Redefinition of Max-Min Fairness in Multi-hop Wireless Networks

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***Abstract*—In this paper, it is shown that it is challenging to evaluate service fairness in multi-hop wireless networks due to intra-flow contention and unequal channel capacity. The conventional fairness criterion in wireline networks in terms of flow rate is not appropriate in the wireless environment. Thus, the channel time in the maximal clique is proposed here as an alternative criterion. Based on this criterion, a new definition of max-min fairness for wireless networks is given. This definition is shown to be general for both wireless and wireline networks. Under certain conditions, it is seen to be equivalent to the proportional fairness definition.**

***Index Terms*—Fairness, multi-hop networks, flow rate.**

I. Introduction

In recent years, a variety of novel wireless network architectures have been proposed, including wireless ad hoc networks [1], sensor networks [2], and wireless mesh networks [3], [4]. Normally these wireless networks cover a large geographical area. However, the radio transmission range of any single node (i.e., the range within which all other nodes can correctly receive frames transmitted from the node in the absence of interference) is usually limited. Thus, multi-hop communications are usually needed, in which traffic from a source to its destination traverses multiple intermediate nodes. Thus, determining ways to fairly and efficiently allocate radio resources to multi-hop flows in wireless networks becomes an interesting and important research issue.

Both service fairness and system throughput are major objectives of resource allocation in wireless networks. It is known that these two objectives may conflict with each other [5]. Maximizing the total system throughput may lead to serious unfairness problems, e.g., some flows may be starved. On the other hand, maintaining strict fairness may significantly reduce system efficiency. Max-min fairness is considered to be a good criterion that balances these two conflicting objectives. It prevents starving of any flow and, at the same time, increases the rate of a flow as much as possible when it does not experience many contentions.

Max-min fairness was originally defined for wireline networks in order to allocate transmission rates to flows sharing a set of links, where a flow can traverse several links and a link can be shared by several flows. Here a link is formed by two nodes directly connected to each other. A rate allocation is said to be max-min fair if the rate of any flow cannot be increased without decreasing the rate of any other flow that has an already smaller (or equal) rate [6]. This definition is henceforth referred to as the *conventional max-min fairness definition*. Max-min fairness has also been extended to wireless networks. In [7], max-min fairness is achieved by computing the appropriate bandwidth shares for communication links (one-hop flows) based on the flow contention graph. A token based scheme is proposed in [8] to provide max-min fair scheduling. However, this and other works in this area focus on single-hop flows. The multi-hop flow case is more complicated for the following two reasons. First, for multi-hop flows, the intra-flow correlations between upstream and downstream hops need to be taken into consideration. For resource allocation at the link layer, the medium access control (MAC) protocol deals only with a single hop of a flow. Without consideration of the intra-flow correlation of a multi-hop flow in resource allocation, it is likely that an upstream node of a multi-hop flow experiences fewer contentions, and transmits faster than a downstream node experiencing more contentions. As a result, buffer overflows at the downstream node, and some packets sent from the upstream node are dropped, leading to waste of resources. Second, in multi-hop wireless networks, along with the inter-flow contention, *intra-flow contention* also exists (i.e., downstream and upstream hops of the same flow contend for the channel) [9]. As a result, it is a challenging and open issue to achieve max-min fairness in multi-hop wireless networks.

The contributions of this paper are three-fold. First, we show that the conventional max-min fairness definition originally proposed for wireline networks is not appropriate for wireless multi-hop networks. Second, we propose a new fairness criterion according to the contending characteristics in multi-hop wireless networks, and a new max-min fairness definition for the multi-hop wireless environment. Third, we demonstrate that our new definition is general for both wireline and wireless networks. In addition, we prove that the new definition is equivalent to proportional fairness under appropriate conditions.

We define the new max-min fairness using contention graphs and cliques (to be explained). Given the network topology and flow locations, a contention graph can be obtained, and subsequently the structure of the cliques as well. The use

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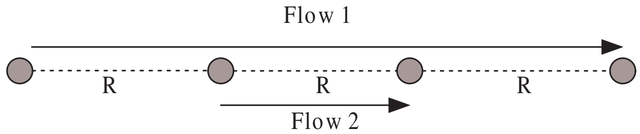


Fig. 1. Multi-hop flows contention: Example 1.

of contention graphs and cliques to capture the contention relationships among links is a commonly used technique for resource allocation in multi-hop wireless networks. Although the clique-based approach has been adopted in many studies (e.g., [10]–[14]), these previous works address different issues from those addressed here. Among these studies, [10] and [13] are related to max-min fairness. However, the focus in [10] and [13] is on achieving conventional rate-based max-min fairness, which is quite different from the goal of our work.

II. Feasibility of Flow Rate as a Fairness Criterion

In the conventional max-min fairness definition, flow rate (i.e., the end-to-end transmission rate of a flow) is used as the criterion to evaluate fairness. This is feasible for wireline networks, in which contentions do not exist among different links. For flows contending for the same link, the link bandwidth is the resource that needs to be fairly allocated to each flow. If allocated the same amount of resources (i.e., link bandwidth), each flow will have the same transmission rate. Thus, it is appropriate to use flow rate as the fairness criterion. However, for a wireless network, this criterion is not feasible due to two characteristics of the wireless environment: intra-flow contention and unequal channel capacity.

A. Effect of Intra-flow Contention

Consider a network topology in Fig. 1, in which a solid line denotes a traffic flow, and the dashed line denotes a wired/wireless link. Two flows contend with each other. Flow 1 has three hops, and flow 2 has one hop. The channel capacity of each link is R. If all the links are wired links, according to the conventional max-min fairness definition, it is max-min fair when the two flows are allocated the same rate (i.e., $R/2$). However, in a wireless environment, the three hops of flow 1 interfere with each other (the immediate neighbors are within the transmission range of each other). When a transmission over one hop occurs, the transmissions over the other hops should defer in order to avoid packet corruption. In contrast to the wired case (where flow 2 contends only with the second hop of flow 1), in the wireless case, flow 2 and the three hops of flow 1 contend with each other. Therefore, it is max-min fair to allocate a rate of $R/4$ to each flow according to the conventional max-min fairness definition. In multi-hop wireless networks, due to the existence of intra-flow contention, transmissions over adjacent hops of a flow contend for common wireless channel time. Thus, the wireless channel time rather than the link bandwidth is the resource that needs to be fairly shared in multi-hop wireless networks.

Note that in wireless multi-hop networks, when allocating the same amount of channel time, the achieved rates of

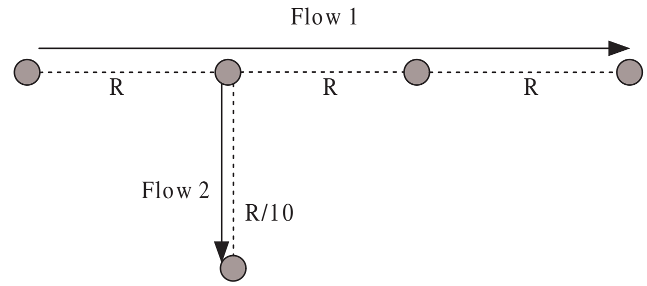


Fig. 2. Multi-hop flows contention: Example 2.

different flows may be different. Consider the example in Fig. 1. When the total channel time are dedicated to flow 2 (i.e., no transmission from flow 1), the maximum rate that can be achieved by flow 2 is R. When the total channel time is dedicated to flow 1, the maximum rate that can be achieved by flow 1 is $R/3$, due to the intra-flow contentions. When the two flows get the same rate of $R/4$, the channel time allocated to flow 1 is three times that of flow 2. The fact that the same flow rate leads to unequal shares of the channel time indicates that it is not appropriate to use the flow rate to evaluate max-min fairness in wireless multi-hop networks.

B. Effect of Unequal Channel Capacity

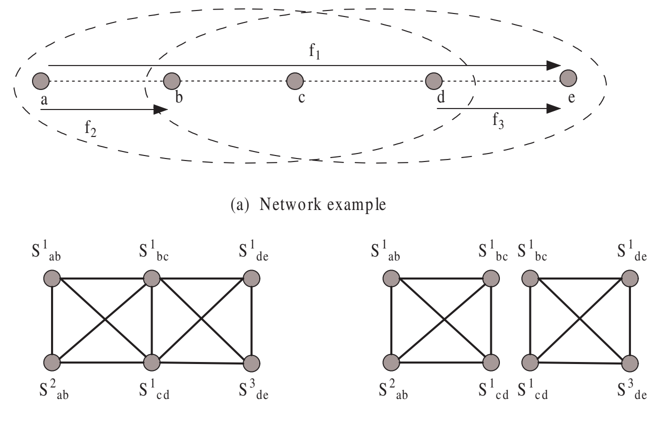
Consider a similar scenario shown in Fig. 2, in which the channel capacity of each link traversed by flow 1 is R, and the channel quality of the link traversed by flow 2 is poor, resulting in a low channel capacity (e.g., $R/10$). To achieve the conventional max-min fairness, equal rate should be allocated to the two flows, resulting in a rate of $R/13$ for each flow. However, the ratio of channel time allocated for flow 1 to that for flow 2 is 3:10. Thus, in this case, flow 2 (with a poor channel) occupies more channel time when the two flows have equal rates. In general, the equal rate allocation tends to allocate a larger fraction of the shared channel time to a flow over a link with lower channel capacity. As a result, all the flows have a very low rate. This is undesirable for resource allocation in wireless networks.

The preceding discussion indicates that shared channel time rather than flow rate should be used to evaluate max-min fairness in wireless networks. The distinction between time-share fairness and throughput (i.e., rate) fairness in multi-rate networks is discussed in [15], where time-share fairness and throughput fairness are considered to be equivalent when a single rate is used by all the nodes. However, the equivalence is valid only for the case of single-hop flows. In the case with multi-hop flows, even with a single rate, the time-share fairness may not be equivalent to the (end-to-end) throughput fairness because of the intra-flow contention. Our work is distinguished from previous related works in that we consider time-share fairness of multi-hop flows.

III. New Definition of Max-Min Fairness for Wireless Networks

A. Network Model

Consider a single-channel multi-hop wireless network. Two nodes that are directly connected to each other (i.e., they are



(a) Network example

(b) Sub-flow contention graph

(c) Decomposition in cliques

Fig. 3. Sub-flow contention graph and cliques.

within the transmission range of each other) form a wireless link. A flow may traverse one or multiple wireless links (hops) from the source to the destination. Each multi-hop flow consists of one or multiple sub-flows, each transmitted over a single wireless link. Note that a packet delivery over a wireless link normally involves two-way communication, i.e., a data packet transmission from the source and an ACK transmission from the destination. To ensure a successful packet delivery, neither transmission should be corrupted. Thus, two sub-flows contend with each other if either the source or destination node of one sub-flow is within the interference range[1] of the source or destination node of the other sub-flow. Contending sub-flows are not allowed to transmit simultaneously; otherwise a collision occurs. Sub-flows that do not contend with each other may transmit simultaneously for frequency reuse.

B. Sub-flow Contention Graphs and Cliques

Each of the multi-hop flows in the network can be divided into several sub-flows, each traversing a single wireless link. To clearly demonstrate the contention relationships among all these sub-flows, a sub-flow contention graph is constructed, using the same method as in [7]. A sub-flow contention graph consists of several vertices and edges. Each vertex represents a sub-flow. If two sub-flows contend with each other, there is an edge connecting the two corresponding vertices in the graph. We illustrate the sub-flow contention graph by using the example shown in Fig. 3(a), where there are three flows f_1, f_2, and f_3, and any two neighboring nodes are within the transmission/interference range of each other (a dashed ellipse represents a clique, to be explained). We use s_{ij}^k to denote the sub-flow of flow f_k from node i to node j. Fig. 3(b) shows the sub-flow contention graph of the network in Fig. 3(a). Sub-flow s_{ab}^2 contends with three sub-flows $s_{ab}^1, s_{bc}^1, s_{cd}^1$, and sub-flow s_{bc}^1 contends with all the other sub-flows. Correspondingly, s_{ab}^2 has an edge to s_{ab}^1, s_{bc}^1, and s_{cd}^1, while s_{bc}^1 has an edge to all the vertices of the graph, as shown in Fig. 3(b).

[1]The interference range of a node is the range within which no other nodes can correctly receive packets if this node is transmitting. Usually the interference range is equal to or larger than the transmission range. For simplicity of presentation, in this paper, we assume that the interference range is equal to the transmission range.

A clique [7] is defined as a subgraph within which any two vertices have an edge, and a maximal clique is a clique which is not included in any other clique. A clique represents a set of vertices (i.e., sub-flows) within which all the sub-flows contend with each other, i.e., at any time, only a single sub-flow within a clique is allowed to transmit. A maximal clique represents a maximal set of those sub-flows that contend with each other. In the following, for simplicity of presentation, a clique means a maximal clique. A sub-flow contention graph can be decomposed into several cliques. We use Fig. 3(a) as an example, where all the sub-flows included in an ellipse contend with each other. In this example, we observe that there are two cliques $\{s_{ab}^1, s_{bc}^1, s_{cd}^1, s_{ab}^2\}$ and $\{s_{bc}^1, s_{cd}^1, s_{de}^1, s_{de}^3\}$. Correspondingly, the sub-flow contention graph can be decomposed into two cliques, as shown in Fig. 3(c). Note that a sub-flow may belong to several cliques (e.g., sub-flows s_{bc}^1 and s_{cd}^1 belong to both of the two cliques).

C. Necessary Condition of Feasible Resource Allocation

Now we investigate condition[2] for feasible resource allocation for all the multi-hop flows. Suppose there are N multi-hop flows in the network, denoted by f_k $(k \in \{1, 2, \dots, N\})$. Denote the end-to-end rate of flow f_k by r_k, and the channel capacity (rate) of the wireless link from node i to node j by R_{ij}. The set of cliques of the network is denoted by $C = \{C_1, C_2, \dots, C_m\}$, where m is the number of cliques. If a flow has at least one sub-flow in a clique, we say the flow belongs to that clique.

If an allocation vector $(r_1, r_2, \dots, r_N)$ to the N multi-hop flows is feasible, then the following condition must be satisfied:

$$\sum_{s_{ij}^k \in C_q} \frac{r_k}{R_{ij}} \leq 1, \quad \forall q \in \{1, 2, \dots, m\} \tag{1}$$

where $\frac{r_k}{R_{ij}}$ represents the fraction of time occupied by sub-flow s_{ij}^k in clique C_q. Since sub-flows within one clique cannot transmit simultaneously, the summation of all the fractions of time cannot exceed 1. From this constraint, the channel time in each clique is treated as the amount of resources in the clique.

D. Overloaded Clique and Bottleneck Clique

For any flow f_k, a clique C_q is called its overloaded clique, if the following three conditions are satisfied

- Flow f_k belongs to clique C_q;
- Suppose the rate vector of the flows (belonging to clique C_q) is $(r_1, r_2, \dots, r_n)$, then there does not exist any other feasible rate vector $(r_1', r_2', \dots, r_n')$ where $\forall j, r_j' \geq r_j$ and $\exists i$ such that $r_i' > r_i$; and
- For any other clique $C_{q'}$ to which flow f_k belongs, $\sum_{s_{ij}^l \in C_q} \frac{r_l}{R_{ij}} \geq \sum_{s_{ij}^l \in C_{q'}} \frac{r_l}{R_{ij}}$.

[2]Here, the feasibility condition is necessary but not sufficient. The purpose of the max-min fairness definition is to provide a criterion for determining whether or not a given resource allocation algorithm is max-min fair, but not for ensuring the implementation feasibility of such an algorithm. In [7], it has been noted that a max-min fair rate allocation may not be feasible to be scheduled by time-division multiple-access (TDMA). The problem of ensuring and/or implementing a feasible max-min fair allocation is beyond the scope of this paper.

The second condition indicates that in an overloaded clique C_q, the rate of any flow in C_q cannot be further increased without decreasing the rate of any other flow in C_q. As a result, the channel time utilization of clique C_q cannot be further increased. The third condition indicates that the channel time utilization of clique C_q is the maximal one (i.e., clique C_q is the most loaded) among all the cliques to which flow f_k belongs.

For clique C_q, we denote T_k^q as the fraction of time occupied by flow f_k within that clique, which is the sum of the fractions of time of all flow f_k's sub-flows belonging to that clique. A clique C_q is called a bottleneck clique of flow f_k, if the following two conditions are satisfied

- C_q is an overloaded clique of flow f_k; and
- $T_k^q \geq T_{k'}^q, \quad \forall k'$ such that $f_{k'}$ belongs to clique C_q, and $f_{k'} \neq f_k$.

The first condition indicates that the resources of the bottleneck clique must be fully-utilized by all the contending sub-flows. The second condition indicates that in C_q, flow f_k gets resources (i.e., a fraction of channel time) not less than those assigned to any other flow.

E. Max-min Fairness in Multi-hop Wireless Networks

Max-min fairness for a multi-hop wireless network can now be re-defined as follows.

Max-min fairness: Given N multi-hop flows in the network, an end-to-end flow rate allocation vector (r_1, r_2,…,r_N) to the N multi-hop flows is max-min fair if the allocation is feasible, and if for any flow f_k, r_k cannot be increased without decreasing the fraction of time $T_{k'}^q$ for any other flow $f_{k'}$ in any overloaded clique C_q of f_k where $T_{k'}^q \leq T_k^q$.

One property of the newly defined max-min fairness is that, when a rate vector is max-min fair (according to the new definition), for every flow, there is at least one bottleneck clique. If a flow f_k does not have a bottleneck clique, r_k could be increased without decreasing the resources (i.e., the fraction of time) shared by any other flows that have a less or equal resource share (compared with flow f_k). Thus, this contradicts the definition of max-min fairness. On the other hand, if all the flows have at least one bottleneck clique, the corresponding allocation must be max-min fair. Observe that when every flow has a bottleneck clique, if any r_k is increased, the resources occupied by f_k in its bottleneck clique will increase as well. Then at least one other flow traversing flow f_k's bottleneck clique must have a resource share decreased from what is already smaller than or equal to that of flow f_k according to the definition of a bottleneck clique.

Consider the examples in Figs. 1 and 2. There are four sub-flows, contending within one clique in the network. According to the new definition, max-min fairness is achieved when flows 1 and 2 get the same fraction of time in the clique (i.e., this clique becomes the bottleneck clique for both flows). Therefore, with the new definition of max-min fairness, for the case in Fig. 1, flow 1 and flow 2 get the rate of $R/6$ and $R/2$, respectively. For the case in Fig. 2, flow 1 and flow 2 get the rate of $R/6$ and $R/20$, respectively. Note that in both cases, the system throughput (aggregate rate) based on the new fairness definition is higher than that based on the conventional max-min fairness definition. This is because the conventional max-min fairness definition tends to allocate more channel time to the flows with more contentions and/or poorer channels. Thus, the system throughput is reduced. On the contrary, by allocating the same amount of channel time instead of the same rate to each flow, the new definition achieves better system efficiency. Also note that when there exists a flow with very poor channel quality, the conventional max-min fairness tends to slow down the rates of all the flows (which is not fair to other flows with good channel quality). By the new definition, other flows are immunized from such rate decreasing.

IV. Further Discussion

A. General Fairness Definition for Wireless/Wireline Networks

For a wireline network, the contention relationship does not exist among different hops. So the sub-flows over a hop form a clique, and also a maximal clique. In each maximal clique, a flow has up to one sub-flow, i.e., there is no intra-flow contention in a maximal clique. In this case, if two flows are assigned the same amount of channel time in a clique, this also means that they achieve the same rate. Thus, our new definition is equivalent to the conventional definition in the wireline case. Therefore, our new fairness definition is a general definition for both wireless and wireline networks.

B. Comparison with Proportional Fairness

Besides max-min fairness, proportional fairness [16] is another commonly used fairness notion, which is based on utility. The utility of a flow is the logarithm maximization of its rate. Proportional fairness is achieved when the system maximizes the sum of the utilities of all flows.

An interesting observation is that under appropriate conditions, our new definition of max-min fairness is equivalent to the proportional fairness.

Proposition 1: When all the flows share a common bottleneck clique, our new definition of max-min fairness is equivalent to proportional fairness.

Proof: Consider a network with N flows. Let r_k denote the allocated rate of flow k ($k \in \{1, 2, \ldots, N\}$). To achieve proportional fairness, the allocated rate vector $(r_1, r_2, \ldots, r_N)$ should be the solution of the following optimization problem

$$\text{Maximize} \left\{ \sum_{k=1}^{N} \log r_k \right\}. \tag{2}$$

When all the flows share a common bottleneck clique (denoted by C_q), the above problem has the following constraint

$$\sum_{k=1}^{N} r_k \cdot \left(\sum_{s_{ij}^k \in C_q} \frac{1}{R_{ij}} \right) = U \tag{3}$$

where U (≤ 1) is the maximal achievable channel time utilization of clique C_q, which is the sum of the time fractions occupied by all the sub-flows belonging to C_q. Let $T_k^q = r_k \cdot (\sum_{s_{ij}^k \in C_q} \frac{1}{R_{ij}})$. For a given network, the term $\sum_{s_{ij}^k \in C_q} \frac{1}{R_{ij}}$ is a constant, denoted by a_k^q. Then problem (2) is rewritten as Maximize $\{\sum_{k=1}^{N} \log \frac{T_k^q}{a_k^q}\}$, and the constraint

(3) is rewritten as $\sum_{i=1}^{N} T_i^q = U$. It is straightforward to see that when $T_1^q = T_2^q = \ldots = T_N^q = U/N$, $\sum_{k=1}^{N} \log \frac{T_k^q}{a_k^q}$ is maximized. Note that T_k^q is actually the fraction of time used by flow k in the clique C_q (i.e., the sum of the fractions of time used by all the sub-flows of flow k within the clique C_q). On the other hand, by the new definition of max-min fairness, we have equal T_k^q for all the flows, which is equivalent to proportional fairness. ■

Proposition 2: When each flow has one and only one bottleneck clique, our new definition of max-min fairness is equivalent to proportional fairness.

Proof: Consider a network with N flows. Without loss of generality, we assume that there are m bottleneck cliques in the network, where flows 1 to N_1 share clique C_1, flows N_1+1 to N_2 share clique $C_2, \ldots$, and flows $N_{m-1}+1$ to N_m (=N) share clique C_m. Thus, we have the following constraints.

$$\begin{cases} \sum_{k=1}^{N_1} r_k \cdot (\sum_{s_{ij}^k \in C_1} \frac{1}{R_{ij}}) = U_1 \\ \sum_{k=N_1+1}^{N_2} r_k \cdot (\sum_{s_{ij}^k \in C_2} \frac{1}{R_{ij}}) = U_2 \\ \vdots \\ \sum_{k=N_{m-1}+1}^{N_m} r_k \cdot (\sum_{s_{ij}^k \in C_m} \frac{1}{R_{ij}}) = U_m \end{cases} \quad (4)$$

where $U_1, \ldots, U_m (\leq 1)$ are the maximal achievable channel time utilization of cliques $C_1, \ldots, C_m$, respectively. The optimization problem (2) can be rewritten as

$$\text{Maximize} \left\{ \sum_{k=1}^{N_1} \log r_k + \sum_{k=N_1+1}^{N_2} \log r_k + \ldots + \sum_{k=N_{m-1}+1}^{N} \log r_k \right\}. \quad (5)$$

Since the m constraints in (4) are independent with each other, the problem (5) can be decomposed into m independent optimization sub-problems. The lth sub-problems is

$$\text{Maximize} \left\{ \sum_{k=N_{l-1}+1}^{N_l} \log r_k \right\}, \quad l = 1, 2, \ldots, m \quad (6)$$

where $N_0 = 0$, with constraint

$$\sum_{k=N_{l-1}+1}^{N_l} r_k \cdot \left(\sum_{s_{ij}^k \in C_l} \frac{1}{R_{ij}} \right) = U_l. \quad (7)$$

For each sub-problem, we have shown in Proposition 1 that the maximum value is obtained when all the contending flows equally share the fraction of time within that clique, which is also the condition for the newly defined max-min fairness. ■

Proportional fairness and max-min fairness are two widely used fairness criteria. In general, they are not related. In this work, we show that when we redefine the max-min fairness as time-based instead of rate-based (i.e., the channel time but not the flow rate is allocated among flows), the proportional fairness and the newly defined max-min fairness are equivalent in some cases. This result illustrates that proportional fairness also has an inherent characteristic of sharing the channel time in a fair manner.

Consider the examples shown in Fig. 4, where the capacity of every wireless link is assumed to be R. In case (a), all the flows share a common bottleneck clique, illustrated by the dashed ellipse. In case (b), flows 1 and 3 share a bottleneck

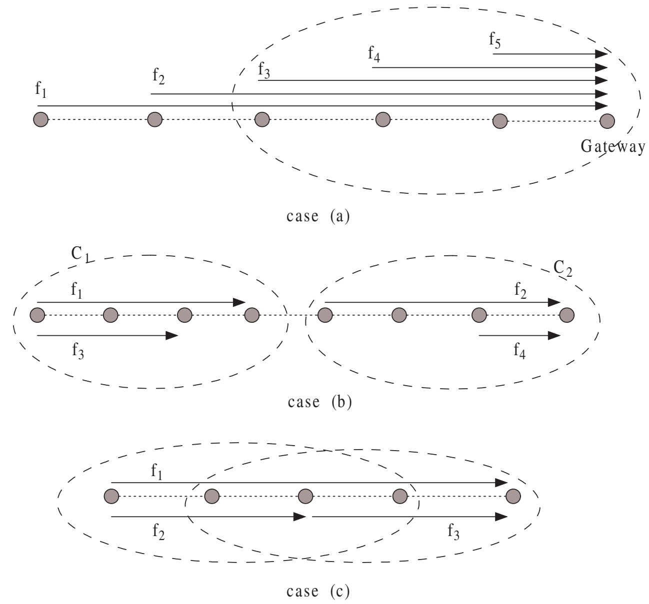


Fig. 4. The cases for comparison of fairness definitions.

clique C_1, while flows 2 and 4 share another bottleneck clique C_2. For cases (a) and (b), in addition to the bottleneck cliques represented by the dashed ellipses, there also exist other cliques that are not shown in the figure. In case (c), the two cliques overlap with each other. Table I shows the allocated rate of each flow in these three cases based on the conventional max-min fairness, our new max-min fairness, and proportional fairness. It can be seen straightforwardly that all the rate vectors are feasible to be scheduled by TDMA. We can see that our new fairness definition and proportional fairness have equivalent rate allocation vectors for cases (a) (i.e., the case of Proposition 1) and (b) (i.e., the case of Proposition 2), but not for case (c). The results also demonstrate that our new max-min fairness definition achieves higher system throughput than the conventional definition in all the cases.

V. CONCLUSION

The characteristics of wireless networks determine that flow rate is not appropriate as a criterion to evaluate max-min fairness in multi-hop wireless networks. Rather, the channel time in a clique better reflects the resources consumed in either wireless networks or wireline networks, and thus is a better and general criterion. This research provides insight into the problem of resource management for multi-hop wireless, wireline, or hybrid networks with fairness consideration.

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TABLE I
RATE ALLOCATION FOR THE THREE CASES IN FIG. 4

Case	Fairness definition	f_1	f_2	f_3	f_4	f_5	Total rate
(a)	New max-min	$R/15$	$R/15$	$R/15$	$R/10$	$R/5$	$R/2$
	Conventional max-min	$R/12$	$R/12$	$R/12$	$R/12$	$R/12$	$5R/12$
	Proportional	$R/15$	$R/15$	$R/15$	$R/10$	$R/5$	$R/2$
(b)	New max-min	$R/6$	$R/6$	$R/4$	$R/2$	-	$13R/12$
	Conventional max-min	$R/5$	$R/4$	$R/5$	$R/4$	-	$9R/10$
	Proportional	$R/6$	$R/6$	$R/4$	$R/2$	-	$13R/12$
(c)	New max-min	$2R/15$	$R/5$	$R/5$	-	-	$8R/15$
	Conventional max-min	$R/6$	$R/6$	$R/6$	-	-	$R/2$
	Proportional	$R/9$	$2R/9$	$2R/9$	-	-	$5R/9$

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