Resource Allocation Robust to Traffic and Channel Variations in Multihop Wireless Networks

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***Abstract*—In a multihop wireless network, the traffic at each node and the channel over each link may fluctuate with time. Thus, traditional optimal resource allocation needs to be computed for each moment with instantaneous information of channel states over all links and the traffic rates at all nodes, leading to huge communication overhead and computation cost. To solve this challenge, in this correspondence, we propose to use robust resource allocation, in which the only needed information is the mean and variance of the wireless channels and the traffic rates. In the formulated problem, there are probabilistic constraints, which are difficult to handle. Effective methods are provided that can transform the probabilistic constraints to convex constraints. As the resource allocation does not need instantaneous channel state information or instantaneous traffic rate information, it is robust to channel and traffic variations, with very little communication and computation overhead.**

***Index Terms*—Multihop wireless networks, resource allocation, robustness.**

I. Introduction

For a multihop wireless network, allocation of resources (power, data rate, bandwidth, etc.) is essential to guarantee quality-of-service and to maximize the network utility. To achieve this, traditionally instantaneous channel state information (CSI) of all the links in each fading block is obtained, and then resource allocation is found by optimizing a defined objective function. The work in [1] considers one fading block. It jointly optimizes flow rates of the source-destination pairs and transmission power levels of the nodes so as to maximize a network utility function, defined as a function of the flow rates, in the fading block. The work in [2] considers an ergodic process of fading blocks. Each source-destination pair communicate with a fixed flow rate over the ergodic process of multiple fading blocks. Transmission power levels of the nodes in the network over fading blocks are jointly optimized with the flow rates of the source-destination pairs, to maximize the difference of the network utility function (i.e., flow rates) and the cost function (i.e., power consumption). The work in [3] also considers an ergodic process of multiple fading blocks, with network utility defined as a function of average flow rates of the source-destination pairs. For a fading block, flow rates of the source-destination pairs and nodes' transmission power are determined based on CSI information of the fading block and *long-term time-averaging throughput* of the flows over prior fading blocks. In works [1]–[3] and many others, all links' instantaneous CSI is required, which means that for dynamic wireless fading channels, resource allocation should be updated frequently, leading to high communication and computation complexity. To overcome this problem, resource allocation approaches robust to channel fluctuations are investigated in [4], [5]. The approaches are robust in that the resource allocation does not need to be updated when the wireless channels fluctuate. Only statistical CSI is needed. As the resource allocation is not based on instantaneous CSI, rate outage (i.e., the data rate over a link is more than the instantaneous link capacity) may happen. Accordingly, a rate outage probability bound is set.

The aforementioned works assume that the route for each source-destination pair is predefined. Actually routing design is also critical to improve system performance in multihop networks. The work in [6] designs a routing strategy in which the minimum flow rate of the source-destination pairs is maximized by using physical-layer signal processing techniques such as successive interference cancellation, superposition coding, etc. Energy consumption is also a consideration in routing design in multihop networks [7]–[10]. The works in [7], [8] both jointly optimize routing strategy, transmission power, and link rates with a target of total energy consumption minimization. The work [7] considers that the total network throughput is given, while the work [8] considers fixed end-to-end flow rates and approximates the non-linear channel capacity function as a linear function. In [9], routing strategy and transmission power are designed sequentially such that the total energy consumption is minimized with a constraint on the spectral efficiency. The work in [10] considers fixed transmission power levels of the nodes. Routing strategy is designed to minimize total energy consumption, by exploiting the broadcasting nature of wireless signals, i.e., a wireless signal may be received by multiple nodes over a route and thus, maximal ratio combining can be used to improve signal reception quality at those nodes.

In a practical multihop wireless network, each node has its own traffic arrival pattern, and the traffic arrival rate may be time-varying, which means that the routing strategy should be updated upon any change of the traffic arrival rates of the nodes. In the literature, routing robust to variations of traffic rates has been investigated for generic (wired) multihop networks. Oblivion routing [11] is based on limited information of the traffic fluctuation pattern of the nodes, and can achieve a static routing strategy for the network when the traffic rates fluctuate. By assuming that the amount of traffic (rather than the rate of traffic) between multiple pairs of nodes in a wireless network is transportable by nodes' available energy, the work in [12] develops oblivious routing strategy to deliver any traffic in a defined interval and at the same time, minimize the energy consumption rate (the ratio of consumed energy to total energy

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of a node). By assuming that the traffic arrival rates are supportable given the existing link capacity, the work in [13] designs robust routing and scheduling strategy that minimizes the worst congestion level. By assuming that the traffic arrival rates follow a hose model, the work in [14] designs robust routing that targets system throughput maximization. Note that in the above surveyed works on traditional and robust routing designs, fixed link capacities are assumed, and thus, the strategies cannot be applied in a wireless environment with channel fading.

In this correspondence, we investigate resource allocation robust to both time-varying channels and time-varying traffic arrival rates. The contributions of this correspondence are summarized as follows. First, we develop a framework of resource allocation that involves power allocation, bandwidth allocation, link rate setting, and routing, which is robust to time-varying channel states and traffic rates. In specific, as long as the channel states and traffic rates fluctuate with a fixed mean and fixed variance, we use a static resource allocation. Second, we do not require instantaneous CSI or instantaneous traffic rate information. Thus, it is possible that outages may happen in the network. Accordingly, we set up constraints for probabilities of outages,[1] which makes the research problem challenging because closed-form expressions of the probabilistic constraints for outages are lacking. To overcome this problem, we develop a method that transforms the probabilistic constraints to closed-form equivalences. We subsequently prove that the problem with the transformed equivalent constraints is convex, and thus, can be solved by traditional convex optimization methods. Third, we demonstrate that for the case when the means and variances of the traffic rates and channel states are unknown and should be estimated based on sampling, robust resource allocation can still be achieved.

The rest of this correspondence is organized as follows. The system model is presented in Section II, as well as formulation of our research problem. Our robust resource allocation solution is derived in Section III. The case when we only know the sampled means and sampled variances of traffic rates and channel states is investigated in Section IV. Performance of our robust resource allocation is evaluated in Section V, and our conclusion remarks are provided in Section VI.

II. System Model and Problem Formulation

Consider a multihop wireless network with N nodes. Denote the set of all nodes as $\mathcal{N} \triangleq \{1, 2, ..., N\}$. When two nodes can communicate directly, we say a link exists between them. Denote the set of all links as $\mathcal{L}$. For link $l \in \mathcal{L}$, at a specific moment, denote its channel gain as h_l, its transmission power as P_l, its transmission bandwidth as w_l, and its transmission rate as r_l. Then the signal-to-noise ratio (SNR) on link l is: $\text{SNR}_l = \frac{P_l h_l}{w_l \sigma^2}$, with σ^2 being the noise power spectrum density, and the capacity of link l is: $C_l = w_l \log(1 + \text{SNR}_l) = w_l \log\left(1 + \frac{P_l h_l}{w_l \sigma^2}\right)$.

[1]Link outage probability issues have been studied extensively in device-to-device (D2D) networks as shown in [15]–[20], all of which assume that the distributions of wireless channel gains are known. Different from the works in D2D networks, our work only assumes knowledge of channel gains' mean and variance, rather than the distributions of channel gains. It is much easier to obtain mean and variance of the channel gains in a practical network (e.g., by using sampling).

For a node $n \in \mathcal{N}$, denote the set of outgoing links as $\mathcal{O}(n)$, and the set of incoming links as $\mathcal{I}(n)$. Denote the maximal allowable transmission bandwidth and power of node n as W_n^T and P_n^T, respectively. Thus, we have the following constraints:

$$\sum_{l \in \mathcal{O}(n)} P_l \leq P_n^T, \forall n \in \mathcal{N}, \quad (1)$$

$$\sum_{l \in \mathcal{O}(n)} w_l \leq W_n^T, \forall n \in \mathcal{N}, \quad (2)$$

$$P_l \geq 0, w_l \geq 0, \forall l \in \mathcal{L}. \quad (3)$$

If a node has traffic to be sent to another node that may be one or more hops away, we say there is a *traffic flow* between the two nodes. Denote the data rate of traffic flow from node i to node j as $t_{i,j}$. A traffic flow can be delivered by different paths in parallel and simultaneously, i.e., each path carries a portion of the traffic flow. For traffic flow $i \to j$ with rate $t_{i,j}$, denote $f_{i,j}(l)$ as the percentage of traffic that is delivered over link l. Thus, set $\{f_{i,j}(l) | i, j \in \mathcal{N}, l \in \mathcal{L}\}$ is the routing strategy for the network, which has the following constraints:

$$0 \leq f_{i,j}(l) \leq 1, \forall l \in \mathcal{L}, \forall i, j \in \mathcal{N}, i \neq j. \quad (4)$$

$$\sum_{l \in \mathcal{O}(k)} f_{i,j}(l) - \sum_{l \in \mathcal{I}(k)} f_{i,j}(l) = \begin{cases} 0 & \text{if} \quad k \neq i, j \\ 1 & \text{if} \quad k = i \\ -1 & \text{if} \quad k = j \end{cases}, \quad \forall l \in \mathcal{L}, \forall i, j, k \in \mathcal{N}, \ i \neq j. \quad (5)$$

In (5), for flow $i \to j$, the term $\sum_{l \in \mathcal{I}(k)} f_{i,j}(l)$ means the percentage of $i \to j$ flow over node k's incoming links, and the term $\sum_{l \in \mathcal{O}(k)} f_{i,j}(l)$ means the percentage of $i \to j$ flow over node k's outgoing links. When $k = i$ (i.e., k is the source of the flow), we have $\sum_{l \in \mathcal{I}(k)} f_{i,j}(l) = 0$ (as the source node of the flow, node k does not get traffic of the flow from other nodes; rather, it generates the flow traffic itself), and $\sum_{l \in \mathcal{O}(k)} f_{i,j}(l) = 1$ (which means that all flow traffic generated by node k is sent out over its outgoing links). When $k = j$ (i.e., k is the destination of the flow $i \to j$), we have $\sum_{l \in \mathcal{I}(k)} f_{i,j}(l) = 1$, and $\sum_{l \in \mathcal{O}(k)} f_{i,j}(l) = 0$ (i.e., destination of the flow does not send the flow traffic out over its outgoing links). When $k \neq i, j$, for node k, the amount of incoming $i \to j$ flow and outgoing $i \to j$ flow should be equal, and thus, $\sum_{l \in \mathcal{O}(k)} f_{i,j}(l) - \sum_{l \in \mathcal{I}(k)} f_{i,j}(l) = 0$.

In the system, the channel gains h_l's of the links and traffic flow rates $t_{i,j}$'s are all random variables. Our target is to design a robust resource allocation strategy over the links. Our target robustness lies in that: our resource allocation strategy is static (i.e., does not need to update over time) if h_l fluctuates following any distribution with fixed mean denoted as μ_{h_l} and fixed variance denoted as Σ_{h_l} and if $t_{i,j}$ fluctuates following any distribution with fixed mean denoted as $\mu_{t_{i,j}}$ and fixed variance denoted as $\Sigma_{t_{i,j}}$. Assume that μ_{h_l}, Σ_{h_l}, $\mu_{t_{i,j}}$, and $\Sigma_{t_{i,j}}$ are known. Let $f(x)$ denote distribution of a random variable x, and let $\mathbb{P}$ denote the collection of all probability distribution functions. Denote $\mathcal{S}_{\text{I}}(h_l) = \{f | \mathbb{E}_f[h_l] = \mu_{h_l}, \mathbb{E}_f[(h_l - \mu_{h_l})^2] = \Sigma_{h_l}, f \in \mathbb{P}\}$ as the set of all possible $f(h_l)$ that have mean equal to μ_{h_l} and variance equal to Σ_{h_l}, and denote $\mathcal{S}_{\text{I}}(t_{i,j}) = \{f | \mathbb{E}_f[t_{i,j}] = \mu_{t_{i,j}}, \mathbb{E}_f[(t_{i,j} - \mu_{t_{i,j}})^2] = \Sigma_{t_{i,j}}, f \in \mathbb{P}\}$ as the set of all possible $f(t_{i,j})$ that have mean equal to $\mu_{t_{i,j}}$ and variance equal to $\Sigma_{t_{i,j}}$, in which $\mathbb{E}_f[\cdot]$ means expectation with respect to the distribution function f. To guarantee quality of the communication, we set the following constraints.

- Each link has a target SNR denoted as γ. Probability of SNR outage (i.e., the SNR of a link falls below γ) should be bounded by a threshold ε_1, i.e.,

$$\sup_{f(h_l)\in\mathcal{S}_{\rm I}(h_l)} \Pr\left(\mathrm{SNR}_l \leq \gamma\right) \leq \varepsilon_1, \forall l \in \mathcal{L}, \tag{6}$$

  in which $\Pr(\cdot)$ means probability of an event.
- The probability of rate outage (i.e., the transmission rate of a link exceeds the link capacity) should be bounded by a threshold ε_2, i.e.,

$$\sup_{f(h_l)\in\mathcal{S}_{\rm I}(h_l)} \Pr\left(r_l \geq C_l\right) \leq \varepsilon_2, \forall l \in \mathcal{L}. \tag{7}$$

- With the constraints in (6) and (7), the *effective data transmission rate* (i.e., the data rate without outage) of link l is $r_l\left(1-\varepsilon_1\right)\left(1-\varepsilon_2\right)$. Denote $T_l = \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\backslash\{i\}} t_{i,j} f_{i,j}(l)$ as the total traffic load on link l. Probability of traffic outage (i.e., for link l, total load T_l exceeds the effective data transmission rate) should be bounded by a threshold ε_3, i.e.,

$$\begin{aligned}\sup_{f(t_{i,j})\in\mathcal{S}_{\rm I}(t_{i,j}), i\in\mathcal{N}, j\in\mathcal{N}\backslash\{i\}} &\Pr(T_l \leq r_l\left(1-\varepsilon_1\right)\left(1-\varepsilon_2\right)) \\ &\geq \left(1-\varepsilon_3\right), \forall l \in \mathcal{L}.\end{aligned} \tag{8}$$

To improve the efficiency of energy and spectrum resource utilization, we minimize a cost function that is a weighted sum of the total transmission power $\sum\limits_{l\in\mathcal{L}} P_l$ and total transmission bandwidth $\sum\limits_{l\in\mathcal{L}} w_l$. Therefore, an optimization problem is formulated as follows.

Problem 1:

$$\begin{aligned}\min_{\{P_l\},\{w_l\},\{r_l\},\{f_{i,j}(l)\}} \ & \alpha\sum_{l\in\mathcal{L}} P_l + \beta\sum_{l\in\mathcal{L}} w_l \\ \text{s.t.} \quad & \text{Constraints (1) – (8)}\end{aligned} \tag{9}$$

where α and β are the weights associated with power consumption and bandwidth consumption, respectively.

Remarks: In this work, we assume knowledge of mean and variance of channel gains and traffic flow rates. The motivations of this setting are as follows. 1) It is hard or costly in a practical network to get the exact distribution functions of channel gains and traffic flow rates. But it is much easier to obtain mean and variance information of channel gains and traffic flow rates, e.g., by using the sampling method to be introduced in Section IV. 2) For a network with knowledge of distribution functions of channel gains and traffic flow rates, it is hard to solve the research problem (i.e., minimize the cost function subject to bounded SNR outage probability, rate outage probability, and traffic outage probability) for the following reason. Recall that for a link $l \in \mathcal{L}$, its total traffic load is expressed as $T_l = \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\backslash\{i\}} t_{i,j} f_{i,j}(l)$, in which $t_{i,j}$ is the rate of traffic flow from node i to node j. Distribution of $t_{i,j}$ is known. However, the distribution of T_l should be the convolution of the distributions of the random variables $t_{i,j} f_{i,j}(l)$, which is hard to derive in closed form. Without closed-form expression of T_l's distribution, the research problem is hard to solve. On the other hand, by only assuming knowledge of mean and variance of channel gains and traffic flow rates, we successfully transform our robustness constraints (6), (7), and (8) to closed-form expressions, and subsequently find an optimal solution of the research problem, as shown in the subsequent section.

III. Solutions of Problem 1

Looking into Problem 1, one major challenge lies in that the left hand-side of constraints (6), (7), and (8) are not closed-form expressions, and thus, Problem 1 cannot be solved by existing numerical optimization methods. To solve the challenge, next we provide a method to transform constraints (6), (7), and (8) into closed-form expressions, which facilitates the solving of Problem 1.

Lemma 1: Given $\mathcal{S}_{\rm I}(h_l)$ for $l \in \mathcal{L}$, constraints (6) and (7) are equivalent to (10) and (11), respectively, as follows.

$$\left(\mu_{h_l} - \sqrt{\frac{(1-\varepsilon_1)}{\varepsilon_1}\Sigma_{h_l}}\right) P_l \geq \gamma\sigma^2 w_l, \forall l \in \mathcal{L}. \tag{10}$$

$$\left(\mu_{h_l} - \sqrt{\frac{(1-\varepsilon_2)}{\varepsilon_2}\Sigma_{h_l}}\right) P_l \geq \sigma^2 w_l\left(e^{\frac{r_l}{w_l}} - 1\right), \forall l \in \mathcal{L}. \tag{11}$$

Proof: We first prove that constraint (6) is equivalent to constraint (10). By replacing SNR_l with $\frac{P_l h_l}{w_l\sigma^2}$, constraint (6) is equivalent to the following constraint

$$\sup_{f(h_l)\in\mathcal{S}_{\rm I}(h_l)} \Pr\left(h_l \leq \frac{\gamma w_l\sigma^2}{P_l}\right) \leq \varepsilon_1, \forall l \in \mathcal{L}. \tag{12}$$

Consider random variable X with distribution $f(X) \in \mathcal{S}(X) = \{f|\mathbb{E}_f\{X\} = \mu_X, \mathbb{E}_f\{(X-\mu_X)^2\} = \Sigma_X, f \in \mathbb{P}\}$. For any threshold value $s > 0$, from the Chebyshev-Cantelli inequality [21], [22], we have

$$\sup_{f(X)\in\mathcal{S}(X)} \Pr\left(X - \mu_X \leq -s\right) = \frac{\Sigma_X}{\Sigma_X + s^2}, \tag{13}$$

$$\sup_{f(X)\in\mathcal{S}(X)} \Pr\left(X - \mu_X \geq s\right) = \frac{\Sigma_X}{\Sigma_X + s^2}. \tag{14}$$

Following (13) and replacing X with h_l and s with $\left(\mu_{h_l} - \frac{\gamma w_l\sigma^2}{P_l}\right)$, we have

$$\sup_{f(h_l)\in\mathcal{S}_{\rm I}(h_l)} \Pr\left(h_l \leq \frac{\gamma w_l\sigma^2}{P_l}\right) = \frac{\Sigma_{h_l}}{\Sigma_{h_l} + \left(\mu_{h_l} - \frac{\gamma w_l\sigma^2}{P_l}\right)^2}. \tag{15}$$

Recall that equation (13) holds on condition $s > 0$. Thus, $s > 0$ is also a condition for equation (15) to hold. Since s is replaced by $\left(\mu_{h_l} - \frac{\gamma w_l\sigma^2}{P_l}\right)$, the condition is

$$\mu_{h_l} - \frac{\gamma w_l\sigma^2}{P_l} > 0, \forall l \in \mathcal{L}, \tag{16}$$

which is equivalent to

$$P_l > \frac{\gamma w_l\sigma^2}{\mu_{h_l}}, \forall l \in \mathcal{L}. \tag{17}$$

Based on (15), to guarantee that constraint (12) holds, we need to set

$$\begin{aligned} & \frac{\Sigma_{h_l}}{\Sigma_{h_l}+(\mu_{h_l}-\frac{\gamma w_l\sigma^2}{P_l})^2} \le \varepsilon_1 \\ \Longleftrightarrow\ & \left(\mu_{h_l}-\frac{\gamma w_l\sigma^2}{P_l}\right)^2 \ge \frac{(1-\varepsilon_1)}{\varepsilon_1}\Sigma_{h_l} \\ \overset{(a)}{\Longleftrightarrow}\ & \left(\mu_{h_l}-\sqrt{\frac{(1-\varepsilon_1)}{\varepsilon_1}\Sigma_{h_l}}\right)P_l \ge \gamma\sigma^2 w_l \end{aligned} \tag{18}$$

where (a) holds because of (16).

Combining (17) and the inequality in the last line of (18), we can get the constraint shown in (10).

Similarly, constraint (7) can be shown to be equivalent to constraint (11). ■

Lemma 2: Given $\mathcal{S}_{\rm I}(t_{i,j})$ for $i\in\mathcal{N}, j\in\mathcal{N}\backslash\{i\}$, constraint (8) is equivalent to

$$\begin{aligned} & \sqrt{\frac{(1-\varepsilon_3)}{\varepsilon_3}}\cdot\sqrt{\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\backslash\{i\}}\Sigma_{t_{i,j}}\left(f_{i,j}(l)\right)^2} \\ & +\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\backslash\{i\}}\mu_{t_{i,j}}f_{i,j}(l)\le r_l\left(1-\varepsilon_1\right)\left(1-\varepsilon_2\right),\forall l\in\mathcal{L}. \end{aligned} \tag{19}$$

Proof:

Take $\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\backslash\{i\}} t_{i,j}f_{i,j}(l)$ as a new random variable. It can be verified that the new random variable has mean being $\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\backslash\{i\}}\mu_{t_{i,j}}f_{i,j}(l)$ and variance being $\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\backslash\{i\}}\Sigma_{t_{i,j}}\left(f_{i,j}(l)\right)^2$, due to the independence of $t_{i,j}$ for $i\in\mathcal{N}, j\in\mathcal{N}\backslash\{i\}$ [23]. Following a similar proof to that of Lemma 1 and using equation (14), constraint (8) can be proved to be equivalent to constraint (19) given $\mathcal{S}_{\rm I}(t_{i,j})$ for $i\in\mathcal{N}, j\in\mathcal{N}\backslash\{i\}$. ■

Lemma 3: Constraints (10), (11) and (19) are convex constraints.

Proof: It is easy to see that constraint (10) is linear with the vector $[P_l,w_l]^T$ (with superscript $[\cdot]^T$ denoting transpose operation) for $l\in\mathcal{L}$, and thus, is a convex constraint.

To prove that constraint (11) is a convex constraint, we need to verify that the function

$$\begin{aligned} & G\left([P_l,w_l,r_l]^T\right) \\ & =\sigma^2 w_l\left(e^{\frac{r_l}{w_l}}-1\right)-\left(\mu_{h_l}-\sqrt{\frac{(1-\varepsilon_2)}{\varepsilon_2}\Sigma_{h_l}}\right)P_l \end{aligned}$$

is convex with respect to the vector $[P_l,w_l,r_l]^T$. Note that function $G\left([P_l,w_l,r_l]^T\right)$ is separable and linear with P_l. Thus, to prove the convexity of $G\left([P_l,w_l,r_l]^T\right)$, we only need to prove the term $\sigma^2 w_l\left(e^{\frac{r_l}{w_l}}-1\right)$ is convex with $[w_l,r_l]^T$. The Hessian matrix of the term $\sigma^2 w_l\left(e^{\frac{r_l}{w_l}}-1\right)$, denoted as $\mathbb{H}$, with respect to $[w_l,r_l]^T$ can be calculated as

$$\mathbb{H}=\frac{\sigma^2 e^{r_l/w_l}}{w_l^3}\cdot\begin{bmatrix} r_l^2 & -r_l w_l \\ -r_l w_l & w_l^2 \end{bmatrix} \tag{20}$$

with eigenvalues 0 and $\frac{\sigma^2 e^{r_l/w_l}\left(r_l^2+w_l^2\right)}{w_l^3}$. With all the eigenvalues being non-negative, the matrix $\mathbb{H}$ is a semi-definite matrix. Therefore, the term $\sigma^2 w_l\left(e^{\frac{r_l}{w_l}}-1\right)$ is convex with respect to vector $[w_l,r_l]^T$.

In (19), the term $\sqrt{\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\backslash\{i\}}\Sigma_{t_{i,j}}\left(f_{i,j}(l)\right)^2}$ can be rewritten as $\sqrt{\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}\backslash\{i\}}\left(\sqrt{\Sigma_{t_{i,j}}}f_{i,j}(l)\right)^2}$, which is a norm function. Since a norm function is convex, the left-hand-side function of constraint (19) is convex, and thus, constraint (19) is convex. ■

Since constraints (10), (11) and (19) are convex, it can be seen that, Problem 1 with (6), (7), and (8) replaced by (10), (11), and (19), respectively, is a convex problem, and thus, can be solved optimally by existing numerical methods [24].

IV. Case when We only Know Sampled Means and Variances of Channel States and Traffic Rates

In this section, we consider that means (i.e., μ_{h_l}, $\mu_{t_{i,j}}$) and variances (i.e., Σ_{h_l}, $\Sigma_{t_{i,j}}$) of channel state h_l and traffic rate $t_{i,j}$ are unknown in advance, but need to be estimated by sampling. Suppose we take Q samples of h_l and $t_{i,j}$, denoted as $h_{l,1},h_{l,2},...,h_{l,Q}$ and $t_{i,j,1},t_{i,j,2},...,t_{i,j,Q}$, respectively, for $l\in\mathcal{L}$ and $i\in\mathcal{N}, j\in\mathcal{N}\backslash\{i\}$. Denote $\hat{\mu}_{h_l}=\frac{1}{Q}\sum_{q=1}^{Q}h_{l,q}$, $\hat{\mu}_{t_{i,j}}=\frac{1}{Q}\sum_{q=1}^{Q}t_{i,j,q}$, $\hat{\Sigma}_{h_l}=\frac{1}{(Q-1)}\sum_{q=1}^{Q}\left(h_{l,q}-\hat{\mu}_{h_l}\right)^2$ and $\hat{\Sigma}_{t_{i,j}}=\frac{1}{(Q-1)}\sum_{q=1}^{Q}\left(t_{i,j,q}-\hat{\mu}_{t_{i,j}}\right)^2$ as the sampled means and sampled variances.

Define

$$\mathcal{S}_{\rm II}(h_l)=\left\{f|\hat{\mathbb{E}}_f\{h_l\}=\hat{\mu}_{h_l},\hat{\mathbb{E}}_f\{(h_l-\hat{\mu}_{h_l})^2\}=\hat{\Sigma}_{h_l},f\in\mathbb{P}\right\}$$

and

$$\begin{aligned} & \mathcal{S}_{\rm II}(t_{i,j}) \\ & =\left\{f|\hat{\mathbb{E}}_f\{t_{i,j}\}=\hat{\mu}_{t_{i,j}},\hat{\mathbb{E}}_f\{(t_{i,j}-\hat{\mu}_{t_{i,j}})^2\}=\hat{\Sigma}_{t_{i,j}},f\in\mathbb{P}\right\} \end{aligned}$$

for $l\in\mathcal{L}$ and $i\in\mathcal{N}, j\in\mathcal{N}\backslash\{i\}$, where $\hat{\mathbb{E}}_f\{X\}$ means the estimated expectation (by samples) of a random variable X with the distribution function f and $\hat{\mathbb{E}}_f\left\{\left(X-\hat{\mathbb{E}}_f\{X\}\right)^2\right\}$ is the unbiased estimation (by samples) of variance of a random variable X with the distribution function f. Then we should solve Problem 1 with $\mathcal{S}_{\rm I}(h_l)$ and $\mathcal{S}_{\rm I}(t_{i,j})$ replaced by $\mathcal{S}_{\rm II}(h_l)$ and $\mathcal{S}_{\rm II}(t_{i,j})$, respectively.

For a set of random samples, $X_1,X_2,...,X_Q$ with sampled mean as $\hat{\mu}_X$ and sampled variance as $\hat{\Sigma}_X$ and any threshold value $s>0$, it is proved in [25] that

$$\Pr\left(X_q-\hat{\mu}_X\ge s\right)\le\frac{\hat{\Sigma}_X}{\hat{\Sigma}_X+\frac{Q}{Q-1}s^2},q=1,2,...,Q. \tag{21}$$

Similarly, we can have

$$\Pr\left(X_q-\hat{\mu}_X\le -s\right)\le\frac{\hat{\Sigma}_X}{\hat{\Sigma}_X+\frac{Q}{Q-1}s^2},q=1,2,...,Q. \tag{22}$$

With the aid of inequalities (21) and (22), and by following a similar proof to those of Lemma 1 and Lemma 2, the following lemma can be expected.

Lemma 4: Given $\mathcal{S}_{\rm II}(h_l)$ for $l\in\mathcal{L}$ and $\mathcal{S}_{\rm II}(t_{i,j})$ for $i\in\mathcal{N}, j\in\mathcal{N}\backslash\{i\}$, constraint (6) holds if constraint (23) holds, constraint (7) holds if constraint (24) holds, and constraint (8) holds if constraint (25) holds, where constraints (23), (24), and (25) are given as follows

$$\left(\hat{\mu}_{h_l}-\sqrt{\frac{(1-\varepsilon_1)(Q-1)}{\varepsilon_1 Q}\hat{\Sigma}_{h_l}}\right)P_l\ge\gamma\sigma^2 w_l,\forall l\in\mathcal{L}, \tag{23}$$

TABLE I
WORST OUTAGE PROBABILITY OF ALL THE LINKS.

	Q=50	Q=500	Q=5000	Perfect Statistics
ε_1=ε_2=ε_3=0.1	1.21×10^{-4}	1.03×10^{-5}	5.82×10^{-6}	3.54×10^{-6}
ε_1=0.1, ε_2=0.2, ε_3=0.1	4.35×10^{-4}	5.23×10^{-5}	7.65×10^{-6}	4.43×10^{-6}

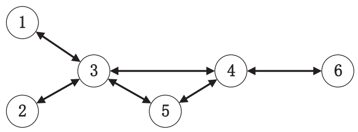

Fig. 1. Network topology.

$$\left(\hat{\mu}_{h_l} - \sqrt{\frac{(1-\varepsilon_2)(Q-1)}{\varepsilon_2 Q}\hat{\Sigma}_{h_l}}\right) P_l \geq \sigma^2 w_l \left(e^{\frac{r_l}{w_l}} - 1\right), \quad \forall l \in \mathcal{L}, \quad (24)$$

$$\sqrt{\tfrac{(1-\varepsilon_3)(Q-1)}{\varepsilon_3 Q}} \cdot \sqrt{\sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}\setminus\{i\}} \hat{\Sigma}_{t_{i,j}} \left(f_{i,j}(l)\right)^2} + \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{N}\setminus\{i\}} \hat{\mu}_{t_{i,j}} f_{i,j}(l) \leq r_l (1-\varepsilon_1)(1-\varepsilon_2), \forall l \in \mathcal{L}. \quad (25)$$

Similar to the proof of Lemma 3, constraints (23), (24) and (25) are convex constraints. Therefore, Problem 1 with constraints (6)-(8) replaced by constraints (23)-(25) is a convex optimization problem and can be solved optimally by existing numerical methods.

V. Performance Evaluation

Consider a multihop wireless network with 6 nodes and 12 directed links as shown in Fig. 1. The distance between two connected nodes is 2000m. Each wireless channel includes path loss attenuation (with path loss exponent being 4) and Nakagami fading (with shape parameter as 15 and spread parameter as 1). The maximal allowable transmission power of every node, $P_n^T, \forall n \in \mathcal{N}$, is 3W. The carrier frequency is at 2.4GHz. The maximal allowable transmission bandwidth of every node, $W_n^T, \forall n \in \mathcal{N}$, is 1MHz. The threshold of received SNR, γ, is set as 5dB. The weights in the cost function of Problem 1 are set as $\alpha = 1$ unit per Watt and $\beta = 1$ unit per MHz. Channel reciprocity is assumed. The traffic flow rates between nodes pairs $(1,6)$, $(2,1)$, $(2,3)$, $(2,6)$, $(3,2)$, $(3,5)$, $(4,3)$, $(4,5)$, $(4,6)$, $(5,1)$, $(5,4)$, $(5,6)$, $(6,1)$, $(6,3)$, and $(6,4)$ are uniform random variables with mean and standard deviation (in the form of mean/standard deviation $\times 10^3$ bps) as 75.50/26.19, 53.17/33.76, 51.90/32.55, 75.81/32.76, 87.61/32.88, 65.69/33.50, 94.38/32.01, 86.15/32.69, 97.30/33.12, 77.05/28.24, 80.36/32.07, 78.58/26.45, 83.84/30.60, 59.48/28.94, and 77.03/31.48, respectively. There are no traffic flows between any other node pair.

A. Performance of the Proposed Resource Allocation

Fig. 2 illustrates the cost function of Problem 1 when one of ε_1, ε_2 and ε_3 changes and the other two are fixed at 0.1. The "Perfect Statistics" stands for the results of Problem 1 when means and variances of channel states and traffic rates are known in advance. The results when sampled means and variances of channel states and traffic rates are known are shown as "$Q = 50$", "$Q = 500$", and "$Q = 5000$", where Q is the number of samples for channel states and traffic rates. When ε_1 grows, the cost function increases. This is because a higher ε_1 has two effects: make constraint (6) looser and make constraint (8) tighter, and the latter effect dominates. When ε_2 grows, the cost function first decreases, and then increases, for the following reason. A higher ε_2 has two effects: make constraint (7) looser and make constraint (8) tighter. The former effect dominates with a small ε_2, while the latter effect dominates with a large ε_2. When ε_3 grows, the cost function always decreases, as a larger ε_3 means that constraint (8) is looser. Additionally, Fig. 2 also shows that when Q increases, the cost function decreases and gets closer to the results with perfect statistics. The is because more samples contribute to more accurate estimation of means and variances of channel states and traffic rates.

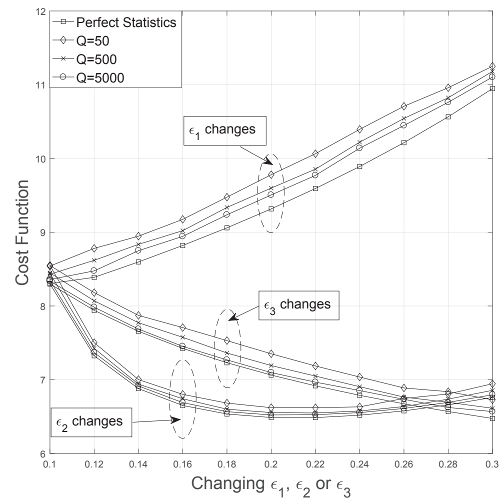

Fig. 2. Cost function when one of ε_1, ε_2, and ε_3 changes while the other two keep unchanged (i.e., fixed at 0.1).

In Problem 1, constraints (6), (7), and (8) are for three outages: SNR outage, rate outage, and traffic outage. To verify the robustness of our solutions, Table I shows the worst outage probability (i.e., the maximal of the SNR outage probability, rate outage probability, and traffic outage probability) of all links for setting (ε_1=0.1, ε_2=0.1, ε_3=0.1) and setting (ε_1=0.1, ε_2=0.2, ε_3=0.1). It can be seen that the worst outage probability is smaller than ε_1, ε_2, or ε_3, which means that our solutions are conservative in outage performance. This also means that we can be very flexible in setting ε_1, ε_2, and ε_3 (i.e., we can set up relatively large ε_1, ε_2, and ε_3). For our example in Fig. 2, we can implement the robust resource allocation for setting (ε_1=0.1, ε_2=0.2, ε_3=0.1), which has cost function value around 6.6. The achieved worst outage probability shown in Table I is in the order of 10^{-4}, which is acceptable.

B. Comparison with a Dynamic Resource Allocation Method

Here we make a comparison with a traditional dynamic optimal resource allocation in which the power, bandwidth, and link rates are adjusted according to instantaneous CSI and instantaneous traffic rates. The dynamic optimal resource allocation is actually Problem 1 with constraints (6), (7), and (8) replaced by the following three constraints: $\text{SNR}_l \geq \gamma; r_l \leq C_l; T_l \leq r_l, \forall l \in \mathcal{L}$.

Cost Function: By running Monte Carlo simulation, the average cost function of the dynamic optimal resource allocation is 1.7341. As aforementioned, when we set (ε_1=0.1, ε_2=0.2, ε_3=0.1), our robust resource allocation has cost function value around 6.6. It can be concluded that our robust resource allocation does not increase the cost function too much.

Communication Overhead: Recall that the number of nodes in the network is N. Then the maximal number of traffic flows is $N(N-1)$, which is used in evaluating overhead. Denote the number of links as L. The dynamic optimal resource allocation needs communication overhead to get instantaneous CSI of L links and instantaneous traffic rates of $N(N-1)$ flows for *each fading block* (for example, in the time scale of millisecond). On the other hand, our robust resource allocation needs communication overhead to get mean and variance of channel gains of L links and traffic rates of $N(N-1)$ flows. After the information is obtained, our robust resource allocation does not need communication overhead any more for a long period (for example, tens of seconds or even hundreds of seconds) in which mean and variance of channel gains and traffic flow rates do not change.

Computation Overhead: For Problem 1 in our robust resource allocation, there are $K \triangleq 3L + LN(N-1)$ variables

- L variables in each of $\{P_l\}$, $\{w_l\}$, and $\{r_l\}$
- $LN(N-1)$ variables in $\{f_{i,j}(l)\}$,

and $M \triangleq 4L + 2N + LN(N-1)(N+1)$ constraints

- N constraints in (1) and N constraints in (2)
- L constraints in each of (3), (10), (11), and (19)
- $LN(N-1)$ constraints in (4), and $LN(N-1)N$ constraints in (5).

Note that $M > K$. For a long period, our robust resource allocation needs to solve Problem 1 once, with complexity $O(M^{3.5})$ [26]. On the other hand, for the dynamic optimal resource allocation, an optimization problem with computation complexity $O(M^{3.5})$ needs to be solved for every fading block.

Overall, the effects of our robust resource allocation lie in the significantly reduced communication and computation overhead with a not-significantly increased cost function.

VI. Conclusion

In this paper, we investigate resource allocation for multihop wireless networks that is robust to variations of channel states and traffic rates. For the cases with known and sampled statistics (mean and variance) of channel states and traffic rates, optimization problems subject to constraints on SNR outage, rate outage, and traffic outage are formulated. We provide methods to transform probabilistic constraints to closed-form constraints. The transformations are proved to make the problems convex, and thus, the problems can be solved by existing convex optimization methods. Compared with dynamic optimal resource allocation, our proposed strategy has a higher cost function (in the same order as that of the dynamic optimal resource allocation), but with much less communication and computation overhead.

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