On the Impact of User Scheduling on Diversity and Fairness in Cooperative NOMA

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***Abstract*—In this correspondence paper, we investigate the problem of user scheduling in a cooperative non-orthogonal multiple access (NOMA) system consisting of a base station, a weak user, and K intermediate users. During each transmission, an intermediate user is scheduled to receive its own message and forward the message destined for the weak user. For this type of cooperative NOMA system, a novel scheduling scheme is proposed to achieve full diversity and scheduling fairness simultaneously. With the consideration that all channels experience independent but non-identically distributed Rayleigh fading, outage probabilities of the weak user and the scheduled intermediate user are derived in closed-form expressions. It is theoretically shown that the proposed scheme provides full diversity for both the weak user and the scheduled intermediate user. Furthermore, theoretical results also demonstrate that the proposed scheme schedules each intermediate user with the same probability $1/K$, demonstrating that scheduling fairness is also guaranteed.**

***Index Terms*—Cooperative non-orthogonal multiple access, fairness, user scheduling.**

I. Introduction

As a promising candidate of multiple access technique for 5G, non-orthogonal multiple access (NOMA) can serve multiple users by the same resource to achieve significant spectrum efficiency gain over conventional orthogonal multiple access [1]–[3]. Recently, cooperative relaying has been incorporated into NOMA to enhance its reliability and coverage, in which the information is forwarded by a dedicated relay [4]–[7]. Actually, as NOMA employs superimposed coding, some users may successfully decode their own messages along with the messages of other users. Thus, recruiting these successful users as potential relays can save the cost of deploying dedicated relays and fully utilize the degree of freedom offered by NOMA. Inspired by this observation, some cooperative NOMA strategies are investigated in recent works [8]–[11]. Considering that NOMA is performed among K users, a cooperative NOMA strategy is designed in [8], where the users that successfully decode their messages by using successive interference cancellation (SIC) sequentially forward other decoded messages. This strategy requires $(K-1)$ extra time-slots for forwarding information, and thus, experiences some spectrum efficiency loss. To enhance the spectrum efficiency, user scheduling for cooperative NOMA is investigated in [9]–[11], where only the scheduled users participate in cooperative NOMA. Three location-based user scheduling schemes are proposed in [9]. However, as the impact of channel fading is not considered, these schemes cannot achieve full diversity. In [10] and [11], two user scheduling based cooperative NOMA strategies are proposed for wireless multicast, illustrating that full diversity can be achieved by properly scheduling users according to instantaneous channel information.

The existing studies [9]–[11] tend to focus on improving the reliability of cooperative NOMA via user scheduling, without considering fairness among users. In fact, these schemes may prefer to schedule some users with certain channel conditions, leading to unfair channel access opportunity among the users who are willing to serve as relays. Motivated by this fact, we investigate user scheduling for cooperative NOMA in this paper, aiming at achieving full diversity and simultaneously ensuring fairness among users.

The main contributions of this paper are summarized as follows. 1) We design a novel cooperation paradigm for a NOMA system consisting of a base station (BS), a weak user (WU) and K intermediate users (IUs), in which each IU is willing to help the WU. An IU scheduling scheme is proposed to achieve a full diversity order while ensuring fairness among all IUs. 2) We derive closed-form outage probabilities for the WU and the scheduled IU, respectively. Further, we prove that the proposed scheduling scheme provides a diversity order of K for both the WU and the scheduled IU. In other words, full diversity is achieved. 3) We theoretically evaluate the scheduling fairness among IUs. It is shown that the proposed scheduling scheme ensures that each IU is scheduled with the same probability $1/K$, indicating that a fair channel access opportunity is provided for each IU.

II. System Model

In cellular communications from a BS denoted as S, it is possible that a user denoted as D may not have direct link

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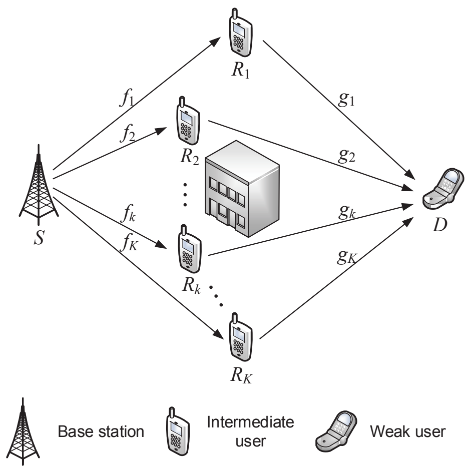

Fig. 1. The system under investigation.

from the BS, for example, due to blockage of a tall building as illustrated in Fig. 1. The user is thus called a WU. This work investigates how the BS can send information to the WU. Since the BS cannot directly communicate to the WU, it seeks relaying services by other users. And to motivate other users to provide relaying services, a user who provides relaying services for the WU can receive its own information from the BS by using NOMA principles. Thus, the system explores the channel conditions of other users, and finds K users denoted as $R_1, \cdots, R_K$ that have links to both the BS and the WU, and are willing to provide relaying services to the WU. These users are called IUs. The BS then recruits one IU, and sends both the WU's and the selected IU's signals to the IU. The selected IU decodes both signals by using SIC, and subsequently forwards the WU's signal to the WU. The details of the cooperation strategy and scheduling scheme are given in Section III.

The BS, WU, and IUs all work in a half-duplex mode with a single antenna. The channel coefficients of link $S \rightarrow R_k$ and link $R_k \rightarrow D$ for $k \in \mathcal{K} \triangleq \{1, ..., K\}$ are denoted as f_k and g_k, respectively. All the links have independent but non-identically distributed (i.n.i.d.) Rayleigh block fading. Thus, the channel gains $|f_k|^2$ and $|g_k|^2$ follow exponential distribution with means F_k and G_k, respectively. The noise at each receiver is modeled as additive white Gaussian noise (AWGN) with identical variance σ^2.

III. Cooperation Strategy and IU Scheduling

A. Cooperation Strategy

Prior to each transmission, one of IUs is scheduled by the IU scheduling scheme that will be proposed in Section III-B. Afterwards, the cooperative NOMA transmission is performed within two phases. Without loss of generality, we assume that IU R_k is scheduled.

During the first phase, the BS sends a superposed message $\sqrt{P_S \alpha} x_0 + \sqrt{P_S \beta} x_1$ to the scheduled IU R_k, where P_S is the transmit power of the BS, x_0 is the desired message of D, x_1 is the desired message of R_k, and α and β are power allocation coefficients with $\alpha + \beta = 1$ and $\alpha > \beta$. The received signal at R_k can be expressed as $y_{R_k} = \sqrt{P_S \alpha} f_k x_0 + \sqrt{P_S \beta} f_k x_1 + n_1$, where n_1 represents the AWGN at R_k. According to the principles of NOMA, R_k decodes message x_0 first by treating message x_1 as interference. Thus, the signal-to-interference-plus-noise ratio (SINR) for R_k to decode x_0 is given by

$$\gamma_{Sk} = \frac{\alpha |f_k|^2}{\beta |f_k|^2 + \rho^{-1}}, \tag{1}$$

where $\rho \triangleq \frac{P_S}{\sigma^2}$ is the *transmit signal-to-noise ratio (SNR)*. If x_0 is correctly decoded, R_k removes x_0 from its observation and further decodes its desired message x_1 with SNR being

$$\tilde{\gamma}_{Sk} = \rho |f_k|^2 \beta. \tag{2}$$

Here we use ˜ for notations related to signal x_1. Denoting the target rates of messages x_0 and x_1 as r_0 and r_1, respectively, the condition for R_k to correctly decode both messages is given by $\{\gamma_{Sk} \geq 2^{2r_0} - 1 \triangleq \gamma_{\text{th}}\} \cap \{\tilde{\gamma}_{Sk} \geq 2^{2r_1} - 1 \triangleq \tilde{\gamma}_{\text{th}}\}$. If R_k correctly decodes both messages, it will forward message x_0 to D in the second phase[1]. Accordingly, the observation at D can be expressed as $y_D = \sqrt{P_R} g_k x_0 + n_0$, where P_R is the transmit power of R_k, and n_0 is the AWGN at D. Defining $\mu \triangleq P_R / P_S$, the SNR for D to decode its desired message x_0 in the second phase is given by

$$\gamma_{kD} = \mu \rho |g_k|^2, \tag{3}$$

and the condition that D correctly decodes x_0 is $\{\gamma_{kD} \geq \gamma_{\text{th}}\}$. On the other hand, if R_k cannot decode both messages, the second phase will be cancelled and the BS will proceed to send new messages in a new transmission block.

Note that α and β should satisfy $\alpha - \beta \gamma_{\text{th}} > 0$ due to the following reason [9]. From (1), it can be seen that $\gamma_{Sk} < \alpha / \beta$ for any $k \in \{1, ..., K\}$. If $\alpha - \beta \gamma_{\text{th}} \leq 0$, then we have $\gamma_{Sk} < \alpha / \beta \leq \gamma_{\text{th}}$, which means that no IU can successfully decode x_0 during the first phase.

B. IU Scheduling

The purpose of our IU scheduling is to achieve full diversity and fairness simultaneously. Recall that the diversity gain is measured in high SNR regime. As seen from (1), when $\rho \rightarrow \infty$, γ_{Sk} approaches α / β for $k = 1, ..., K$. Since $\alpha - \beta \gamma_{\text{th}} > 0$, we further have $\gamma_{Sk} \simeq \alpha / \beta > \gamma_{\text{th}}$ in the high SNR regime. This fact indicates that, in the high SNR regime, message x_0 can be decoded by any IU. On the other hand, from (2) and (3), when $\rho \rightarrow \infty$, $\tilde{\gamma}_{Sk}$ and γ_{kD} are still related to the channel gains $|f_k|^2$ and $|g_k|^2$, respectively. This observation demonstrates that, even with a sufficiently high transmit SNR, it is still possible that $\tilde{\gamma}_{Sk} < \tilde{\gamma}_{\text{th}}$ and $\gamma_{kD} < \gamma_{\text{th}}$ under certain channel conditions, which means that the scheduled IU and the WU cannot successfully decode their desired messages. Thus, for achieving diversity gain at the scheduled IU and the WU, we only need to improve the reception quality for decoding their desired messages. Since the target rates of messages x_0 and x_1 are different in general, we consider the normalized SNRs $\tilde{\Gamma}_{Sk} \triangleq \tilde{\gamma}_{Sk} / \tilde{\gamma}_{\text{th}}$

[1]Here we assume that the scheduled IU forwards x_0 only when it has correctly decoded both messages. If the scheduled IU cannot decode its own message, it may not have any incentive to help the WU.

and $\Gamma_{kD} \triangleq \gamma_{kD}/\gamma_{\rm th}$ as the reception metrics for R_k and D to decode their desired messages. Then, to simultaneously achieve diversity gain for the scheduled IU and the WU's decoding of their desired messages, the IU R_{k^*} can be scheduled from all IUs as $k^* = \arg\max_{k\in\mathcal{K}}\left[\min\left\{\tilde{\Gamma}_{Sk}, \Gamma_{kD}\right\}\right] = \arg\max_{k\in\mathcal{K}}\left[\min\left\{\frac{\beta|f_k|^2}{\tilde{\gamma}_{\rm th}}, \frac{\mu|g_k|^2}{\gamma_{\rm th}}\right\}\right]$. However, since the channel gains $|f_k|^2$ and $|g_k|^2$ are non-identically distributed, the above scheduling favors IUs with larger F_k and G_k. Thus, to ensure fairness among IUs, we normalize $|f_k|^2$ and $|g_k|^2$ with respect to their average values F_k and G_k, respectively, leading to the following scheduling scheme

$$k^* = \arg\max_{k\in\mathcal{K}}\left[\min\left\{\frac{\beta|f_k|^2}{\tilde{\gamma}_{\rm th}F_k}, \frac{\mu|g_k|^2}{\gamma_{\rm th}G_k}\right\}\right]. \tag{4}$$

IV. Performance Analysis

A. Outage Probability

1) Exact outage probability of the WU: Recall that the scheduled IU R_{k^*} forwards message x_0 only when it correctly decodes both messages. Thus, the complementary event of D experiencing an outage is that R_{k^*} correctly decodes both messages and D correctly decodes message x_0, i.e., $\{\gamma_{Sk^*} \geq \gamma_{\rm th}, \tilde{\gamma}_{Sk^*} \geq \tilde{\gamma}_{\rm th}, \gamma_{k^*D} \geq \gamma_{\rm th}\}$. Based on this fact, the outage probability of D can be expressed as

$$\begin{aligned} P_{out,D} &= 1 - \Pr\left[\gamma_{Sk^*} \geq \gamma_{\rm th}, \tilde{\gamma}_{Sk^*} \geq \tilde{\gamma}_{\rm th}, \gamma_{k^*D} \geq \gamma_{\rm th}\right] \\ &= 1 - \Pr\left[\frac{\alpha|f_{k^*}|^2}{\beta|f_{k^*}|^2+\rho^{-1}} \geq \gamma_{\rm th}, \rho|f_{k^*}|^2\beta \geq \tilde{\gamma}_{\rm th}, \right. \\ &\qquad \left. \mu\rho|g_{k^*}|^2 \geq \gamma_{\rm th}\right] \\ &= 1 - \Pr\left[\rho|f_{k^*}|^2 \underbrace{\min\left\{\frac{\alpha-\beta\gamma_{\rm th}}{\gamma_{\rm th}}, \frac{\beta}{\tilde{\gamma}_{\rm th}}\right\}}_{\triangleq\eta} \geq 1, \right. \\ &\qquad \left. \frac{\rho|g_{k^*}|^2}{\gamma_{\rm th}/\mu} \geq 1\right] \\ &= \Pr\left[\min\left\{\frac{\rho|f_{k^*}|^2}{\eta}, \frac{\rho|g_{k^*}|^2}{\gamma_{\rm th}/\mu}\right\} < 1\right] \\ &= \sum_{i=1}^{K} \Pr\left[\min\left\{\frac{\rho|f_i|^2}{\eta}, \frac{\rho|g_i|^2}{\gamma_{\rm th}/\mu}\right\} < 1, k^* = i\right], \end{aligned} \tag{5}$$

where $\Pr[\cdot]$ means probability. According to the proposed scheduling scheme shown in (4), the event $\{k^* = i\}$ can be equivalently expressed as

$$\bigcap_{k\in\mathcal{K}\backslash\{i\}}\left\{Z_k < \frac{1}{\rho}\min\left\{\frac{\beta X}{\tilde{\gamma}_{\rm th}F_i}, \frac{\mu Y}{\gamma_{\rm th}G_i}\right\}\right\}, \tag{6}$$

where $X \triangleq \rho|f_i|^2$, $Y \triangleq \rho|g_i|^2$, and $Z_k \triangleq \min\left\{\frac{\beta|f_k|^2}{\tilde{\gamma}_{\rm th}F_k}, \frac{\mu|g_k|^2}{\gamma_{\rm th}G_k}\right\}$ for $k \in \mathcal{K}$. Applying (6) into (5), we have $P_{out,D} = \sum_{i=1}^{K} P_{out,D,i}$ with $P_{out,D,i}$ being expressed as

$$\begin{aligned} P_{out,D,i} &= \Pr\left[\min\left\{\frac{X}{\eta}, \frac{Y}{\gamma_{\rm th}/\mu}\right\} < 1, \right. \\ &\qquad \left. \bigcap_{k\in\mathcal{K}\backslash\{i\}}\left\{Z_k < \frac{1}{\rho}\min\left\{\frac{\beta X}{\tilde{\gamma}_{\rm th}F_i}, \frac{\mu Y}{\gamma_{\rm th}G_i}\right\}\right\}\right] \\ &= \iint\limits_{\min\{\frac{x}{\eta}, \frac{\mu y}{\gamma_{\rm th}}\}<1} p_{X,Y}(x,y) \prod_{k\in\mathcal{K}\backslash\{i\}} F_{Z_k}\left(\frac{1}{\rho}\right. \\ &\qquad \left. \times \min\left\{\frac{\beta x}{\tilde{\gamma}_{\rm th}F_i}, \frac{\mu y}{\gamma_{\rm th}G_i}\right\}\right) {\rm d}x{\rm d}y, \end{aligned} \tag{7}$$

where $p_{X,Y}(\cdot,\cdot)$ is the joint probability density function of X and Y, and $F_{Z_k}(\cdot)$ is the cumulative distribution function of Z_k. As channel gains $|f_k|^2$ and $|g_k|^2$ follow exponential distributions with means F_k and G_k for $k = 1, ..., K$, we have

$$\begin{cases} p_{X,Y}(x,y) = \frac{1}{\rho^2 F_i G_i} e^{-(x/F_i + y/G_i)/\rho} \\ F_{Z_k}(z) = 1 - e^{-\Xi z} \end{cases} \tag{8}$$

where $\Xi \triangleq \tilde{\gamma}_{\rm th}/\beta + \gamma_{\rm th}/\mu$. An expression of $P_{out,D,i}$ is derived in the Appendix, shown as

$$P_{out,D,i} = \begin{cases} I_{i,1} + I_{i,2} + I_{i,3}, & a \leq b_i \\ Q_{i,1} + Q_{i,2} + Q_{i,3}, & a > b_i \end{cases} \tag{9}$$

where $a \triangleq \frac{\mu\eta}{\gamma_{\rm th}}$ and $b_i \triangleq \frac{\tilde{\gamma}_{\rm th}\mu F_i}{\beta\gamma_{\rm th}G_i}$. Here, the closed-form expressions for $I_{i,1}$, $I_{i,2}$ and $I_{i,3}$ are given by (A.6), (A.7) and (A.9), and the closed-form expressions for $Q_{i,1}$, $Q_{i,2}$ and $Q_{i,3}$ are given by (A.10), (A.11) and (A.13).

2) Exact outage probability of the scheduled IU: The complementary event of R_{k^*} experiencing an outage is that R_{k^*} correctly decodes both messages, i.e., $\{\gamma_{Sk^*} \geq \gamma_{\rm th}, \tilde{\gamma}_{Sk^*} \geq \tilde{\gamma}_{\rm th}\}$, indicating that the outage probability of R_{k^*} can be expressed as $P_{out,R_{k^*}} = 1 - \Pr\left[\gamma_{Sk^*} \geq \gamma_{\rm th}, \tilde{\gamma}_{Sk^*} \geq \tilde{\gamma}_{\rm th}\right]$. Following the derivations in (5), (6) and (7), we further have

$$\begin{aligned} &P_{out,R_{k^*}} \\ &= \sum_{i=1}^{K} \Pr\left[X < \eta, \bigcap_{k\in\mathcal{K}\backslash\{i\}}\left\{Z_k < \frac{1}{\rho}\right.\right. \\ &\qquad \left.\left. \times \min\left\{\frac{\beta X}{\tilde{\gamma}_{\rm th}F_i}, \frac{\mu Y}{\gamma_{\rm th}G_i}\right\}\right\}\right] \\ &= \sum_{i=1}^{K} \iint\limits_{x<\eta} \prod_{k\in\mathcal{K}\backslash\{i\}} F_{Z_k}\left(\frac{1}{\rho}\min\left\{\frac{\beta x}{\tilde{\gamma}_{\rm th}F_i}, \frac{\mu y}{\gamma_{\rm th}G_i}\right\}\right) \\ &\qquad \times p_{X,Y}(x,y){\rm d}x{\rm d}y \\ &= \sum_{i=1}^{K} (J_{i,1} + J_{i,2}), \end{aligned} \tag{10}$$

with $J_{i,1} \triangleq \int_0^\eta \int_0^{x/b_i} \prod_{k\in\mathcal{K}\backslash\{i\}} F_{Z_k}\left(\frac{\mu y}{\rho\gamma_{\rm th}G_i}\right) p_{X,Y}(x,y){\rm d}y{\rm d}x$ and $J_{i,2} \triangleq \int_0^\eta \int_{x/b_i}^\infty \prod_{k\in\mathcal{K}\backslash\{i\}} F_{Z_k}\left(\frac{\beta x}{\rho\tilde{\gamma}_{\rm th}F_i}\right) p_{X,Y}(x,y){\rm d}y{\rm d}x$. Then, following the derivations in the Appendix, closed-form expressions for $J_{i,1}$ and $J_{i,2}$ are derived as

$$\begin{aligned} J_{i,1} &= \sum_{j=0}^{K-1} \binom{K-1}{j} \frac{(-1)^j}{1+j\Xi\mu/\gamma_{\rm th}} \left(1 - e^{-\frac{\eta}{\rho F_i}}\right. \\ &\quad - \frac{1/F_i}{1/F_i + 1/(G_i b_i) + j\mu\Xi/(\gamma_{\rm th}G_i b_i)} \\ &\quad \left. \times \left(1 - e^{-\frac{\eta}{\rho}\left(\frac{1}{F_i} + \frac{1}{G_i b_i} + \frac{j\Xi\mu}{\gamma_{\rm th}G_i b_i}\right)}\right)\right), \end{aligned} \tag{11}$$

$$\begin{aligned} J_{i,2} &= \sum_{j=0}^{K-1} \binom{K-1}{j} \frac{(-1)^j/F_i}{1/F_i + 1/(G_i b_i) + j\Xi\beta/(\tilde{\gamma}_{\rm th}F_i)} \\ &\quad \times \left(1 - e^{-\frac{\eta}{\rho}\left(\frac{1}{F_i} + \frac{1}{G_i b_i} + \frac{j\Xi\beta}{\tilde{\gamma}_{\rm th}F_i}\right)}\right). \end{aligned} \tag{12}$$

B. Diversity Orders

When transmit SNR $\rho \to \infty$, we have

$$\begin{aligned} &\prod_{k\in\mathcal{K}\backslash\{i\}} F_{Z_k}\left(\frac{1}{\rho}\min\left\{\frac{\beta x}{\tilde{\gamma}_{\rm th}F_i}, \frac{\mu y}{\gamma_{\rm th}G_i}\right\}\right) \\ &= \left(1 - e^{-\frac{\Xi}{\rho}\min\left\{\frac{\beta x}{\tilde{\gamma}_{\rm th}F_i}, \frac{\mu y}{\gamma_{\rm th}G_i}\right\}}\right)^{K-1} \\ &\overset{\rho\to\infty}{\simeq} \left[\frac{\Xi}{\rho}\min\left\{\frac{\beta x}{\tilde{\gamma}_{\rm th}F_i}, \frac{\mu y}{\gamma_{\rm th}G_i}\right\}\right]^{K-1}, \end{aligned} \tag{13}$$

where the asymptotic equality uses the fact that $e^{-x} \simeq 1-x$ holds for $x \to 0$. Applying (13) into (7) and following the derivations in the Appendix, we have

$$\begin{cases} I_{i,1} \overset{\rho\to\infty}{\simeq} Q_{i,1} \overset{\rho\to\infty}{\simeq} \frac{\gamma_{\rm th}}{\mu(G_i)^K} \times \frac{\Xi^{K-1}}{K\rho^K}, \\ I_{i,2} \overset{\rho\to\infty}{\simeq} \frac{\beta^{K-1}(b_i^K - a^K)\gamma_{\rm th}^{K+1}}{\mu^{K+1}\tilde{\gamma}_{\rm th}^{K-1}(F_i)^K G_i} \times \frac{\Xi^{K-1}}{(K+1)K\rho^{K+1}}, \\ Q_{i,2} \overset{\rho\to\infty}{\simeq} \frac{\mu^{K-1}(b_i^{-K} - a^{-K})\eta^{K+1}}{\gamma_{\rm th}^{K-1}(G_i)^K F_i} \times \frac{\Xi^{K-1}}{(K+1)K\rho^{K+1}}, \\ I_{i,3} \overset{\rho\to\infty}{\simeq} Q_{i,3} \overset{\rho\to\infty}{\simeq} \frac{\beta^{K-1}\eta^K}{\tilde{\gamma}_{\rm th}^{K-1}(F_i)^K} \times \frac{\Xi^{K-1}}{K\rho^K}. \end{cases} \tag{14}$$

Applying the results in (14) into (9) with ignoring high order infinitesimals (i.e., the asymptotic expressions for $I_{i,2}$ and $Q_{i,2}$), and then combining the result with the fact $P_{out,D} = \sum_{i=1}^K P_{out,D,i}$, the high-SNR asymptotic expression for $P_{out,D}$ is obtained as

$$P_{out,D} \overset{\rho\to\infty}{\simeq} \frac{\Xi^{K-1}}{K\rho^K} \sum_{i=1}^K \left(\frac{\gamma_{\rm th}}{\mu(G_i)^K} + \frac{\beta^{K-1}\eta^K}{\tilde{\gamma}_{\rm th}^{K-1}(F_i)^K} \right) \propto \frac{1}{\rho^K}. \tag{15}$$

Similarly, applying (13) into (10) and ignoring high order infinitesimals yields

$$P_{out,R_{k^*}} \overset{\rho\to\infty}{\simeq} \frac{\Xi^{K-1}\beta^{K-1}\eta^K}{K\rho^K \tilde{\gamma}_{\rm th}^{K-1}} \sum_{i=1}^K \frac{1}{(F_i)^K} \propto \frac{1}{\rho^K}. \tag{16}$$

From (15) and (16) we know that a diversity order of K is achieved by both the WU and the scheduled IU, demonstrating that the proposed scheduling scheme fully exploits the spatial diversity at both users.

C. Scheduling Fairness among IUs

Differing from [12] and [13] which enable fairness by the rate maximization of the worst-case user, we here guarantee the fairness among users from the perspective of providing each IU with a fair opportunity for channel access. With the proposed scheduling scheme shown in (4), the probability that an IU $R_i (i \in \mathcal{K})$ is scheduled can be derived as follows.

$$\begin{aligned} &\Pr[k^* = i] \\ &\overset{\rm (i)}{=} \Pr\left[\bigcap_{k\in\mathcal{K}\setminus\{i\}} \left\{ Z_k < \tfrac{1}{\rho} \min\left\{ \tfrac{\beta X}{\tilde{\gamma}_{\rm th} F_i}, \tfrac{\mu Y}{\gamma_{\rm th} G_i} \right\} \right\} \right] \\ &= \int_0^\infty \int_0^\infty \prod_{k\in\mathcal{K}\setminus\{i\}} F_{Z_k}\left(\tfrac{1}{\rho} \min\left\{ \tfrac{\beta x}{\tilde{\gamma}_{\rm th} F_i}, \tfrac{\mu y}{\gamma_{\rm th} G_i} \right\} \right) \\ &\qquad \times p_{X,Y}(x,y) \mathrm{d}x \mathrm{d}y \\ &\overset{\rm (ii)}{=} \int_0^\infty \int_0^\infty \sum_{j=0}^{K-1} \tbinom{K-1}{j} (-1)^j e^{-\frac{j\Xi}{\rho} \min\left\{ \frac{\beta x}{\tilde{\gamma}_{\rm th} F_i}, \frac{\mu y}{\gamma_{\rm th} G_i} \right\}} \\ &\qquad \times \tfrac{1}{\rho^2 F_i G_i} e^{-\frac{1}{\rho}\left(\frac{x}{F_i} + \frac{y}{G_i} \right)} \mathrm{d}x \mathrm{d}y \\ &= \sum_{j=0}^{K-1} \tbinom{K-1}{j} \tfrac{(-1)^j}{\rho^2 F_i G_i} \Big(\int_0^\infty \int_0^{b_i y} e^{-\frac{j\Xi\beta x}{\rho\tilde{\gamma}_{\rm th} F_i} - \frac{x}{\rho F_i} - \frac{y}{\rho G_i}} \mathrm{d}x \mathrm{d}y \\ &\qquad + \int_0^\infty \int_{b_i y}^\infty e^{-\frac{j\Xi\mu y}{\rho\gamma_{\rm th} G_i} - \frac{x}{\rho F_i} - \frac{y}{\rho G_i}} \mathrm{d}x \mathrm{d}y \Big) \\ &= \sum_{j=0}^{K-1} \tbinom{K-1}{j} (-1)^j \Big[\tfrac{b_i/F_i}{b_i/F_i + 1/G_i + j\Xi\beta b_i/(\tilde{\gamma}_{\rm th} F_i)} \\ &\qquad + \tfrac{1/G_i}{b_i/F_i + 1/G_i + j\Xi\mu/(\gamma_{\rm th} G_i)} \Big], \end{aligned} \tag{17}$$

where step (i) comes from (6), and step (ii) comes from (8). Applying $b_i = \frac{\tilde{\gamma}_{\rm th}\mu F_i}{\beta\gamma_{\rm th} G_i}$ and $\Xi = \frac{\tilde{\gamma}_{\rm th}}{\beta} + \frac{\gamma_{\rm th}}{\mu}$ into the last equality of (17) with some algebraic manipulations, we further have

$$\begin{aligned} &\Pr[k^* = i] = \sum_{j=0}^{K-1} \tbinom{K-1}{j} \tfrac{(-1)^j}{1+j} \\ &= 1 - \sum_{j=1}^{K-1} \tbinom{K-1}{j} \tfrac{(-1)^{j+1}}{1+j} \overset{\rm (iii)}{=} 1 - \tfrac{K-1}{K} = \tfrac{1}{K}, \end{aligned} \tag{18}$$

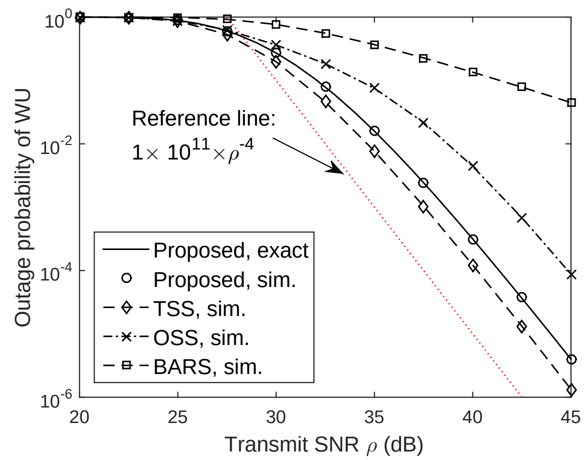

Fig. 2. Outage probability of the WU D.

where step (iii) comes from [14, eq. (0.155.1)]. As observed from (18), each IU is scheduled with the same probability $1/K$, demonstrating that the proposed scheduling scheme can ensure scheduling fairness among IUs.

V. Numerical Results and Discussion

This section verifies the theoretical results by simulation. The BS and the WU are located at coordinates $(0,0)$ and $(0,100)$, respectively. There are $K=4$ IUs. Locations of the IUs are randomly generated within a circle centered at (50,0) with radius being 25, and the resulted IU locations are $(44.1655, -5.6186)$, $(38.7878, 6.7263)$, $(72.7884, -3.4627)$ and $(57.3251, 7.4178)$. The path-loss attenuation is modeled as $1/[1+(d/d_0)^\kappa]$ [9], where d is the distance, $d_0 = 20$ is the path-loss reference distance and $\kappa = 2.2$ is the path-loss exponent. The target rates of messages x_0 and x_1 are $r_0 = 0.5$ and $r_1 = 1$, respectively. Thus, the SINR/SNR thresholds for successfully decoding messages x_0 and x_1 are $\gamma_{\rm th} = 2^{2r_0} - 1 = 1$ and $\tilde{\gamma}_{\rm th} = 2^{2r_1} - 1 = 3$, respectively. Other parameters are $\alpha = 0.9$, $\beta = 0.1$ and $\mu = P_R/P_S = 0.01$. The simulated values are obtained from 1×10^8 independent numerical trials.

In our simulation, we compare the proposed scheme with the following existing schemes for cooperative NOMA. 1) *Two-stage scheduling* (TSS), which follows the rationale of two-stage relay selection in [7] and operates in two stages: the first stage is to find the set of IUs that guarantee the reliable reception of message x_0, i.e., $\mathcal{S}_r \triangleq \{R_k | \min\{\gamma_{Sk}, \gamma_{kD}\} \geq \gamma_{\rm th}\}$, while the second stage is to schedule an IU from $\mathcal{S}_r$ to maximize the received SNR of message x_1, i.e., $k^* = \arg\max_{R_k\in\mathcal{S}_r} \tilde{\gamma}_{Sk}$. 2) *One-stage scheduling* (OSS), which follows the rationale of single-stage relay selection in [15] and schedules an IU to maximize the reception quality of message x_0 in both phases, i.e., $k^* = \arg\max_{k=1,\ldots,K} \min\{\gamma_{Sk}, \gamma_{kD}\}$. 3) *Best average reception scheduling* (BARS) [16], which schedules an IU with the best average channel gain from the BS, i.e., $k^* = \arg\max_{k=1,\ldots,K} F_k$.

Figs. 2 and 3 depict the outage probabilities of the WU D and the scheduled IU R_{k^*}, respectively. As shown in

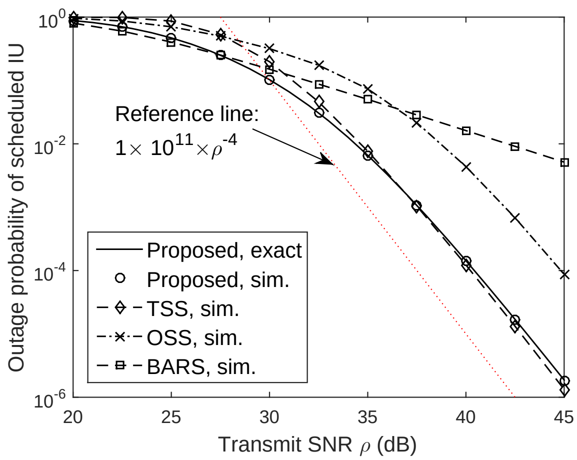

Fig. 3. Outage probability of scheduled IU R_{k^*}.

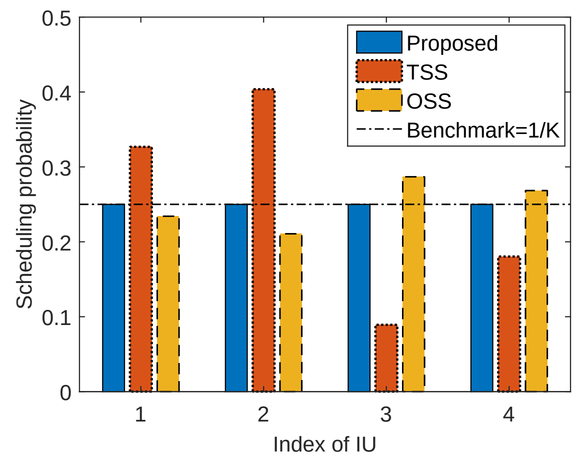

Fig. 4. Scheduling probabilities of IUs.

both figures, the derived exact outage probabilities of our scheme exactly match the simulated values, which verifies our theoretical analysis. Further, by comparing the derived outage probabilities of our proposed scheme with the reference line $1 \times 10^{11} \times \rho^{-4}$ in both figures, it is observed that, in high SNR regime ($\rho \geq 40$ dB), the outage probabilities of the WU and the scheduled IU decrease at a rate ρ^{-4}, demonstrating that full diversity is achieved by them.

Figs. 2 and 3 also compare our proposed scheme with the TSS, OSS and BARS schemes. As shown in Fig. 2, compared with the TSS scheme, the proposed scheme provides a higher outage probability for the WU. As shown in Fig. 3, for the scheduled IU, both schemes have similar outage performance: the proposed scheme performs better when $\rho < 37.5$ dB, while the TSS scheme slightly performs better when $\rho > 37.5$ dB. Thus, compared to the TSS scheme, the proposed scheme has some outage performance loss at the WU. The reason is as follows. The TSS scheme guarantees the reliability of the WU first, and then, maximizes the reception quality of the scheduled IU. Thus, the TSS scheme is expected to provide better reliability for the WU than the proposed scheme. Furthermore, the proposed scheme needs to ensure scheduling fairness among the IUs, which may lead to some outage performance loss at the WU. Thus, better scheduling fairness of our proposed scheme is achieved at the cost of some outage performance loss at the WU.[2] From Figs. 2 and 3, the proposed scheme outperforms the OSS scheme in the outage probabilities of both the WU and the scheduled IU. The reason is two-fold. First, the OSS scheme does not consider the reception of message x_1 intended by the scheduled IU, thus providing worse outage performance for the scheduled IU. Second, if the scheduled IU does not successfully decode its own message, it has no incentive to forward message x_0 intended by the WU, thus leading to an outage at the WU. For the BARS scheme, it schedules an IU based on average channel gains. Thus, when the topology of the system does not change, the BARS scheme always schedules the same IU, resulting in worse outage performance than that of our scheme, as seen in Figs. 2 and 3.

Fig. 4 presents simulation results of the probabilities that the IUs are scheduled, where transmit SNR is $\rho = 40$ dB. Recall that the BARS scheme always schedules the same IU when the system topology does not change, meaning that the BARS scheme provides no fairness for the IU scheduling. Thus, the BARS scheme is not included in the comparison of scheduling fairness. As seen in Fig. 4, the proposed scheme schedules each IU with the same probability equal to $1/K$, verifying our theoretical result in (18). This fact indicates that the proposed scheme can guarantee scheduling fairness among IUs while providing full diversity for both the WU and the scheduled IU. On the other hand, the TSS and OSS schemes cannot schedule each IU with the same probability, implying that these two schemes cannot provide fair opportunity for each IU to receive its desired information.

VI. Conclusion

In this paper, we present a novel user scheduling scheme for a cooperative NOMA system, in which an IU is scheduled to receive its own message and forward the message destined for the WU. With the consideration of i.n.i.d. Rayleigh fading, the outage performance and scheduling fairness are theoretically evaluated. The derived results indicate that the proposed scheme achieves full diversity and scheduling fairness simultaneously.

Appendix A
Derivations of Equation (9)

Defining $a \triangleq \frac{\mu\eta}{\gamma_{\text{th}}}$ and $b_i \triangleq \frac{\tilde{\gamma}_{\text{th}}\mu F_i}{\beta\gamma_{\text{th}}G_i}$ for $i = 1, ..., K$, the minimum operations in (7) can be further expressed as

$$\min\left\{\frac{x}{\eta}, \frac{\mu y}{\gamma_{\text{th}}}\right\} = \begin{cases} \mu y/\gamma_{\text{th}}, & x \geq ay, \\ x/\eta, & x < ay, \end{cases} \tag{A.1}$$

$$\min\left\{\frac{\beta x}{\tilde{\gamma}_{\text{th}}F_i}, \frac{\mu y}{\gamma_{\text{th}}G_i}\right\} = \begin{cases} \mu y/(\gamma_{\text{th}}G_i), & x \geq b_i y, \\ \beta x/(\tilde{\gamma}_{\text{th}}F_i), & x < b_i y. \end{cases} \tag{A.2}$$

[2]In the TSS scheme, IU R_3 may not be motivated to participate in the cooperation, as the probability that R_3 is scheduled is below 10% as shown in Fig. 4. On the other hand, in our proposed scheme, all IUs are motivated to participate in the cooperation, as each of them has an equal opportunity to be scheduled.

When $a \le b_i$, by applying (A.1) and (A.2) into (7), $P_{out,D,i}$ in (7) can be expressed as $P_{out,D,i} = I_{i,1} + I_{i,2} + I_{i,3}$ with

$$I_{i,1} \triangleq \iint\limits_{\substack{x>b_i y \\ \mu y/\gamma_{\rm th}<1}} \prod_{k\in\mathcal{K}\backslash\{i\}} F_{Z_k}\left(\frac{\mu y}{\rho\gamma_{\rm th}G_i}\right) p_{X,Y}(x,y){\rm d}x{\rm d}y, \tag{A.3}$$

$$I_{i,2} \triangleq \iint\limits_{\substack{ay<x\le b_i y \\ \mu y/\gamma_{\rm th}<1}} \prod_{k\in\mathcal{K}\backslash\{i\}} F_{Z_k}\left(\frac{\beta x}{\rho\tilde{\gamma}_{\rm th}F_i}\right) p_{X,Y}(x,y){\rm d}x{\rm d}y, \tag{A.4}$$

$$I_{i,3} \triangleq \iint\limits_{\substack{x\le ay \\ x/\eta<1}} \prod_{k\in\mathcal{K}\backslash\{i\}} F_{Z_k}\left(\frac{\beta x}{\rho\tilde{\gamma}_{\rm th}F_i}\right) p_{X,Y}(x,y){\rm d}x{\rm d}y. \tag{A.5}$$

Substituting (8) into (A.3), we have

$$\begin{aligned} I_{i,1} &= \int_0^{\gamma_{\rm th}/\mu}\int_{b_i y}^{\infty}\sum_{j=0}^{K-1}\binom{K-1}{j}(-1)^j e^{-\frac{j\Xi\mu y}{\rho\gamma_{\rm th}G_i}} \\ &\quad\times\frac{1}{\rho^2 F_i G_i}e^{-(x/F_i+y/G_i)/\rho}{\rm d}x{\rm d}y \\ &= \sum_{j=0}^{K-1}\binom{K-1}{j}\frac{(-1)^j}{\rho G_i}\int_0^{\gamma_{\rm th}/\mu} e^{-\frac{y}{\rho}\left(\frac{j\Xi\mu}{\gamma_{\rm th}G_i}+\frac{b_i}{F_i}+\frac{1}{G_i}\right)}{\rm d}y \\ &= \sum_{j=0}^{K-1}\binom{K-1}{j}\frac{(-1)^j/G_i}{b_i/F_i+1/G_i+j\Xi\mu/(\gamma_{\rm th}G_i)} \\ &\quad\times\left(1-e^{-\frac{\gamma_{\rm th}}{\mu\rho}\left(\frac{b_i}{F_i}+\frac{1}{G_i}+\frac{j\Xi\mu}{\gamma_{\rm th}G_i}\right)}\right). \end{aligned} \tag{A.6}$$

Similarly, combining (8) with (A.4) and (A.5) yields

$$I_{i,2} = \sum_{j=0}^{K-1}\binom{K-1}{j}\frac{(-1)^j[\varphi_i(a)-\varphi_i(b_i)]}{1+j\Xi\beta/\tilde{\gamma}_{\rm th}} \tag{A.7}$$

with $\varphi(x)$ defined as

$$\begin{aligned} \varphi_i(x) &\triangleq \frac{1/G_i}{1/G_i+x/F_i+j\Xi\beta x/(\tilde{\gamma}_{\rm th}F_i)} \\ &\quad\times\left(1-e^{-\frac{\gamma_{\rm th}}{\mu\rho}\left(\frac{1}{G_i}+\frac{x}{F_i}+\frac{j\Xi\beta x}{\tilde{\gamma}_{\rm th}F_i}\right)}\right), \end{aligned} \tag{A.8}$$

and

$$\begin{aligned} I_{i,3} &= \sum_{j=0}^{K-1}\binom{K-1}{j}\frac{(-1)^j/F_i}{1/F_i+1/(G_i a)+j\beta\Xi/(\tilde{\gamma}_{\rm th}F_i)} \\ &\quad\times\left(1-e^{-\frac{\eta}{\rho}\left(\frac{1}{F_i}+\frac{1}{G_i a}+\frac{j\beta\Xi}{\tilde{\gamma}_{\rm th}F_i}\right)}\right). \end{aligned} \tag{A.9}$$

On the other hand, when $a > b_i$, by following the derivations in (A.3)$\sim$(A.9), we have $P_{out,D,i} = Q_{i,1} + Q_{i,2} + Q_{i,3}$ with $Q_{i,1}$, $Q_{i,2}$ and $Q_{i,3}$ derived as

$$\begin{aligned} Q_{i,1} &= \sum_{j=0}^{K-1}\binom{K-1}{j}\frac{(-1)^j/G_i}{a/F_i+1/G_i+j\Xi\mu/(\gamma_{\rm th}G_i)} \\ &\quad\times\left(1-e^{-\frac{\gamma_{\rm th}}{\mu\rho}\left(\frac{a}{F_i}+\frac{1}{G_i}+\frac{j\Xi\mu}{\gamma_{\rm th}G_i}\right)}\right), \end{aligned} \tag{A.10}$$

$$Q_{i,2} = \sum_{j=0}^{K-1}\binom{K-1}{j}\frac{(-1)^j[\psi_i(a)-\psi_i(b_i)]}{1+j\Xi\mu/\gamma_{\rm th}} \tag{A.11}$$

with $\psi_i(x)$ defined as

$$\begin{aligned} \psi_i(x) &= \frac{1/F_i}{1/F_i+1/(G_i x)+j\mu\Xi/(\gamma_{\rm th}G_i x)} \\ &\quad\times\left(1-e^{-\frac{\eta}{\rho}\left(\frac{1}{F_i}+\frac{1}{G_i x}+\frac{j\mu\Xi}{\gamma_{\rm th}G_i x}\right)}\right), \end{aligned} \tag{A.12}$$

and

$$\begin{aligned} Q_{i,3} &= \sum_{j=0}^{K-1}\binom{K-1}{j}\frac{(-1)^j/F_i}{1/F_i+1/(G_i b_i)+j\beta\Xi/(\tilde{\gamma}_{\rm th}F_i)} \\ &\quad\times\left(1-e^{-\frac{\eta}{\rho}\left(\frac{1}{F_i}+\frac{1}{G_i b_i}+\frac{j\beta\Xi}{\tilde{\gamma}_{\rm th}F_i}\right)}\right). \end{aligned} \tag{A.13}$$

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