T = \alpha_0^2 P_0 + P_0\|\mathbf{w} \circ \mathbf{a}\|^2$.

According to our definition in (1), the PN-SNR of the relay network is

$$\eta \triangleq \frac{\text{SNR}}{P_T} = \frac{\alpha_0^2 P_0 (\mathbf{w}^T\mathbf{h})^2}{(1 + \|\mathbf{w} \circ \mathbf{g}\|^2)(\alpha_0^2 P_0 + P_0\|\mathbf{w} \circ \mathbf{a}\|^2)}. \tag{6}$$

Denote the amplitude and the phase of w_i as α_i and θ_i, respectively, i.e., $w_i = \alpha_i e^{j\theta}$. Let $\boldsymbol{\alpha} \triangleq [\alpha_1 \ \cdots \ \alpha_R]^T$ and $\boldsymbol{\theta} \triangleq [\theta_1 \ \cdots \ \theta_R]^T$. Note that both $\|\mathbf{w} \circ \mathbf{g}\|^2$ and $\|\mathbf{w} \circ \mathbf{a}\|^2$ are independent of the phase vector $\boldsymbol{\theta}$. Thus, the denominator of η given in (6) is independent of $\boldsymbol{\theta}$. It is obvious that the numerator is maximized when $\theta_i = -\angle h_i$ for any given $\boldsymbol{\alpha}$, where $h_i = f_i g_i$ is the ith entry of $\mathbf{h}$. We can also see that η is an increasing function of α_0. Thus, it is optimal to choose $\alpha_0 = 1$, which means that the transmitter should always transmit with its maximum power.

With the maximum power consumed at the transmitter ($\alpha_0 = 1$) and the optimal phase adjustment at the relays, the end-to-end received SNR in (5) reduces to

$$\text{SNR} = \frac{P_0(\boldsymbol{\alpha}^T\mathbf{b})^2}{1 + \|\boldsymbol{\alpha} \circ \mathbf{d}\|^2}, \tag{7}$$

and the PN-SNR in (6) reduces to

$$\eta = \frac{(\boldsymbol{\alpha}^T\mathbf{b})^2}{(1 + \|\boldsymbol{\alpha} \circ \mathbf{d}\|^2)(1 + \|\boldsymbol{\alpha} \circ \mathbf{a}\|^2)}, \tag{8}$$

where $\mathbf{b} \triangleq [|f_1g_1| \ \cdots \ |f_Rg_R|]$, $\mathbf{d} \triangleq [|g_1|, \cdots, |g_R|]$ and $\mathbf{a} = \left[\sqrt{\frac{1}{P_0} + |f_1|^2} \ \ \cdots \ \ \sqrt{\frac{1}{P_0} + |f_R|^2}\right]$ when $\alpha_0 = 1$.

Our design problem is thus finding the relay power control vector $\boldsymbol{\alpha}$ such that the PN-SNR in (8) is maximized. In the subsequent sections, we will first analyze the PN-SNR in single-relay networks, then solve the PN-SNR maximization problem for multi-relay networks under a sum relay power constraint and under separate relay power constraints.

III. SNR-Per-Unit-Power Optimization in Single-Relay Networks

In this section, we consider a single-relay network, i.e., $R = 1$, which is a special case of the system model in Section II. For a given transmit power P_0, we derive the optimal relay power that maximizes the PN-SNR in closed-form and analytically evaluate the network performance. We then compare the performance of the proposed PN-SNR-maximizing scheme with that of the existing schemes.

In single-relay networks, the relay power control vector and channel vectors reduce to scalars, i.e., $\boldsymbol{\alpha}$ reduces to α, $\mathbf{f}$ reduces to f, and $\mathbf{g}$ reduces to g. We denote the power constraint on the relay as $P_{R,\text{lim}}$, and denote the actual transmit power on the relay as P. Thus, $P \leq P_{R,\text{lim}}$. From (7), the end-to-end received SNR can be expressed as

$$\text{SNR} = \frac{|fg|^2 P P_0}{1 + |f|^2 P_0 + |g|^2 P} \approx \frac{|fg|^2 P P_0}{|f|^2 P_0 + |g|^2 P}. \tag{9}$$

In the second equality in (9), we have used an approximation which has been shown to be tight in the high SNR regime [29]. The corresponding PN-SNR of the network is thus

$$\eta = \frac{|fg|^2 P P_0}{(1 + |f|^2 P_0 + |g|^2 P)(P + P_0)} \approx \frac{|fg|^2 P P_0}{(|f|^2 P_0 + |g|^2 P)(P + P_0)}. \tag{10}$$

A. The PN-SNR-Maximizing Solution

In this subsection, we solve the PN-SNR maximization problem. We first consider the ideal case that the relay power is unlimited, i.e., $P_{R,\text{lim}} = \infty$, then consider the practical case of finite $P_{R,\text{lim}}$. Infinite power constraint is of course ideal, as any device has a finite power limit. We consider this ideal case to better understand the behavior of the PN-SNR efficiency measure. Studying this ideal case also helps us to find the solution to the finite power constraint case.

Using (10), our PN-SNR maximization problem is thus given as

$$\max_{P>0} \frac{|fg|^2 P P_0}{(1+|f|^2 P_0+|g|^2 P)(P+P_0)}. \tag{11}$$

Differentiating the objective function in (11) with respect to P and equating it to zero, the optimal relay power, denoted as P_{opt}, is obtained as

$$P_{\text{opt}} = \frac{\sqrt{P_0(1+|f|^2 P_0)}}{|g|}. \tag{12}$$

When the transmitter power is high ($P_0 \gg 1$), this solution can be approximated as

$$P_{\text{opt}} \approx P_{\text{approx}} = \frac{|f|}{|g|} P_0. \tag{13}$$

The same result can be obtained if the approximate SNR formula in (9) is used in the PN-SNR formula. From (12) and (13), we can see that although the relay power constraint is assumed to be infinity, for the highest PN-SNR, the relay should only use a finite amount of power. This is different from the SNR-maximizing scheme, where the optimal solution is easily seen to be $P = P_{R,\text{lim}} = \infty$. Also, we can see that the optimal relay power in (12) and (13) is channel dependent, meaning that for the highest PN-SNR, the relay should adjust its transmit power according to the channel qualities. When the ratio of the transmitter-relay channel quality ($|f|$) to the relay-receiver channel quality ($|g|$) is larger, the relay should use more power; and vice versa. This property complies with the relay noise suppression idea. When this ratio is high, the transmitter-relay channel has a better quality than the relay-receiver channel, the received signal at the relay contains a small noise component and it should use a relatively large amount of power to forward. On the other hand, when the ratio is low, the transmitter-relay channel has lower quality than the relay-receiver channel, the received signal at the relay is highly noisy and the relay should use low power to suppress relay noise amplification.

Now, we consider the practical case that $P_{R,\text{lim}}$ is finite, i.e., $P_{R,\text{lim}} < \infty$. It is straightforward to show that $\frac{d\eta}{dP} > 0$ when $P \leq \frac{|f|}{|g|} P_0$ and $\frac{d\eta}{dP} < 0$ when $P \geq \frac{|f|}{|g|} P_0$. Thus, the PN-SNR increases with P when $P \leq \frac{|f|}{|g|}P_0$ and decreases with P when $P \geq \frac{|f|}{|g|}P_0$. So for the finite power constraint case, the PN-SNR maximizing solution can be easily extended from (13) as

$$P_{\rm opt} \approx P_{\rm approx} = \min\left(\frac{|f|}{|g|}P_0, P_{R,\rm lim}\right). \tag{14}$$

In the following subsection, performance of the proposed power control in (14) is evaluated.

B. Performance of the PN-SNR-Maximizing Scheme

Now, we analyze the network performance under the proposed PN-SNR-maximizing solution to further understand the adopted PN-SNR measure. For the performance, we consider the average relay transmit power, the average PN-SNR, the average end-to-end received SNR, and the outage probability. We summarize our performance analysis results in the following theorem.

Theorem 1. *With the relay power design in* (14)*, the average power consumed by the relay is*

$$P_{\rm ave} = P_0 \tan^{-1}\left(\frac{P_{R,\rm lim}}{P_0}\right). \tag{15}$$

Define $\xi \triangleq \frac{P_{R,\rm lim}}{P_0}$*. When* $P_0 \gg 1$ *and using the SNR approximation in* (9)*, the average PN-SNR of the network is given as*

$$\eta_{\rm ave} \approx \frac{3}{8}\pi - \frac{3}{4}\tan^{-1}\left(\frac{1}{\xi}\right) - \frac{4\xi^3 - 7\xi^2 - \xi}{4(\xi+1)(\xi-1)^2} + \frac{2\xi^2 \ln\left(\frac{\xi^2+1}{\xi(\xi+1)}\right)}{(\xi-1)^3(\xi+1)}, \tag{16}$$

and the corresponding average end-to-end received SNR is

$$\mathrm{SNR}_{\rm ave} \approx P_0\left[\frac{\pi}{8} - \frac{1}{4}\tan^{-1}\left(\frac{1}{\xi}\right) - \frac{3\xi^3+5\xi}{4(\xi-1)^2(\xi^2+1)} - \frac{1}{4}\ln\left(\frac{(\xi+1)^2}{\xi^2+1}\right) + \frac{2\xi^2\ln\left(\frac{\xi^2+1}{\xi(\xi+1)}\right)}{(\xi-1)^3}\right]. \tag{17}$$

Also, with SNR threshold $\gamma_{\rm th}$*, the outage probability, denoted as* $\mathcal{O}$*, can be bounded as*

$$\left(1+\frac{1}{\xi}\right)\frac{\gamma_{\rm th}}{P_0} + \mathcal{O}\left(\frac{1}{P_0^2}\right) \lesssim O \lesssim \left(1+\frac{1}{\xi}\right)\frac{\gamma_{\rm th}}{P_0} + \mathcal{O}\left(\frac{1}{P_0^{4/3}}\right). \tag{18}$$

Proof. See Appendix A. □

From (15) in Theorem 1, we can see that, under the PN-SNR-maximizing design, the average relay power is non-decreasing in P_0. But it is always finite, regardless of the relay power constraint. It can also be seen from (16) and (17) that η_{ave} and $\frac{\text{SNR}_{\text{ave}}}{P_0}$ are continuous and non-decreasing in ξ (it can be shown that $\xi = 1$ is not a singular point). The outage probability in (18) decreases as ξ increases. We can conclude that increasing the ratio of the maximum relay power and the maximum transmitter power improves the average PN-SNR, the average received SNR, and the outage probability. For a given P_0, larger ξ means that more power is available at the relay, which results in better performance. However, the performance is bounded by the extreme case where ξ is infinity, i.e., the relay power constraint $P_{R,\lim}$ is unlimited. The performance of the extreme case can be summarized in the following corollary.

Corollary 1. *When $\xi = \infty$, with the relay power design in (14) and using the SNR approximation in (9), the average power consumed on the relay is*

$$P_{\text{ave}} = \frac{\pi}{2} P_0, \tag{19}$$

the average PN-SNR of the network is

$$\eta_{\text{ave}} \approx \frac{3}{8}\pi - 1, \tag{20}$$

the corresponding average end-to-end received SNR is

$$\text{SNR}_{\text{ave}} \approx \frac{\pi}{8} P_0, \tag{21}$$

and the outage probability with SNR threshold γ_{th} can be bounded as

$$\frac{\gamma_{\text{th}}}{P_0} + \mathcal{O}\left(\frac{1}{P_0^2}\right) \lesssim O \lesssim \frac{\gamma_{\text{th}}}{P_0} + \mathcal{O}\left(\frac{1}{P_0^{4/3}}\right). \tag{22}$$

Proof. This corollary can be easily obtained from Theorem 1 by setting $\xi = \infty$. It is also derived in [30]. □

C. Comparison with Fixed Relay Power Design

In this subsection, we compare the performance of the proposed scheme with the fixed relay power scheme, where the relay power P is fixed regardless of the channel quality. The network performance in fixed relay power scheme can be summarized as follows.

Lemma 1. *When $P_0 \gg 1$, with the relay transmit power fixed as P for each transmission, the average PN-SNR of the network is*

$$\eta_{\text{ave\_fix}} \approx \frac{PP_0}{(P-P_0)^2} - \frac{2P^2P_0^2}{(P-P_0)^3(P+P_0)} \ln\left(\frac{P}{P_0}\right), \tag{23}$$

the corresponding average end-to-end received SNR is

$$\text{SNR}_{\text{ave\_fix}} \approx \frac{PP_0(P+P_0)}{(P-P_0)^2} - \frac{2P^2P_0^2}{(P-P_0)^3} \ln\left(\frac{P}{P_0}\right). \tag{24}$$

If P has the same scaling as P_0, i.e., $P = \zeta P_0$, the outage probability with SNR threshold γ_{th} is

$$O_{\text{fix}} \approx \left(1 + \frac{1}{\zeta}\right)\frac{\gamma_{\text{th}}}{P_0} + \mathcal{O}\left(\frac{\ln P_0}{P_0^2}\right). \tag{25}$$

Proof. See Appendix B.

For a given P_0, it can be derived from (23) that the channel-independent optimal relay power that maximizes the average PN-SNR is P_0, i.e., the relay power should be the same as the transmitter power for the highest $\eta_{\text{ave\_fix}}$.

In what follows, we compare the performance of the proposed PN-SNR maximizing scheme with that of the fixed relay power scheme. For fairness, we set the average relay power used in the two schemes to be the same, i.e., they have the same power resource. Thus, for the fixed relay power scheme, we have $P = P_{\text{ave}} = P_0 \tan^{-1}(P_{R,\text{lim}}/P_0)$. Recalling that $\xi = P_{R,\text{lim}}/P_0$, we simplify the average PN-SNR, the average SNR, and the outage probability in (23), (24), and (25), respectively, as

$$\eta_{\text{ave\_fix}} \approx \frac{\tan^{-1}(\xi)}{[\tan^{-1}(\xi)-1]^2} - \frac{2\tan^{-1}(\xi)^2 \ln[\tan^{-1}(\xi)]}{[\tan^{-1}(\xi)-1]^3[\tan^{-1}(\xi)+1]}, \tag{26}$$

$$\text{SNR}_{\text{ave\_fix}} \approx \left[\frac{\tan^{-1}(\xi)[\tan^{-1}(\xi)+1]}{[\tan^{-1}(\xi)-1]^2} - \frac{2\tan^{-1}(\xi)^2 \ln[\tan^{-1}(\xi)]}{(\tan^{-1}(\xi)-1)^3}\right] P_0, \tag{27}$$

and

$$O_{\text{fix}} \approx \left[1 + \frac{1}{\tan^{-1}(\xi)}\right]\frac{\gamma_{\text{th}}}{P_0} + \mathcal{O}\left(\frac{\ln P_0}{P_0^2}\right). \tag{28}$$

Comparing (16) with (26), we discover that the average PN-SNR in the proposed scheme is always higher than the fixed relay power scheme. For the extreme case of $P_{R,\text{lim}} = \infty$, our scheme is 11.3% better in the average PN-SNR. With respect to the outage probability, comparing (18) with (28), the proposed scheme has the following gain:

$$\mathrm{G}_{\text{outage}} \triangleq 10\log\frac{\mathcal{O}_{\text{fix}}}{\mathcal{O}} = 10\log\left[\frac{\xi\tan^{-1}(\xi)+\xi}{\xi\tan^{-1}(\xi)+\tan^{-1}(\xi)}\right]. \tag{29}$$

Note that as $\xi \geq \tan^{-1}(\xi)$ for $\xi \geq 0$, the numerator in (29) is larger than the denominator, meaning that $\mathrm{G}_{\text{outage}}$ is always non-negative. Thus, our scheme outperforms the fixed relay power scheme in outage probability. It can also be shown that $\mathrm{G}_{\text{outage}} \leq 2.14$ dB with equality when $P_{R,\text{lim}} = \infty$.

We can conclude that our proposed scheme is more power efficient than the fixed relay power scheme. Furthermore, it also outperforms the fixed relay power scheme in outage probability with the same relay power consumption. These advantages are due to the PN-SNR measure used in our scheme, leading to a channel-dependent relay power control.

Another existing scheme is the SNR-maximizing scheme, where the relay uses its maximum power for the highest received SNR, i.e., $P = P_{R,\text{lim}}$. In fact, the SNR-maximizing scheme can be viewed as a fixed relay power scheme and its performance can be obtained from (23)-(25) by setting $P = P_{R,\text{lim}}$ and $\zeta = 1$. Compared to our proposed method, the SNR-maximizing scheme is expected to have better average received SNR but significantly lower average PN-SNR. Its outage probability (given in (28) with $\zeta = 1$) has the same dominant term as that of the proposed method, indicating that for high P_0, the two schemes have the same outage probability. Thus, the proposed scheme achieves significantly better efficiency in terms of the PN-SNR with about the same outage probability.

IV. Multi-Relay Networks with a Sum Power Constraint

In this section, we investigate the PN-SNR maximization problem in general multi-relay networks with a sum power constraint on relays, where the total power consumed by all relays, denoted as P, is no larger than $P_{R,\text{lim}}$, i.e., $P = \sum_{i=1}^{R} P_i \leq P_{R,\text{lim}}$. This sum-power constraint model has been widely used in the literature, e.g., [9], [14], [31]–[33].

From (4) and (8), the PN-SNR maximization problem can be expressed as

$$\begin{aligned} \max_{\boldsymbol{\alpha}\succeq 0} \quad & \frac{(\boldsymbol{\alpha}^T \mathbf{b})^2}{(1+\|\boldsymbol{\alpha}\circ\mathbf{d}\|^2)(1+\|\boldsymbol{\alpha}\circ\mathbf{a}\|^2)} \\ \text{s. t.} \quad & \sum_{i=1}^{R}(1+\alpha_0^2 P_0|f_i|^2)|\alpha_i|^2 \le P_{R,\lim}. \end{aligned} \tag{30}$$

The problem in (30) is a non-convex optimization problem since the objective function is non-convex. In this section, we first simplify the problem into a one-dimensional problem using the results in [9], then prove that the maximum of the simplified problem is unique. Thus, we propose to use a gradient-ascent algorithm to find the optimal solution.

To simplify the problem, we can rewrite (30) as follow:

$$\begin{aligned} \max_{P} \quad & \frac{1}{P_0+P}\left(\max_{\boldsymbol{\alpha}\succeq 0}\frac{P_0(\boldsymbol{\alpha}^T\mathbf{b})^2}{1+\|\boldsymbol{\alpha}\circ\mathbf{d}\|^2}\right) \\ \text{s. t.} \quad & 0 < P \le P_{R,\lim}\ , P = P_0\|\boldsymbol{\alpha}\circ\mathbf{a}\|^2, \end{aligned} \tag{31}$$

With any fixed sum relay power P, the inner problem in (31) is an SNR optimization problem with a sum relay power constraint. This problem is solved in [9] where the optimal power coefficient of the ith relay is

$$\alpha_i = \frac{|f_i g_i|}{|f_i|^2 P_0 + |g_i|^2 P + 1}\sqrt{\frac{P}{\sum_{i=1}^{R}\frac{|f_i|^2|g_i|^2(|f_i|^2P_0+1)}{(|f_i|^2P_0+|g_i|^2P+1)^2}}}, \tag{32}$$

and the corresponding maximum end-to-end received SNR is

$$\text{SNR}_{\max}(P) = \max_{\boldsymbol{\alpha}\succeq 0}\frac{P_0(\boldsymbol{\alpha}^T\mathbf{b})^2}{1+\|\boldsymbol{\alpha}\circ\mathbf{d}\|^2} = \sum_{i=1}^{R}\frac{|f_i|^2|g_i|^2P_0P}{|f_i|^2P_0+|g_i|^2P+1}. \tag{33}$$

Substituting (33) into (31), our PN-SNR maximization problem is reduced to the following one-dimensional problem of finding the optimal sum power P consumed on all relays:

$$\max_{0<P\le P_{R,\lim}}\sum_{i=1}^{R}\frac{|f_i|^2|g_i|^2P_0P}{(|f_i|^2P_0+|g_i|^2P+1)(P+P_0)}. \tag{34}$$

The second order derivative of the objective function can be calculated as

$$\frac{d^2\eta}{dP^2} = \sum_{i=1}^{R}\frac{2|f_i|^2|g_i|^2P_0(|g_i|^4P^3 - |g_i|^2P_0(|f_i|^2P_0^2+1)(3P+P_0) - P_0(|f_i|^2P_0^2+1)^2)}{(P+P_0)^3(|f_i|^2P_0+|g_i|^2P+1)^3}. \tag{35}$$

It can be seen from (35) that $\frac{d^2\eta}{dP^2}$ is not always negative, so the objective function in (34) is not concave in general. We first consider the case when the sum power constraint on relays is unlimited, i.e., $0 < P < \infty$. The following lemma is proved.

Lemma 2. *The objective function in (34) is a semi-strictly quasi-concave function and has only one maximum for* $0 < P < \infty$.

Proof. The objective function in (34) can be expressed as $\frac{\mathrm{SNR}_{\max}(P)}{P+P_0}$ where the nominator is provided in (33). It is easy to verify that

$$\frac{d^2\mathrm{SNR}_{\max}}{dP^2} = -\sum_{i=1}^{R} \frac{2|f_i|^2|g_i|^4 P_0(|f_i|^2P_0+1)}{(|f_i|^2P_0+|g_i|^2P+1)^3} < 0.$$

In other words, the numerator is a strict concave function of P. It is obvious that $P + P_0$ is a convex function and both $\mathrm{SNR}_{\max}$ and $P+P_0$ are positive for $P > 0$. According to Theorem 2.3.8 in [34], the objective function is a semi-strictly quasi-concave function. Moreover, it is shown in [35] that a semi-strictly quasi-concave function has a unique maximum if the numerator is strictly concave. Thus, the maximum of (34) is unique for $P > 0$. □

Denote the optimal sum power as P^*. To find P^*, we propose to use a gradient-ascent algorithm. It has been shown in [36] that by proper step size selection, gradient-ascent algorithm will converge to a stationary point that satisfies $\frac{d\eta}{dP} = 0$. According to Lemma 2, this is also the only stationary point for $P > 0$. Thus, gradient-ascent algorithm will converge to the optimal solution. The complexity of such algorithm is low. Newton's algorithm, for example, has quadratic convergence. In each iteration, Newton's algorithm only needs to calculate the second order derivative in (35).

We next consider the case when $P_{R,\lim}$ is finite. Since the objective function in (34) has a unique maximum at P^*, it is non-decreasing when $P \leq P^*$ and non-increasing when $P \geq P^*$. Thus, the optimal sum relay power in this case can be expressed as

$$P_{\text{opt}} = \min(P^*, P_{R,\lim}). \tag{36}$$

Finally, the optimal power control coefficient for each relay under sum relay power constraint can be easily obtained by using (36) in (32).

Unfortunately, we are unable to analytically investigate the network performance in this case since the optimal solution can only be found numerically. Numerical simulation results of the network performance will be shown in Section VI.

V. Multi-Relay Networks with Separate Power Constraints

In this section, we consider multi-relay networks with separate relay power constraints, where the ith relay has its own power constraint denoted as $P_{i,\lim}$.

According to (4) and (8), our PN-SNR maximization problem can be described as

$$\begin{aligned} \max_{\boldsymbol{\alpha}\succeq 0} \quad & \frac{(\boldsymbol{\alpha}^T\mathbf{b})^2}{(1+\|\boldsymbol{\alpha}\circ\mathbf{d}\|^2)(1+\|\boldsymbol{\alpha}\circ\mathbf{a}\|^2)} \\ \text{s. t.} \quad & 0 \le \alpha_i \le \sqrt{\frac{P_{i,\lim}}{1+P_0|f_i|^2}}, \text{ for } i = 1, \cdots, R, \end{aligned} \tag{37}$$

where $\mathbf{a} = \left[\sqrt{\frac{1}{P_0}+|f_1|^2} \quad \cdots \quad \sqrt{\frac{1}{P_0}+|f_R|^2}\right]$ as defined in Section II.

This is a non-convex optimization problem in which finding the globally optimal solution is usually sophisticated. We first propose a numerical algorithm to obtain the optimal solution. Next, we provide a low-complexity algorithm to find a suboptimal solution for the problem. The performance of the suboptimal solution is simulated in Section VI and compared with that of the optimal solution.

A. *Optimal Solution*

We first examine the Karush-Kuhn-Tucker (KKT) conditions for (37) to better understand the problem. In general, the KKT conditions are not sufficient optimality conditions for non-convex problems. With linear constraint, however, the KKT conditions are necessary optimality conditions. With straightforward calculations, KKT conditions of the problem in (37) can be derived as

$$\alpha_i \left(\alpha_i - \sqrt{\frac{P_{i,\lim}}{1+P_0|f_i|^2}}\right)\frac{\partial \eta}{\partial \alpha_i} = 0.$$

The optimal solution will either be an inner point of the feasible set satisfying $\nabla_{\boldsymbol{\alpha}}\eta = 0$ or be a boundary point meaning that there exists at least one i, such that $\alpha_i = 0$ or $\alpha_i = \sqrt{\frac{P_{i,\lim}}{1+P_0|f_i|^2}}$. If the optimal solution is an inner point, we will show later with Lemma 3 that it can be easily found by gradient-ascent algorithm. However, if it is on the boundary, gradient-ascent algorithm can only converge to a stationary point which may not even be locally optimal [36]. The uniqueness of locally optimal solution of (37) is not guaranteed either.

When the optimal solution is on the boundary, the constraints in (37) are satisfied with equality for some i, which means some relays will transmit with zero or maximum power. The difficulty lies in determining the relays that transmit with zero or maximum power. Exhaustive search for these relays has exponential complexity in the number of relays, and thus, it is obviously impractical. The same problem is encountered in [10] and [15], where optimal and suboptimal relay ordering criteria are proposed to reduce the complexity. In our problem, however, optimal relay ordering criteria may not exist.

Thus, we combine sequential quadratic programming (SQP) algorithm [37] with scatter search to obtain the globally optimal solution. The former algorithm is guaranteed to converge to a locally optimal solution [38] and the latter search starts SQP algorithm from different randomly selected initial points for a number of times to find the globally optimal solution.

SQP algorithm is widely used in solving nonlinear optimization problems whose main idea is to solve a non-convex problem by successive convex approximation [39]. It is also a gradient based iterative algorithm. At each major iteration, a Taylor series approximation of the objective function (or Lagrangian function if nonlinear constraints are involved) at a local iteration point is made. Then, an approximation of the Hessian matrix of the objective function is used to generate a convex quadratic programming (QP) subproblem whose solution is used to form a direction for the next iteration. With properly selected step size, SQP algorithm will converge to a local optimum in finite iterations for arbitrarily small error tolerance. In our simulations, we use Matlab's optimization toolbox to implement the SQP algorithm.

This SQP algorithm is more computationally complex compared with the gradient-ascent algorithm. According to [38], the rate of convergence of SQP algorithm is at best super-linear. Meanwhile, a QP subproblem is involved in each iteration and the SQP algorithm is run several times to find the globally optimal solution. The optimal solution proposed in this subsection is mainly used as a benchmark for performance evaluation.

B. Suboptimal Solution

In this subsection, we will discover a computationally more affordable algorithm to find a suboptimal solution. Recall that in single-relay and multi-relay networks with a sum relay power constraint, while solving the PN-SNR maximization problems, we first find the optimal solutions without any power constraint then project the optimal solution into the feasible set. In both cases, it is either a one-dimensional problem or it can be simplified into a one-dimensional problem in which the projection preserves the optimality. In the PN-SNR maximization problem with separate power constraints on relays, however, projection no longer preserves optimality. Nevertheless, the same methodology can be used to obtain a suboptimal solution. We first ignore the power constraints in (37) and focus on the following problem

$$\max_{\boldsymbol{\alpha}\succeq 0} \frac{(\boldsymbol{\alpha}^T\mathbf{b})^2}{(1+\|\boldsymbol{\alpha}\circ\mathbf{d}\|^2)(1+\|\boldsymbol{\alpha}\circ\mathbf{a}\|^2)}. \tag{38}$$

The following property for the objective function in (38) is proved.

Lemma 3. *The objective function in (38) has unique maximum for* $\boldsymbol{\alpha}\succeq 0$.

Proof. The problem in (38) and (30) have the same objective function while the constraint in (30) is $P_0\|\boldsymbol{\alpha}\circ\mathbf{a}\|^2 < P_{R,\lim}$. Thus, (38) can be viewed as a special case of (30) when $P_{R,\lim}$ is infinity, where the sum relay power constraint is eliminated. According to Lemma 2, the problem in (30) has a unique maximum for all $P_{R,\lim}$. Thus, the maximum for (38) is also unique. □

With Lemma 3, the globally optimal solution for problem (38) can be easily located with the gradient-ascent algorithm used in Section IV. We denote the optimal solution for (38) as $\boldsymbol{\alpha}^*$.

Our suboptimal solution for (37), denoted as $\boldsymbol{\alpha}^{\rm sub}$, is obtained by truncating those entries of $\boldsymbol{\alpha}^*$ whose amplitudes exceed $\sqrt{\frac{P_{i,\rm lim}}{1+P_0|f_i|^2}}$, i.e.,

$$\alpha_i^{\rm sub} = \min\left(\alpha_i^*, \sqrt{\frac{P_{i,\rm lim}}{1+P_0|f_i|^2}}\right) \text{ for } i = 1, \cdots, R.$$

In fact, we know from previous discussion that $\boldsymbol{\alpha}^{\rm sub}$ is the optimal solution if it is an inner point of the feasible set. If it is a boundary point, $\boldsymbol{\alpha}^{\rm sub}$ is suboptimal. We will see in the next section that this suboptimal solution actually has close-to-optimal performance.

VI. Numerical Simulation

In this section, we present the simulated performance of our proposed PN-SNR-maximizing scheme. We also compare the proposed scheme with the fixed relay power scheme and the SNR-maximizing scheme. Channels are randomly generated as i.i.d. circularly symmetric complex Gaussian with zero mean and unit variance in our simulation. The main criterion we use to evaluate the network is the average PN-SNR. Meanwhile, we also simulate the average end-to-end received SNR and the outage probability as alternative criteria for the performance evaluation.

A. Single-Relay Networks

In this subsection, we present the simulation results for single-relay networks. We simulate the average PN-SNR, end-to-end received SNR and outage probability with threshold $\gamma_{\rm th} = 0$ dB for the proposed PN-SNR-maximizing scheme (denoted as "Proposed"), the SNR-maximizing scheme (denoted as "SNR-max") and the fixed relay power scheme (denoted as "Fixed power"). For the relay power constraint, three cases are considered: 1) $P_{R,\rm lim} = \infty$, 2) $P_{R,\rm lim} = 4P_0$, and 3) $P_{R,\rm lim} = P_0$. For fair comparison, in the fixed relay power scheme, the relay power is set to be $P_{\rm ave}$ provided in (15). So the proposed and the fixed relay power schemes use the same amount of power resource. In the SNR-maximizing scheme, the relay always uses its maximum power $P_{R,\rm lim}$, and thus, it consumes more relay power than the other two schemes. Also, the case $P_{R,\rm lim} = \infty$ does not apply.

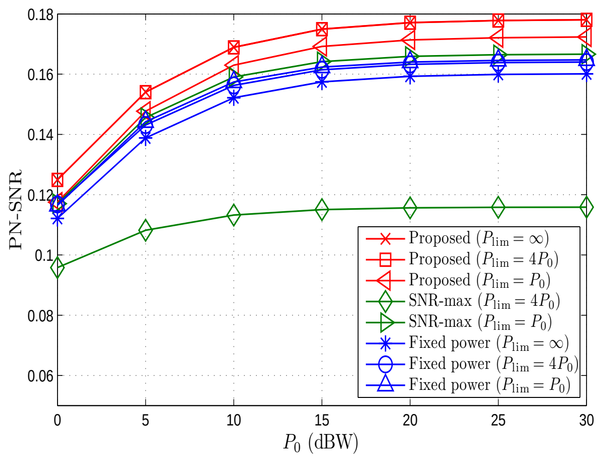

Fig. 2. Average PN-SNR versus P_0 for a single-relay network.

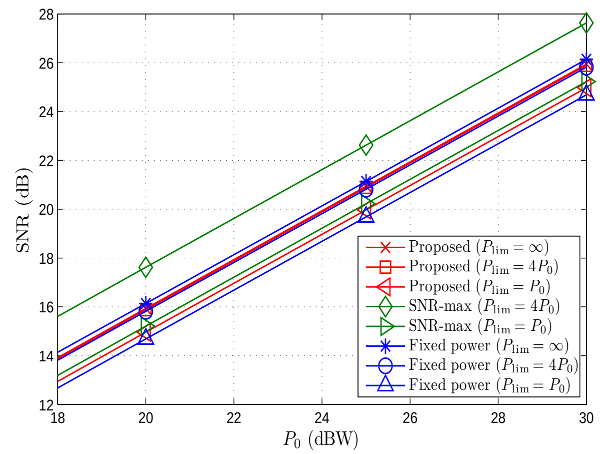

Fig. 3. Average received SNR versus P_0 for a single-relay network.

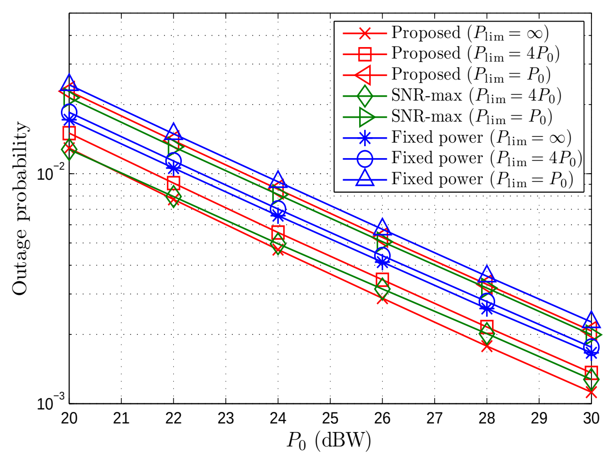

Fig. 4. SNR outage probability versus P_0 for a single-relay network.

Fig. 2 shows the average PN-SNR versus P_0 for the three schemes. We can see that in the proposed PN-SNR-maximizing scheme, the PN-SNR is non-decreasing as $P_{R,\lim}$ increases. In the SNR-maximizing scheme, however, PN-SNR decreases sharply as $P_{R,\lim}$ increases. It can be shown that the PN-SNR decreases to 0 as $P_{R,\lim}$ tends to infinity. In the fixed relay power scheme, the PN-SNR slowly decreases as $P_{R,\lim}$ increases. Among the three schemes, our proposed scheme always achieves the highest PN-SNR. When $P_{R,\lim} = P_0$, the proposed scheme is 5.4% better than the fixed relay power scheme and 4.2% better than the SNR-maximizing scheme at $P_0 = 30$ dBW. When $P_{R,\text{lim}} = 4P_0$, it is 9.2% better than the fixed relay power scheme and 53% better than the SNR-maximizing scheme at $P_0 = 30$ dBW. These observations comply with the theoretical analysis in Section III.

Fig. 3 shows the average end-to-end received SNR versus P_0 for the three schemes. We can see that in the SNR-maximizing scheme, the average SNR increases rapidly as $P_{R,\text{lim}}$ increases, and grows out of bound when $P_{R,\text{lim}}$ approaches infinity. But in both the proposed scheme and the fixed relay power scheme, the average SNR increases but saturates quickly. This is because for these two schemes, only partial relay power is used. When $P_{R,\text{lim}} = P_0$ and $P_0 = 30$ dBW, the average SNR in the proposed scheme is 0.25 dB better than the fixed relay power scheme but 0.2 dB worse than the SNR-maximizing scheme. When $P_{R,\text{lim}} = 4P_0$ and $P_0 = 30$ dBW, the average SNR in the proposed scheme is 0.1 dB better than the fixed relay power scheme, while about 2 dB worse than the SNR-maximizing scheme. When $P_{R,\text{lim}} = \infty$ and $P_0 = 30$ dBW, the average SNR in the proposed scheme is inferior to the fixed relay power scheme by 0.2 dB. The simulation results comply with the analysis in Section III and indicate that our proposed scheme has comparable performance in average SNR with the fixed relay power scheme but it is inferior to the SNR-maximizing scheme.

Fig. 4 shows the outage probability versus P_0 for the three schemes. We can see that as $P_{R,\text{lim}}$ increases, all three schemes have better performance in outage probability. When $P_{R,\text{lim}} = P_0$, our proposed scheme is 0.5 dB superior to the fixed relay power scheme but is 0.1 dB inferior to the SNR-maximizing scheme. When $P_{R,\text{lim}} = 4P_0$, our proposed scheme is 1.2 dB superior to the fixed relay power scheme but is inferior, by 0.25 dB, to the SNR-maximizing scheme. For the extreme case $P_{R,\text{lim}} = \infty$, our proposed scheme is superior by about 1.8 dB to the fixed relay power scheme. The two curves in our scheme and the SNR-maximizing scheme become closer to each other as P_0 increases. These observations are in accordance with the analysis in Section III. We can conclude that the proposed PN-SNR-maximizing scheme outperforms the fixed relay power scheme in outage probability and is comparable to the SNR-maximizing scheme.

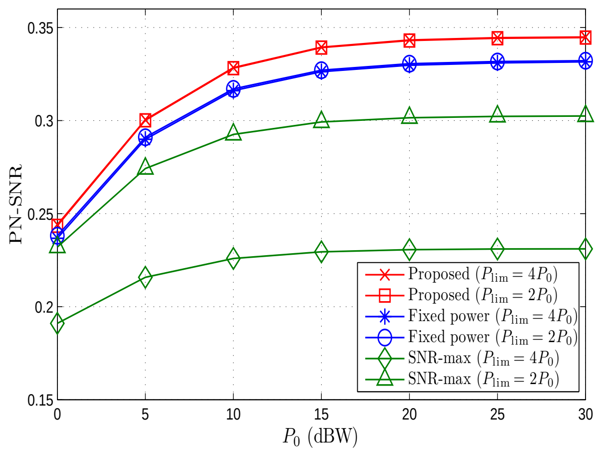

Fig. 5. Average PN-SNR versus P_0 for a two-relay network with sum relay power constraint.

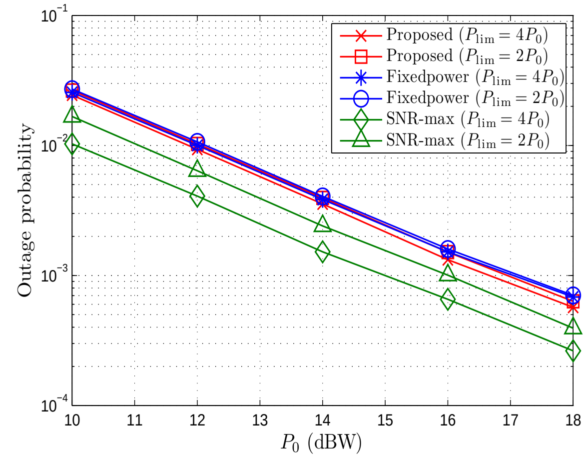

Fig. 6. SNR outage probability versus P_0 for a two-relay network with sum relay power constraint.

We can see from Figs. 2 to 4 that the proposed PN-SNR-maximizing scheme is more efficient in single-relay networks compared with the other two schemes. Meanwhile, with the same power resource, the proposed scheme has comparable performance in the average SNR and is better in outage probability compared with fixed relay power scheme. Naturally, the SNR-maximizing scheme achieves higher SNR than the proposed scheme. But its advantage in outage probability is small and negligible in the high SNR regime. And it has significant lower PN-SNR than the proposed scheme, implying that the power is not efficiently used in this method.

B. Multi-Relay Networks with a Sum Power Constraint on Relays

In this subsection, we present the simulation results for multi-relay networks with a sum relay power constraint. We simulate the average PN-SNR and the outage probability with threshold $\gamma_{\text{th}} = 0$ dB for the proposed PN-SNR-maximizing scheme, the SNR-maximizing scheme, and the fixed relay power scheme. In the fixed relay power scheme, the sum power on relays is fixed for each transmission regardless of the channel quality. For fair comparisons, this fixed power is set to be the average sum relay power P in the proposed scheme. In the SNR-maximizing scheme, the relays always use the maximum sum power $P_{R,\text{lim}}$ to achieve the maximum SNR. The power control coefficient for each relay is obtained according to (32). We simulate a two-relay network with the sum power constraint $P_{R,\lim} = 2P_0$ and $P_{R,\lim} = 4P_0$.

Fig. 5 shows the average PN-SNR versus P_0 for the three schemes. In the PN-SNR-maximizing scheme, the average PN-SNR slightly increases as $P_{R,\lim}$ changes from $2P_0$ to $4P_0$. In the fixed relay power scheme, the average PN-SNR slightly decreases as $P_{R,\lim}$ increases. In the SNR-maximizing scheme, the average PN-SNR sharply decreases as $P_{R,\lim}$ increases. This is the same trend as in single-relay networks. Among the three schemes, the proposed scheme always achieves the highest PN-SNR. When $P_{R,\lim} = 2P_0$ and $P_0 = 30$ dBW, the proposed scheme outperforms the fixed relay power scheme and the SNR-maximizing scheme in term of PN-SNR by 3.8% and 14%, respectively, and the percentage turns to 4% and 48% when $P_{R,\lim} = 4P_0$.

Fig. 6 shows the outage probability versus P_0 for the three schemes. Note that for all three schemes, the outage probabilities decreases as $P_{R,\lim}$ grows from $2P_0$ to $4P_0$. When $P_{R,\lim} = 2P_0$, our proposed scheme outperforms the fixed relay power scheme by about 0.4 dB, but it is 1 dB inferior to the SNR-maximizing scheme. When $P_{R,\lim} = 4P_0$, our proposed scheme is 0.6 dB superior to the fixed relay power scheme but is about 1.8 dB inferior to the SNR-maximizing scheme. We can see that the gap between the proposed scheme and the SNR-maximizing scheme grows larger as $P_{R,\lim}$ increases.

We can conclude from Figs. 5 and 6 that the PN-SNR-maximizing scheme is more power efficient than the other two schemes. Compared with the fixed relay power scheme with the same power resource, the proposed scheme also outperforms in outage probability. In the SNR-maximizing scheme, more power is used to achieve a better outage probability compared with our proposed scheme. But the efficiency of consumed power is low in the sense of producing received SNR. Our results indicate that there is a tradeoff between PN-SNR and received SNR.

C. Multi-Relay Networks with Separate Relay Power Constraints

In this subsection, we investigate the performance of a multi-relay network with separate power constraints on relays. We simulate the average PN-SNR and outage probability with threshold

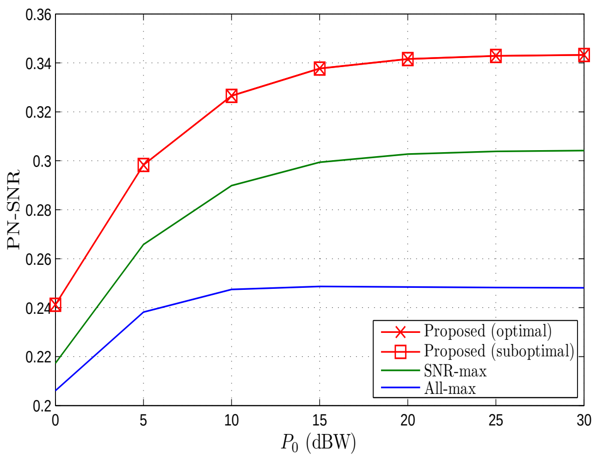

Fig. 7. Average PN-SNR versus P_0 for a two-relay network with separate relay power constraints.

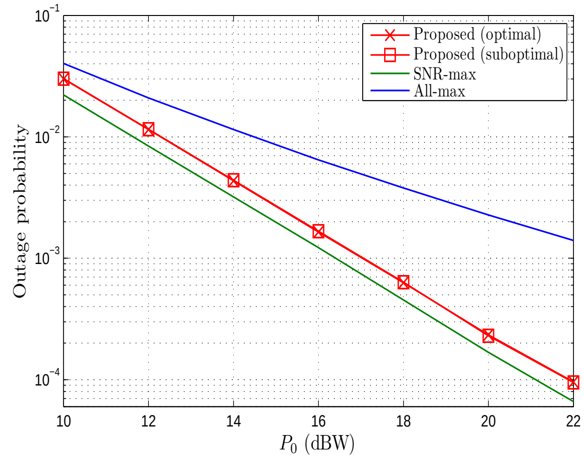

Fig. 8. SNR outage probability versus P_0 for a two-relay network with separate relay power constraints.

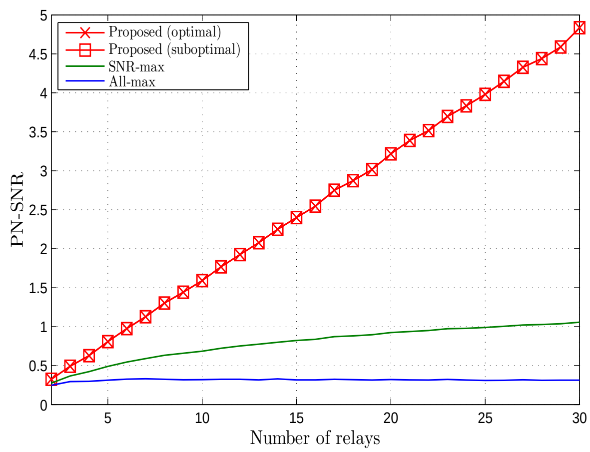

Fig. 9. Average PN-SNR versus number of relays for networks with separate relay power constraints.

$\gamma_{\text{th}} = 0$ dB for the PN-SNR-maximizing scheme (denoted as "Proposed") and compare them with the SNR-maximizing scheme (denoted as"SNR-max") and the all maximum scheme (denoted as "All-max"). In the SNR-maximizing scheme, the optimal beamforming design proposed in [10] is employed to maximize the end-to-end received SNR. In the all maximum scheme, all relays transmit with their maximum power. We first simulate a two-relay network and assume that all nodes have the same power constraint i.e, $P_{i,\text{lim}} = P_0$ for $i = 1, 2$. Next, the average PN-SNR in networks with more that two relays is also simulated.

Fig. 7 shows the average PN-SNR versus P_0 for the three schemes in two-relay networks. First We can see that the PN-SNR of the suboptimal solution is almost the same as the optimal solution. Also, the proposed PN-SNR-maximizing scheme outperforms the other two schemes in terms of the PN-SNR. When $P_0 = 30$ dBW, we can read from the plot that our proposed scheme is superior by 20.4% and 40% compared with the other two schemes.

Fig. 8 shows the outage probability versus P_0 for the three schemes in two-relay networks. We can see that the proposed scheme is 0.7 dB worse in outage probability than the SNR-maximizing scheme. By reading from the slopes of the outage curves, we can also see that the all-maximum scheme loses diversity order while the other two schemes achieve full diversity.

Fig. 9 shows the average PN-SNR in networks with different numbers of relays. The transmit power on transmitter and each relay is set to be 10 dBW. We can first see that our suboptimal solution performs as well as the optimal solution. In the proposed PN-SNR-maximizing scheme, the average PN-SNR increases linearly with the number of the relays. In the SNR-maximizing scheme, however, the average PN-SNR increases with a significant smaller rate and saturates as the number of the relays increases. For the all-maximum scheme, the average PN-SNR remains unchanged as the number of the relays increases.

We can conclude from Figs. 7 to 9 that our PN-SNR-maximizing scheme is more efficient than the other two schemes in using transmit power to provide the received SNR. This scheme also has comparable network performance with the SNR-maximizing scheme in two-relay networks with $P_{i,\lim} = P_0$ for $i = 1, 2$. Even though there is a trade-off between the PN-SNR and the received SNR, the PN-SNR can be a promising measure in designing energy efficient networks.

VII. Conclusion

In this paper, we adopted a new metric, namely power normalized SNR (PN-SNR) to design efficient relay networks, and proposed an optimal relay power control scheme which maximizes this metric. Performance of the proposed scheme is analyzed and compared with existing schemes. Our studies showed that the proposed scheme achieves better PN-SNR while having comparable or even better outage performance compared with the fixed relay power scheme. Compared with the SNR-maximizing design, it has significantly higher PN-SNR with moderate degradation in outage performance. The work discovered the potential of the PN-SNR as an efficiency measure in relay network design.

APPENDIX

A. Proof of Theorem 1

We herein provide the proof of Theorem 1. Define $X \triangleq |f|$ and $Y \triangleq |g|$, then X and Y are Rayleigh distributed whose probability density function (pdf) is $f(x) = 2xe^{-x^2}$ for $x \geq 0$. The average of the relay power in (14) can be calculated as

$$\begin{aligned}
\mathbb{E}\{P_{\text{approx}}\} &= \int_0^{+\infty}\int_0^{+\infty} \min\left(\frac{x}{y}P_0, P_{R,\text{lim}}\right) 4xy \cdot e^{-(x^2+y^2)} dxdy \\
&= \int_0^{+\infty}\left[\int_0^{\frac{P_{R,\text{lim}}}{P_0}y} 4P_0 x^2 e^{-(x^2+y^2)}dx + \int_{\frac{P_{R,\text{lim}}y}{P_0}}^{+\infty} 4P_{R,\text{lim}} xye^{-(x^2+y^2)}dx\right] dy \\
&= \int_0^{+\infty}\left\{2P_0 e^{-y^2}\left[\frac{\sqrt{\pi}}{2}\text{erf}\left(\frac{P_{R,\text{lim}}}{P_0}y\right) - \frac{P_{R,\text{lim}}}{P_0}ye^{-\left(\frac{P_{R,\text{lim}}y}{P_0}\right)^2}\right] + 2P_{R,\text{lim}}ye^{-y^2}e^{-\left(\frac{P_{R,\text{lim}}}{P_0}y\right)^2}\right\} dy \\
&= \int_0^{+\infty}\sqrt{\pi}P_0 e^{-y^2}\text{erf}\left(\frac{P_{R,\text{lim}}}{P_0}y\right)dy = P_0 \tan^{-1}\left(\frac{P_{R,\text{lim}}}{P_0}\right).
\end{aligned}$$

Recall the expression of the PN-SNR in (10) and also $\xi = \frac{P_{R,\text{lim}}}{P_0}$. With the relay power design in (14), the average PN-SNR can be calculated as

$$\begin{aligned}
\eta_{\text{ave}} &\approx \int_0^{+\infty}\int_0^{+\infty} \frac{x^2y^2 \min\left(\frac{x}{y}P_0, P_{R,\text{lim}}\right)}{\left[x^2P_0 + y^2\min\left(\frac{x}{y}P_0, P_{R,\text{lim}}\right)\right]\left[P_0 + \min\left(\frac{x}{y}P_0, P_{R,\text{lim}}\right)\right]} 4xye^{-(x^2+y^2)}dxdy \\
&= 4\int_0^{+\infty}\int_0^{\xi y} \frac{x^3y^3}{(x+y)^2}e^{-(x^2+y^2)}dxdy + 4\frac{\xi}{1+\xi}\int_0^{+\infty}\int_{\xi y}^{+\infty} \frac{x^3y^3}{x^2+\xi y^2}e^{-(x^2+y^2)}dxdy \\
&= 4\int_{\tan^{-1}\left(\frac{1}{\xi}\right)}^{\frac{\pi}{2}} \frac{\cos^3\theta\sin^3\theta}{(\cos\theta+\sin\theta)^2}d\theta + \frac{4\xi}{1+\xi}\int_0^{\tan^{-1}\left(\frac{1}{\xi}\right)} \frac{\cos^3\theta\sin^3\theta}{\cos^2\theta + \xi\sin^2\theta}d\theta \\
&= \frac{3}{8}\pi - \frac{3}{4}\tan^{-1}\left(\frac{1}{\xi}\right) - \frac{4\xi^3 - 7\xi^2 - \xi}{4(\xi+1)(\xi-1)^2} + \frac{2\xi^2\ln\left(\frac{\xi^2+1}{\xi(\xi+1)}\right)}{(\xi-1)^3(\xi+1)}.
\end{aligned}$$

In step three, we use the polar coordinate system and the fact that $\int_0^{+\infty} r^5 e^{-r^2}dr = 1$. From (9), the corresponding average SNR can be obtained in the same way as

$$\begin{aligned}
\text{SNR}_{\text{ave}} &\approx \int_0^{+\infty}\int_0^{+\infty} \frac{x^2y^2\min\left(\frac{x}{y}P_0, P_{R,\text{lim}}\right)}{x^2P_0 + y^2\min\left(\frac{x}{y}P_0, P_{R,\text{lim}}\right)} 4xye^{-(x^2+y^2)}dxdy \\
&= 4P_0\int_0^{+\infty}\int_0^{\xi y} \frac{x^3y^2}{x+y}e^{-(x^2+y^2)}dxdy + 4P_0\xi\int_0^{+\infty}\int_{\xi y}^{+\infty} \frac{x^3y^3}{x^2+\xi y^2}e^{-(x^2+y^2)}dxdy
\end{aligned}$$

$$=4P_0\int_{\tan^{-1}\left(\frac{1}{\xi}\right)}^{\frac{\pi}{2}}\frac{\cos^3\theta\sin^2\theta}{\cos\theta+\sin\theta}d\theta+4P_0\xi\int_0^{\tan^{-1}\left(\frac{1}{\xi}\right)}\frac{\cos^3\theta\sin^3\theta}{\cos^2\theta+\xi\sin^2\theta}d\theta$$
$$=P_0\left[\frac{\pi}{8}-\frac{1}{4}\tan^{-1}\left(\frac{1}{\xi}\right)-\frac{3\xi^3+5\xi}{4(\xi-1)^2(\xi^2+1)}-\frac{1}{4}\ln\left(\frac{(\xi+1)^2}{\xi^2+1}\right)+\frac{2\xi^2\ln\left(\frac{\xi^2+1}{\xi(\xi+1)}\right)}{(\xi-1)^3}\right].$$

Defining $A\triangleq|f|^2$ and $B\triangleq|g|^2$, the outage probability can be expressed as

$$\begin{aligned}
O=&\mathbb{P}\left(\mathrm{SNR}\le\gamma_{\mathrm{th}}\right)=\mathbb{P}\left(\mathrm{SNR}\le\gamma_{\mathrm{th}}\cap X\le\xi Y\right)+\mathbb{P}\left(\mathrm{SNR}\le\gamma_{\mathrm{th}}\cap X\ge\xi Y\right)\\
\approx&\mathbb{P}\left(\frac{X^2YP_0}{X+Y}\le\gamma_{\mathrm{th}}\cap X\le\xi Y\right)+\mathbb{P}\left(\frac{ABP_{R,\lim}}{A+\xi B}\le\gamma_{\mathrm{th}}\cap A\ge\xi^2B\right)\\
=&\mathbb{P}\left(Y\le\frac{\gamma_{\mathrm{th}}X}{P_0X^2-\gamma_{\mathrm{th}}}\cap Y\ge\frac{X}{\xi}\cap P_0X^2\ge\gamma_{\mathrm{th}}\right)+\mathbb{P}\left(X\le\xi Y\cap P_0X^2\le\gamma_{\mathrm{th}}\right)\\
&+\mathbb{P}\left(A\le\frac{\gamma_{\mathrm{th}}\xi B}{P_{R,\lim}B-\gamma_{\mathrm{th}}}\cap A\ge\xi^2B\cap B\ge\frac{\gamma_{\mathrm{th}}}{P_{R,\lim}}\right)+\mathbb{P}\left(A\ge\xi^2B\cap B\le\frac{\gamma_{\mathrm{th}}}{P_{R,\lim}}\right)\\
=&\int_{\sqrt{\frac{\gamma_{\mathrm{th}}}{P_0}}}^{\sqrt{\frac{(1+\xi)\gamma_{\mathrm{th}}}{P_0}}}f_X(x)dx\left(\int_{\frac{x}{\xi}}^{\frac{\gamma_{\mathrm{th}}x}{P_0x^2-\gamma_{\mathrm{th}}}}f_Y(y)dy\right)+\int_0^{\sqrt{\frac{\gamma_{\mathrm{th}}}{P_0}}}f_X(x)dx\left(\int_{\frac{x}{\xi}}^{+\infty}f_Y(y)dy\right)\\
&+\int_{\frac{\gamma_{\mathrm{th}}}{P_{R,\lim}}}^{\frac{\gamma_{\mathrm{th}}}{P_{R,\lim}}\left(1+\frac{1}{\xi}\right)}f_B(b)db\left(\int_{\xi^2b}^{\frac{\gamma_{\mathrm{th}}\xi b}{P_{R,\lim}b-\gamma_{\mathrm{th}}}}f_A(a)da\right)+\int_0^{\frac{\gamma_{\mathrm{th}}}{P_{R,\lim}}}f_B(b)db\left(\int_{\xi^2b}^{+\infty}f_A(a)da\right)\\
=&\frac{\xi^2}{\xi^2+1}\left[1-e^{-\left(1+\frac{1}{\xi^2}\right)(1+\xi)\frac{\gamma_{\mathrm{th}}}{P_0}}\right]-e^{\frac{\gamma_{\mathrm{th}}}{P_0}}\int_0^{\frac{\gamma_{\mathrm{th}}\xi}{P_0}}e^{-\frac{\gamma_{\mathrm{th}}^3}{P_0^3u_1^2}-\frac{\gamma_{\mathrm{th}}^2}{P_0^2u_1}-u_1}du_1 \qquad (39)\\
&+\frac{1}{\xi^2+1}\left[1-e^{-\left(1+\frac{1}{\xi}\right)(1+\xi^2)\frac{\gamma_{\mathrm{th}}}{\xi P_0}}\right]-e^{\left(1+\frac{1}{\xi}\right)\frac{\gamma_{\mathrm{th}}}{P_0}}\int_0^{\frac{\gamma_{\mathrm{th}}}{\xi^2P_0}}e^{-\frac{\gamma_{\mathrm{th}}^2}{\xi P_0^2u_2}-u_2}du_2. \qquad (40)
\end{aligned}$$

The integral in (39) can be upper bounded by

$$\int_0^{\frac{\gamma_{\mathrm{th}}\xi}{P_0}}e^{-\frac{\gamma_{\mathrm{th}}^3}{P_0^3u_1^2}-\frac{\gamma_{\mathrm{th}}^2}{P_0^2u_1}-u_1}du_1\le\int_0^{\frac{\gamma_{\mathrm{th}}\xi}{P_0}}e^{-u_1}du_1=1-e^{-\frac{\gamma_{\mathrm{th}}\xi}{P_0}}=\frac{\gamma_{\mathrm{th}}\xi}{P_0}+\mathcal{O}\left(\frac{1}{P_0^2}\right).$$

It can also be lower bounded by

$$\int_0^{\frac{\gamma_{\mathrm{th}}\xi}{P_0}}e^{-\frac{\gamma_{\mathrm{th}}^3}{P_0^3u_1^2}-\frac{\gamma_{\mathrm{th}}^2}{P_0^2u_1}-u_1}du_1\ge\int_{P_0^{-4/3}}^{\frac{\gamma_{\mathrm{th}}\xi}{P_0}}e^{-\frac{\gamma_{\mathrm{th}}^3}{P_0^3u_1^2}-\frac{\gamma_{\mathrm{th}}^2}{P_0^2u_1}-u_1}du_1\ge e^{-\frac{\gamma_{\mathrm{th}}^3}{P_0^3\left(P_0^{-4/3}\right)^2}-\frac{\gamma_{\mathrm{th}}^2}{P_0^2P_0^{-4/3}}}\int_{P_0^{-4/3}}^{\frac{\gamma_{\mathrm{th}}\xi}{P_0}}e^{-u_1}du_1$$
$$=\left(1-\frac{\gamma_{\mathrm{th}}^3}{P_0^3\left(P_0^{-4/3}\right)^2}-\frac{\gamma_{\mathrm{th}}^2}{P_0^2P_0^{-4/3}}\right)\left(\frac{\gamma_{\mathrm{th}}\xi}{P_0}-P_0^{-4/3}\right)+\mathcal{O}\left(\frac{1}{P_0^{4/3}}\right)=\frac{\gamma_{\mathrm{th}}\xi}{P_0}+\mathcal{O}\left(\frac{1}{P_0^{4/3}}\right).$$

We can see that the dominant terms are the same for the upper and the lower bound. For the second integral, it can be shown in the same manner that

$$\frac{\gamma_{\mathrm{th}}}{\xi^2P_0}+\mathcal{O}\left(\frac{1}{P_0^2}\right)\le\int_0^{\frac{\gamma_{\mathrm{th}}}{\xi^2P_0}}e^{-\frac{\gamma_{\mathrm{th}}^2}{\xi P_0^2u_2}-u_2}du_2\le\frac{\gamma_{\mathrm{th}}}{\xi^2P_0}+\mathcal{O}\left(\frac{1}{P_0^{3/2}}\right)$$

By using Taylor series expansion for large P_0, the outage probability can be bounded as (18).

B. Proof of Lemma 1

With the relay transmit power fixed as P for each transmission, the PN-SNR and the end-to-end received SNR can be expressed as

$$\eta_{\text{fix}} \approx \frac{|fg|^2 PP_0}{(|f|^2P_0+|g|^2P)(P_0+P)}, \quad \text{SNR}_{\text{fix}} \approx \frac{|fg|^2PP_0}{|f|^2P_0+|g|^2P}.$$

The average PN-SNR thus is

$$\begin{aligned}
\eta_{\text{ave_fix}} &\approx \frac{P}{P+P_0}\int_0^{+\infty}\int_0^{+\infty}\frac{P_0x^2y^2}{P_0x^2+Py^2}4xy\cdot e^{-(x^2+y^2)}dxdy\\
&=4\frac{P}{P+P_0}\int_0^{+\infty}\int_0^{+\infty}\frac{x^3y^3}{x^2+P/P_0y^2}e^{-(x^2+y^2)}dxdy\\
&=4\frac{P}{P+P_0}\int_0^{\frac{\pi}{2}}\frac{\cos^3\theta\sin^3\theta}{\cos^2\theta+P/P_0\sin^2\theta}\left(\int_0^{+\infty}r^5e^{-r^2}dr\right)d\theta\\
&=\frac{PP_0}{(P-P_0)^2}-\frac{2P^2P_0^2}{(P-P_0)^3(P+P_0)}\ln\left(\frac{P}{P_0}\right).
\end{aligned}$$

The average end-to-end received SNR can be easily derived as

$$\text{SNR}_{\text{ave\_fix}} \approx \eta_{\text{ave\_fix}}(P_0+P) = \frac{PP_0(P+P_0)}{(P-P_0)^2}-\frac{2P^2P_0^2}{(P-P_0)^3}\ln\left(\frac{P}{P_0}\right).$$

The outage probability with SNR threshold γ_{th} can be derived as [28], [40]

$$\begin{aligned}
O_{\text{fix}} &\approx 1-e^{-\frac{\gamma_{\text{th}}}{P}}e^{-\frac{\gamma_{\text{th}}}{P_0}}2\frac{\gamma_{\text{th}}}{P}\sqrt{\frac{P}{P_0}}K_1\left(2\frac{\gamma_{\text{th}}}{P}\sqrt{\frac{P}{P_0}}\right)\\
&=1-e^{-\frac{\gamma_{\text{th}}}{P}-\frac{\gamma_{\text{th}}}{P_0}}\left[1-\mathcal{O}\left(\frac{\ln(PP_0)}{PP_0}\right)\right]=\left(1+\frac{1}{\zeta}\right)\frac{\gamma_{\text{th}}}{P_0}+\mathcal{O}\left(\frac{\ln(P_0)}{P_0^2}\right).
\end{aligned}$$

This ends the proof. □

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