When NOMA Meets Multiuser Cognitive Radio: Opportunistic Cooperation and User Scheduling

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***Abstract*—This correspondence paper investigates a novel non-orthogonal multiple access (NOMA) assisted overlay spectrum sharing framework for multiuser cognitive radio networks towards an enhanced spectrum utilization. In particular, one secondary user is scheduled to help forward the primary signal and convey its own signals as well by applying the NOMA principle. A reliability oriented secondary user scheduling (R-SUS) scheme is first proposed, with a target at minimal primary and secondary outage probabilities. Then a fairness oriented secondary user scheduling (F-SUS) scheme is proposed, such that all candidate secondary users have an equal opportunity to be scheduled for the cooperation. Expressions of primary and secondary outage probabilities are derived in closed form to evaluate the resultant network reliability performance. The results reveal that: 1) the proposed R-SUS and F-SUS schemes can achieve a full diversity order for the primary and secondary transmissions, and 2) although the F-SUS scheme enhances user fairness, it suffers a higher secondary outage probability compared with the R-SUS scheme.**

***Index Terms*—Non-orthogonal multiple access, cognitive radio, user scheduling, cooperation, diversity.**

I. INTRODUCTION

Driven by its high spectral efficiency, non-orthogonal multiple access (NOMA) has received increasing research activities [1]–[5]. Utilizing power domain multiplexing, multiple users' signals can be transmitted simultaneously in the same resource block (i.e., time/frequency/code domain) with successive interference cancellation (SIC) employed at receivers to separate the multiplexed signals. On the other hand, cognitive radio (CR) is another emerging technology to improve the wireless spectrum utilization, by allowing secondary users to access spectrum that is initially licensed to primary users. The combination of NOMA and CR has a promising potential to allow more nodes (i.e., more secondary users and/or more primary users) to transmit concurrently, and thus, can meet the requirements of high spectrum efficiency, massive connectivity, and low latency for fifth-generation (5G) mobile networks [1].

The application of NOMA to underlay CR is investigated in [6], and the exact outage performance is evaluated by using stochastic geometry tool. A CR-inspired NOMA scenario is investigated in [7], [8], in which the wireless spectrum of a user with weak channel condition (called primary user) is shared by a user with strong channel condition (called secondary user) by using NOMA signaling to enhance spectrum efficiency. No cooperation is considered in [7], [8]. The work in [9] introduces cooperation to CR-inspired NOMA, in which a base station sends a unicast message to a primary user, and simultaneously sends a multicast message to a group of secondary users. For cooperation, one secondary user that successfully decodes the unicast and multicast messages helps to forward its received messages to the primary user and other secondary users.

In this correspondence paper, we develop a NOMA assisted cooperative overlay spectrum sharing framework that considers multiple secondary users and exploits multiuser diversity for the mutual cooperation between the primary and secondary networks. We focus on a spectrum overlay paradigm wherein the primary network allocates the available spectrum resources to secondary users in exchange for boosting the primary reception by secondary cooperative relaying. A secondary user is scheduled to invoke NOMA principle to simultaneously convey the primary signal along with its own signals. The target of the secondary user scheduling is to achieve the best outage performance for the primary and secondary transmissions. To the best of our knowledge, this is the first effort to study multiuser CR with NOMA. The difference of our work from existing cooperative NOMA works (such as [2]) and cooperative CR NOMA works (such as [9]) is as follows. In those existing works, the relay only forwards its received signals, and it does not generate its own signals. In this paper, a relay (a secondary user) forwards its received signals, and sends its own secondary signals as well by NOMA signaling. We focus on user scheduling to select one secondary user to cooperate, to achieve our target outage performance for both primary and secondary systems.

The main contributions of this work are summarized as follows.

- *Novel framework*: We propose a novel NOMA assisted cooperative spectrum sharing framework, where one secondary user serves as a relay to recover the primary signal and then superimpose it with the secondary signals for the NOMA transmission. On this basis, two secondary user scheduling schemes, i.e., reliability oriented secondary user scheduling (R-SUS) targeted at minimal primary and secondary outage probabilities, and fairness oriented secondary user scheduling (F-SUS) targeted at superior user fairness, are then developed to facilitate the NOMA assisted cooperation.
- *Tractable analysis*: We rigorously prove that the R-SUS scheme minimizes the primary and secondary outage probabilities (i.e., achieving the outage-optimal performance), and the F-SUS scheme ensures that all candidate secondary

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users have an equal opportunity to be scheduled for the cooperation. For each scheme, we derive expressions of primary and secondary outage probabilities in closed form and investigate the network diversity order. The results show that the two proposed schemes can achieve a full diversity order for both primary and secondary transmissions.

II. SYSTEM MODEL

We consider a cooperative overlay CR scenario as illustrated in Fig. 1, which consists of one pair of primary transceivers, and N secondary transmitters denoted as $s_1, s_2, \ldots, s_N$, respectively. Each secondary transmitter, say s_n ($n \in \{1, 2, \ldots, N\}$), serves M_n secondary receivers with NOMA pricinple, denoted as $\{r_1^n, r_2^n, \ldots, r_{M_n}^n\}$. In the system, each node has a single antenna, working in a half-duplex mode (which means that the node cannot transmit and receive simultaneously). We consider a scenario with no direct link between the primary transmitter and receiver, due to heavy shadowing and/or physical obstacles. Therefore, the primary network tries to seek cooperation from the nearby secondary transmitters by recruiting one of them as a relay. For a reward, the primary network grants to the secondary transmitter spectrum access opportunities. The signal for the primary transmitter is denoted as x_0. If secondary transmitter s_n is selected to help, its signals to its M_n receivers are denoted as $x_1^n, x_2^n, \ldots, x_{M_n}^n$, respectively. Each signal has zero mean and unit variance. The transmit power of each transmitter is P_t. The additive noise at each receiver is modeled by a complex Gaussian random variable with mean being zero and variance being N_0.

All channels in the network undergo independent block fading, i.e., the channels remain unchanged within each fading block, but vary independently from one fading block to another. The length of a fading block is one time unit. Consider one fading block, the channel coefficient from the primary transmitter to secondary transmitter s_n is denoted as h_{p,s_n}, the channel coefficient from secondary transmitter s_n to the primary receiver is denoted as $h_{s_n,p}$, and the channel coefficient from secondary transmitter s_n to secondary receiver r_m^n is denoted as h_{s_n,r_m^n} ($m \in \{1, 2, \ldots, M_n\}$). We consider independent but non-identically distributed (i.n.i.d.) Rayleigh fading with parameters $\mathbb{E}[|h_{p,s_n}|^2] = \lambda_{ps,n}$, $\mathbb{E}[|h_{s_n,p}|^2] = \lambda_{sp,n}$, and $\mathbb{E}[|h_{s_n,r_m^n}|^2] = \lambda_{ss,nm}$, where $\mathbb{E}[\cdot]$ means the expectation operation. Without loss of generality, we assume that the secondary receivers connected to secondary transmitter s_n are ordered as $\mathbb{E}[|h_{s_n,r_1^n}|^2] < \mathbb{E}[|h_{s_n,r_2^n}|^2] < \cdots < \mathbb{E}[|h_{s_n,r_{M_n}^n}|^2]$. Similar to [4], [5], NOMA signal detection for secondary signals with SIC follows the order of the mean channel gains of the secondary receivers. Benefits of this setting include: no need of communication overhead to obtain the instantaneous channel gains of all the secondary receivers; no need of communication overhead to update to secondary receivers signal detection order in each fading block. Thus, signal detection for secondary signals with SIC should follow the order $x_1^n \rightarrow \cdots \rightarrow x_{M_n}^n$.

We develop a NOMA assisted cooperative overlay spectrum sharing framework, where the primary and secondary messages are superimposed by using NOMA principle for the simultaneous cooperative communications. In each fading block, the opportunistic cooperation has two phases with equal duration: the primary transmission phase, and the NOMA assisted cooperation phase, detailed as follows.

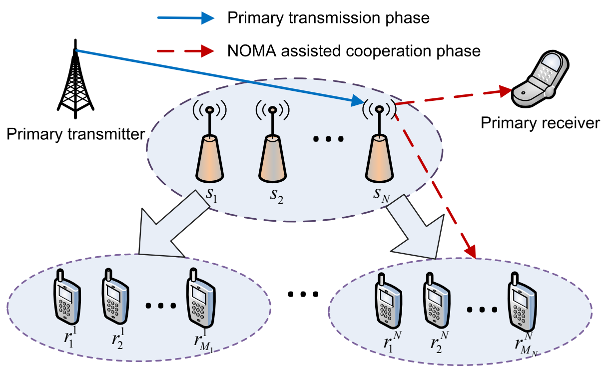

Fig. 1. The NOMA assisted cooperative overlay CR scenario.

During the primary transmission phase, the primary transmitter sends x_0 to all the secondary transmitters. The received signal at s_n is expressed as $y_{s_n} = \sqrt{P_t} h_{p,s_n} x_0 + \eta_{s_n}$, where η_{s_n} is the additive noise observed by s_n. The achievable data rate at s_n is given by $R_{s_n} = \frac{1}{2}\log_2\left(1 + \rho_t |h_{p,s_n}|^2\right)$ where $\rho_t = P_t/N_0$ is called the transmit signal-to-noise ratio (SNR).

During the NOMA assisted cooperation phase, a secondary transmitter, say s_n, can be opportunistically scheduled (detailed scheduling schemes are discussed in Section III) to serve the primary receiver and its own receivers simultaneously with NOMA signaling. More specifically, s_n first regenerates x_0 and superimposes it with x_1^n, ..., $x_{M_n}^n$, and then broadcasts the signal mixture. The observations at the primary receiver and the m-th secondary receiver are written as

$$y_p = \sqrt{\alpha_0^n P_t} h_{s_n,p} x_0 + \sum_{i=1}^{M_n} \sqrt{\alpha_i^n P_t} h_{s_n,p} x_i^n + \eta_p, \qquad (1)$$

$$y_{r_m^n} = \sqrt{\alpha_0^n P_t} h_{s_n,r_m^n} x_0 + \sum_{i=1}^{M_n} \sqrt{\alpha_i^n P_t} h_{s_n,r_m^n} x_i^n + \eta_{r_m^n}, \qquad (2)$$

where η_p and $\eta_{r_m^n}$ are the additive noise at the primary receiver and the m-th secondary receiver, respectively, and α_0^n and α_i^n are the power allocation coefficients for signals x_0 and x_i^n, respectively, with condition $\alpha_0^n + \sum_{i=1}^{M_n} \alpha_i^n = 1$. Consider higher priority of primary user and fairness of secondary receivers, the power allocation coefficients should satisfy $\alpha_0^n \geq \alpha_1^n \geq \cdots \geq \alpha_{M_n}^n$ [7]–[9]. At the end of the NOMA assisted cooperation phase, primary receiver retrieves x_0 by treating the secondary signals as noise, and the achievable data rate for x_0 is given by

$$R_p = \frac{1}{2}\log_2\left(1 + \frac{\alpha_0^n |h_{s_n,p}|^2}{\sum_{i=1}^{M_n} \alpha_i^n |h_{s_n,p}|^2 + 1/\rho_t}\right). \qquad (3)$$

At the m-th secondary receiver r_m^n, SIC is carried out to separate the multiplexed signals and combat the negative impacts of the inter-user interference. The m-th secondary receiver first decodes x_0 and then moves towards $x_1^n, \ldots, x_m^n$ [7]. For presentation simplicity, next we use x_0^n to represent primary signal x_0. The achievable data rate for r_m^n to retrieve x_k^n ($0 \leq k < m$) can be expressed as

$$R_{r_m^n, x_k^n} = \frac{1}{2}\log_2\left(1 + \frac{\alpha_k^n |h_{s_n,r_m^n}|^2}{\sum_{i=k+1}^{M_n} \alpha_i^n |h_{s_n,r_m^n}|^2 + 1/\rho_t}\right). \qquad (4)$$

Conditioned on $R_{r_m^n, x_k^n} \geq R_k^n$, where R_k^n is the preset target data rate of signal x_k^n, the m-th secondary receiver can first decode

x_k^n and then subtract this component from its received signals. Finally, the achievable data rate for its own x_m^n is given by

$$R_{r_m^n,x_m^n} = \frac{1}{2}\log_2\left(1+\frac{\alpha_m^n|h_{s_n,r_m^n}|^2}{\sum_{i=m+1}^{M_n}\alpha_i^n|h_{s_n,r_m^n}|^2+1/\rho_t}\right). \quad (5)$$

III. SECONDARY USER SCHEDULING

Next we address two challenging issues: how could a secondary transmitter be scheduled in a distributed manner such that the primary and secondary outage probabilities are minimized? And how to further achieve fairness among the secondary transmitters? Here a *primary outage* is defined as the event that the primary network cannot achieve its preset target data rate R_0; and a *secondary outage* is defined as the event that no secondary transmitter is scheduled, or if a secondary transmitter (say s_n) is scheduled, one of its receivers, say the k-th receiver, has an achievable data rate smaller than its preset target data rate R_k^n. To address the challenging issues, we develop the R-SUS scheme and the F-SUS scheme below.

A. R-SUS Scheme

Here we introduce the R-SUS scheme for primary and secondary outage probabilities minimization, which encompasses the following two stages.

- Stage-I: For each secondary transmitter s_n, if it can decode the primary transmitter's signal (i.e., $R_{s_n} \geq R_0$), and it can help the primary network to achieve target rate R_0 by NOMA signaling (i.e., $R_p \geq R_0$), then s_n is called a candidate helper. The set of all candidate helpers is expressed as

$$\mathcal{S}_c \triangleq \{s_n : R_{s_n} \geq R_0,\ R_p \geq R_0\}. \quad (6)$$

  If $\mathcal{S}_c$ is empty, no secondary transmitter is scheduled, and we declare primary outage and secondary outage.
- Stage-II: If $\mathcal{S}_c$ is nonempty, a secondary transmitter s_{n^*} from $\mathcal{S}_c$ will be scheduled for the cooperation, as

$$n^* = \arg\max_{s_n\in\mathcal{S}_c}\left\{\min_{m=1,\dots,M_n}\left(\pi_m^n|h_{s_n,r_m^n}|^2\right)\right\}, \quad (7)$$

  where $\pi_m^n = \min_{k=0,\dots,m}\zeta_k^n$, and $\zeta_k^n = \frac{\alpha_k^n}{2^{2R_k^n}-1} - \sum_{i=k+1}^{M_n}\alpha_i^n$. Note that the condition $\zeta_k^n > 0$ should be guaranteed in application of NOMA [6].

The R-SUS scheme can be implemented at the secondary transmitters in a distributed fashion as follows. Each secondary transmitter s_n belonging to $\mathcal{S}_c$ maintains a virtual timer [10] and sets an initial value for the timer in inversely proportional to $\min_{m=1,\dots,M_n}\left(\pi_m^n|h_{s_n,r_m^n}|^2\right)$. Then secondary transmitter s_{n^*} will have the smallest initial timer value. Thus, its timer will expire first, and upon its timer expiration, it will broadcast a control message to other secondary transmitters to notify its existence for the NOMA assisted cooperation.

Channel state information requirement of R-SUS is discussed below. 1) Each secondary transmitter should know its channel gain to the primary receiver (to decide whether it can help the primary network to achieve the target rate R_0). This requirement can be fulfilled by a pilot signal sending from the primary receiver to all secondary transmitters. 2) Each secondary transmitter needs to know its mean channel gain to each of its secondary receivers. 3) Each candidate helper, say s_n, needs to know $\min_{m=1,\dots,M_n}\left(\pi_m^n|h_{s_n,r_m^n}|^2\right)$. This can be achieved as follows. For s_n, its secondary receiver r_m^n maintains a virtual timer with initial value in proportional to $\pi_m^n|h_{s_n,r_m^n}|^2$. The secondary receiver, say r_k^n, whose timer expires first can send to s_n its information of $\pi_k^n|h_{s_n,r_k^n}|^2$, which is exactly $\min_{m=1,\dots,M_n}\left(\pi_m^n|h_{s_n,r_m^n}|^2\right)$.

***Theorem** 1:* The R-SUS scheme minimizes the primary and secondary outage probabilities.

Proof: A primary outage happens if and only if $\mathcal{S}_c$ is an empty set. When $\mathcal{S}_c$ is nonempty, any scheduled user from $\mathcal{S}_c$ always ensures that the primary network can achieve the preset target data rate R_0. Thus, the proposed R-SUS scheme minimizes the primary outage probability.

On the other hand, suppose that secondary transmitter s_n is scheduled for the cooperation. Based on (4) and (5), one can compute the conditional secondary outage probability by

$$\begin{aligned} P^{\text{s}}_{\text{R-SUS}|s_n} &= 1-\Pr\left(\bigcap_{m=1}^{M_n}\left(\bigcap_{k=0}^{m}\left\{R_{r_m^n,x_k^n}\geq R_k^n\right\}\right)\right) \\ &= 1-\Pr\left(\bigcap_{m=1}^{M_n}\left(\bigcap_{k=0}^{m}\left\{\zeta_k^n|h_{s_n,r_m^n}|^2\geq\frac{1}{\rho_t}\right\}\right)\right) \\ &= \Pr\left(\min_{m=1,\dots,M_n}\left(\pi_m^n|h_{s_n,r_m^n}|^2\right)<\frac{1}{\rho_t}\right), \end{aligned} \quad (8)$$

where $\Pr(\cdot)$ means probability of an event, and the superscript "s" in $P^{\text{s}}_{\text{R-SUS}|s_n}$ means "secondary outage." It can be observed that, to minimize the conditional secondary outage probability, it is optimal to choose the secondary transmitter that maximizes $\min_{m=1,\dots,M_n}\left(\pi_m^n|h_{s_n,r_m^n}|^2\right)$, which validates the criterion of (7). This completes the proof. ■

B. F-SUS Scheme

To further achieve fairness among the candidate helpers, we introduce the F-SUS scheme detailed as follows.

- Stage-I: Same as Stage-I of the R-SUS scheme.
- Stage-II: If $\mathcal{S}_c$ is nonempty, a secondary transmitter $s_{n^\dagger}$ from $\mathcal{S}_c$ will be scheduled for the cooperation, as

$$n^\dagger = \arg\max_{s_n\in\mathcal{S}_c}\left\{\min_{m=1,\cdots,M_n}\left(\frac{M_n|h_{s_n,r_m^n}|^2}{\lambda_{ss,nm}}\right)\right\}. \quad (9)$$

The F-SUS scheme can also be implemented in a distributed fashion following a similar procedure to that of the R-SUS scheme, where the initial value of secondary transmitter s_n's virtual timer is set in inversely proportional to $\min_{m=1,\dots,M_n}\left(M_n|h_{s_n,r_m^n}|^2/\lambda_{ss,nm}\right)$.

Channel state information requirement of F-SUS is similar to that of R-SUS, and thus, the discussion is omitted.

***Theorem** 2:* The F-SUS scheme ensures that the probability of scheduling any $s_n \in \mathcal{S}_c$ as a cooperative relay is the same, i.e., $\Pr(n^\dagger = n) = \frac{1}{|\mathcal{S}_c|}$ with $|\mathcal{S}_c|$ being the size of $\mathcal{S}_c$.

Proof: Suppose that secondary transmitter s_n is scheduled to assist the NOMA cooperation. For notational convenience, we let $Z_n = \min_{m=1\dots,M_n}\left(M_n|h_{s_n,r_m^n}|^2/\lambda_{ss,nm}\right)$ for $s_n \in \mathcal{S}_c$. The probability $\Pr(n^\dagger = n)$ can be expressed as

$$\Pr\left(n^\dagger = n\right) = \Pr\left(\bigcap_{s_k\in\mathcal{S}_c,k\neq n}\left(Z_n > Z_k\right)\right)$$

$$= \int_{0}^{\infty} \prod_{s_k \in \mathcal{S}_c, k \neq n} F_{Z_k}(z) \cdot f_{Z_n}(z) dz, \tag{10}$$

where $f_{Z_n}(\cdot)$ and $F_{Z_k}(\cdot)$ denote the probability density function (PDF) of Z_n and cumulative distribution function (CDF) of Z_k, respectively. The CDF of Z_k can be computed by

$$F_{Z_k}(z) = 1 - \prod_{m=1}^{M_k} \Pr\left(|h_{s_k, r_m^k}|^2 > \frac{\lambda_{ss,km} z}{M_k}\right) = 1 - e^{-z}. \tag{11}$$

This implies that all Z_k for $s_k \in \mathcal{S}_c$ have the same CDF expression. By substituting this result into (10), the probability $\Pr(n^\dagger = n)$ can be further obtained as

$$\Pr\left(n^\dagger = n\right) = \int_{0}^{\infty} \left[1 - e^{-z}\right]^{|\mathcal{S}_c| - 1} e^{-z} dz = \frac{1}{|\mathcal{S}_c|}. \tag{12}$$

This completes the proof. ■

IV. ANALYSIS OF OUTAGE PROBABILITY AND DIVERSITY ORDER

In this section, we characterize the primary and secondary outage probabilities, and diversity orders achieved by the R-SUS and F-SUS schemes.

A. Outage Behavior of Primary Transmission

1) Exact outage probability: A primary outage occurs if and only if $\mathcal{S}_c$ is empty, and is not affected by the secondary user scheduling schemes. Therefore, the R-SUS scheme and the F-SUS scheme will exhibit the same primary outage probability, which is written as

$$\begin{aligned} P_{\text{R-SUS}}^{\text{p}} &= P_{\text{F-SUS}}^{\text{p}} = \Pr\left(\mathcal{S}_c = \emptyset\right) \\ &= \prod_{n=1}^{N} [1 - \Pr(R_{s_n} \geq R_0) \Pr(R_p \geq R_0)] \\ &= \prod_{n=1}^{N} \left(1 - e^{-\frac{1}{\rho_t}\left(\frac{\epsilon_0}{\lambda_{ps,n}} + \frac{\xi_0^n}{\lambda_{sp,n}}\right)}\right), \end{aligned} \tag{13}$$

where $\xi_0^n = \frac{\epsilon_0}{\alpha_0^n - \epsilon_0 \sum_{m=1}^{M_n} \alpha_m^n}$, $\epsilon_0 = 2^{2R_0} - 1$, and the superscript "p" in $P_{\text{R-SUS}}^{\text{p}}$ and $P_{\text{F-SUS}}^{\text{p}}$ means "primary outage". The condition of $\alpha_0^n > \epsilon_0 \sum_{m=1}^{M_n} \alpha_m^n$ should be satisfied, because otherwise, the primary outage will happen for sure [6], [7].

2) Asymptotic outage probability: Using the fact that $1 - e^{-x} \simeq x$ when $x \to 0$, the asymptotic primary outage probability in high ρ_t regime can be obtained as

$$P_{\text{R-SUS}}^{\text{p,asy}} = P_{\text{F-SUS}}^{\text{p,asy}} = \rho_t^{-N} \prod_{n=1}^{N} \left(\frac{\epsilon_0}{\lambda_{ps,n}} + \frac{\xi_0^n}{\lambda_{sp,n}}\right). \tag{14}$$

***Remark** 1:* As observed from (14), both the R-SUS and F-SUS schemes can achieve a diversity order of N (i.e., a full diversity order given N secondary transmitters) for the primary transmission.

B. Outage Behavior of Secondary Transmission

1) Exact outage probability: The secondary outage probability with the R-SUS scheme can be written as

$$P_{\text{R-SUS}}^{\text{s}} = \Pr\left(\mathcal{S}_c = \emptyset\right) + \Pr\left(E_s^{\text{R-SUS}}, \mathcal{S}_c \neq \emptyset\right), \tag{15}$$

where $E_s^{\text{R-SUS}}$ denotes the event that for the scheduled secondary transmitter s_{n^*} by the R-SUS scheme, at least one of its receivers has an achievable data rate smaller than its target data rate. The first term on the right-hand side of (15) is given by (13), and the second term can be rewritten as

$$\begin{aligned} &\Pr\left(E_s^{\text{R-SUS}}, \mathcal{S}_c \neq \emptyset\right) \\ &= \sum_{\substack{\mathcal{A} \subseteq \{s_1, \ldots, s_N\} \\ |\mathcal{A}| \geq 1}} \Pr\left(\mathcal{S}_c = \mathcal{A}\right) \Pr\left(E_s^{\text{R-SUS}} | \mathcal{S}_c = \mathcal{A}\right). \end{aligned} \tag{16}$$

$\Pr(\mathcal{S}_c = \mathcal{A})$ can be computed by

$$\begin{aligned} \Pr\left(\mathcal{S}_c = \mathcal{A}\right) &= e^{-\sum_{s_n \in \mathcal{A}} \frac{1}{\rho_t}\left(\frac{\epsilon_0}{\lambda_{ps,n}} + \frac{\xi_0^n}{\lambda_{sp,n}}\right)} \\ &\quad \times \prod_{s_n \in \bar{\mathcal{A}}} \left(1 - e^{-\frac{1}{\rho_t}\left(\frac{\epsilon_0}{\lambda_{ps,n}} + \frac{\xi_0^n}{\lambda_{sp,n}}\right)}\right), \end{aligned} \tag{17}$$

where $\bar{\mathcal{A}} = \{s_1, s_2, ..., s_N\} \backslash \mathcal{A}$. With the R-SUS scheme in (7), $\Pr(E_s^{\text{R-SUS}} | \mathcal{S}_c = \mathcal{A})$ is derived as

$$\begin{aligned} &\Pr\left(E_s^{\text{R-SUS}} | \mathcal{S}_c = \mathcal{A}\right) \\ &\overset{\text{(i)}}{=} \Pr\left(\max_{s_n \in \mathcal{A}} \left(\min_{m=1,\ldots,M_n} \pi_m^n |h_{s_n, r_m^n}|^2\right) < \frac{1}{\rho_t}\right) \\ &\overset{\text{(ii)}}{=} \prod_{s_n \in \mathcal{A}} \Pr\left(\min_{m=1,\ldots,M_n} \pi_m^n |h_{s_n, r_m^n}|^2 < \frac{1}{\rho_t}\right) \\ &= \prod_{s_n \in \mathcal{A}} \left(1 - e^{-\sum_{m=1}^{M_n} \frac{1}{\rho_t \pi_m^n \lambda_{ss,nm}}}\right), \end{aligned} \tag{18}$$

where step (i) is from (8), and step (ii) follows the independence of the candidate helpers. Substituting (13), (16)–(18) into (15), the closed-form secondary outage probability of the R-SUS scheme is derived.

Next we focus on the secondary outage probability achieved by the F-SUS scheme. Similar to (15), we have

$$\begin{aligned} P_{\text{F-SUS}}^{\text{s}} = \Pr\left(\mathcal{S}_c = \emptyset\right) + &\sum_{\substack{\mathcal{A} \subseteq \{s_1, \ldots, s_N\} \\ |\mathcal{A}| \geq 1}} \Pr\left(\mathcal{S}_c = \mathcal{A}\right) \\ &\times \Pr\left(E_s^{\text{F-SUS}} | \mathcal{S}_c = \mathcal{A}\right) \end{aligned} \tag{19}$$

where $E_s^{\text{F-SUS}}$ denotes the event that for $s_{n^\dagger}$ scheduled by the F-SUS scheme, at least one of its receivers has an achievable data rate smaller than its target data rate. According to (9), $\Pr\left(E_s^{\text{F-SUS}} | \mathcal{S}_c = \mathcal{A}\right)$ can be calculated as

$$\begin{aligned} &\Pr\left(E_s^{\text{F-SUS}} | \mathcal{S}_c = \mathcal{A}\right) \\ &= \sum_{s_i \in \mathcal{A}} \Pr\left(\min_{m=1,\ldots,M_i} \pi_m^i |h_{s_i, r_m^i}|^2 < \frac{1}{\rho_t},\ n^\dagger = i\right) \\ &= \sum_{s_i \in \mathcal{A}} \Pr\Bigg(\min_{m=1,\ldots,M_i} \pi_m^i |h_{s_i, r_m^i}|^2 < \frac{1}{\rho_t}, \\ &\qquad \bigcap_{s_k \in \mathcal{A}, k \neq i} \left(Z_k < \min_{m=1,\ldots,M_i} \frac{M_i |h_{s_i, r_m^i}|^2}{\lambda_{ss,im}}\right)\Bigg) \\ &\overset{\text{(iii)}}{=} \sum_{s_i \in \mathcal{A}} \int \cdots \int_{\mathcal{R}} \prod_{m=1}^{M_i} \frac{1}{\rho_t \lambda_{ss,im}} e^{-\frac{y_m}{\rho_t \lambda_{ss,im}}} \end{aligned}$$

$$\times \prod_{s_k\in\mathcal{A},k\neq i} F_{Z_k}\left(\frac{1}{\rho_t}\min_{m=1,\dots,M_i}\frac{M_i y_m}{\lambda_{ss,im}}\right)dy_1\cdots dy_{M_i}$$

$$\overset{\text{(iv)}}{=} \sum_{s_i\in\mathcal{A}}\sum_{j=0}^{|\mathcal{A}|-1}\binom{|\mathcal{A}|-1}{j}(-1)^j\int\cdots\int_{\mathcal{R}}\Theta dy_1\cdots dy_{M_i}. \quad (20)$$

In step (iii), $y_1, ..., y_{M_i}$ stand for exponentially distributed random variables $\rho_t|h_{s_i,r_1^i}|^2, ..., \rho_t|h_{s_i,r_{M_i}^i}|^2$, and the integration is over $\mathcal{R} = \{y_1, \dots, y_{M_i} : \min_{m=1,\dots,M_i}(\pi_m^i y_m) < 1\}$. In step (iv), we use the CDF of Z_k in (11), and Θ is defined as

$$\Theta \triangleq e^{-\frac{j}{\rho_t}\min\limits_{m=1,\dots,M_i}\frac{M_i y_m}{\lambda_{ss,im}}}\prod_{m=1}^{M_i}\frac{1}{\rho_t\lambda_{ss,im}}e^{-\frac{y_m}{\rho_t\lambda_{ss,im}}}.$$

It is rather challenging to derive a closed-form expression for (20) when $M_i > 2$, due to the complicated multi-dimension integration domain $\mathcal{R}$. Thus, here we focus on a special case $M_i = 2$ for $i \in \{1, 2, ..., N\}$. Note that the outage analysis for this special case is practically meaningful, because the two-user NOMA is one most typical NOMA scenario, and is recommended to be used in practical systems, such as multiuser superposition transmission (MUST) in Third Generation Partnership Project-Long Term Evolution (3GPP-LTE) [1].

With $M_i = 2$ for $i \in \{1, 2, ..., N\}$, we define constant $a_i \triangleq \frac{\pi_2^i}{\pi_1^i}$ and constant $b_i \triangleq \frac{\lambda_{ss,i1}}{\lambda_{ss,i2}}$. Here we assume $a_i > b_i$ (noting that the case with $a_i \leq b_i$ can be treated similarly). Then $\mathcal{R}$ can be divided into sub-region $\mathcal{R}_1 = \{y_1, y_2 : y_1 \geq a_i y_2, \pi_2^i y_2 < 1\}$ and sub-region $\mathcal{R}_2 = \{y_1, y_2 : y_1 < a_i y_2, \pi_1^i y_1 < 1\}$. Sub-region $\mathcal{R}_2$ can be further divided into $\mathcal{R}_{21} = \{y_1, y_2 : y_1 < b_i y_2, \pi_1^i y_1 < 1\}$ and $\mathcal{R}_{22} = \{y_1, y_2 : b_i y_2 \leq y_1 < a_i y_2, \pi_1^i y_1 < 1\}$. Thus, the integration of Θ over $\mathcal{R}$ is equal to sum of integrations of Θ over $\mathcal{R}_1$, $\mathcal{R}_{21}$ and $\mathcal{R}_{22}$, as follows.

$$\iint\limits_{\mathcal{R}_1}\Theta dy_1 dy_2 = \iint\limits_{\substack{a_i y_2\leq y_1\\ \pi_2^i y_2<1}}\frac{e^{-\frac{y_1}{\rho_t\lambda_{ss,i1}}-\frac{(2j+1)y_2}{\rho_t\lambda_{ss,i2}}}}{\rho_t^2\lambda_{ss,i1}\lambda_{ss,i2}}dy_1 dy_2$$
$$= \frac{b_i}{a_i+(2j+1)b_i}\left(1-e^{-\frac{a_i+(2j+1)b_i}{b_i\rho_t\pi_2^i\lambda_{ss,i2}}}\right). \quad (21)$$

$$\iint\limits_{\mathcal{R}_{21}}\Theta dy_1 dy_2 = \iint\limits_{\substack{y_1<b_i y_2\\ \pi_1^i y_1<1}}\frac{e^{-\frac{(2j+1)y_1}{\rho_t\lambda_{ss,i1}}-\frac{y_2}{\rho_t\lambda_{ss,i2}}}}{\rho_t^2\lambda_{ss,i1}\lambda_{ss,i2}}dy_1 dy_2$$
$$= \frac{1}{2j+2}\left(1-e^{-\frac{2j+2}{\rho_t\pi_1^i\lambda_{ss,i1}}}\right). \quad (22)$$

$$\iint\limits_{\mathcal{R}_{22}}\Theta dy_1 dy_2 = \iint\limits_{\substack{b_i y_2\leq y_1<a_i y_2\\ \pi_1^i y_1<1}}\frac{e^{-\frac{y_1}{\rho_t\lambda_{ss,i1}}-\frac{(2j+1)y_2}{\rho_t\lambda_{ss,i2}}}}{\rho_t^2\lambda_{ss,i1}\lambda_{ss,i2}}dy_1 dy_2$$
$$= \frac{a_i\left(1-e^{-\frac{a_i+(2j+1)b_i}{a_i\rho_t\pi_1^i\lambda_{ss,i1}}}\right)}{(2j+1)a_i+(2j+1)^2 b_i} - \frac{1-e^{-\frac{2j+2}{\rho_t\pi_1^i\lambda_{ss,i1}}}}{(2j+1)(2j+2)}. \quad (23)$$

Applying the above three integration results into (20) and (19), secondary outage probability of the F-SUS scheme in the special case $M_i = 2$ ($i \in \{1, 2, ..., N\}$) is attained in closed form.

2) Asymptotic outage probability: In high ρ_t regime, the asymptotic secondary outage probability achieved by the R-SUS scheme is given by

$$P_{\text{R-SUS}}^{\text{s,asy}} = \rho_t^{-N}\Bigg[\prod_{n=1}^{N}\left(\frac{\epsilon_0}{\lambda_{ps,n}}+\frac{\xi_0^n}{\lambda_{sp,n}}\right) + \sum_{\substack{\mathcal{A}\subseteq\{s_1,\dots,s_N\}\\ |\mathcal{A}|\geq 1}}$$
$$\prod_{s_n\in\bar{\mathcal{A}}}\left(\frac{\epsilon_0}{\lambda_{ps,n}}+\frac{\xi_0^n}{\lambda_{sp,n}}\right)\prod_{s_n\in\mathcal{A}}\left(\sum_{m=1}^{M_n}\frac{1}{\pi_m^n\lambda_{ss,nm}}\right)\Bigg]. \quad (24)$$

For the F-SUS scheme, according to high ρ_t regime approximations, we have $\prod_{s_k\in\mathcal{A},k\neq i}F_{Z_k}\left(\frac{1}{\rho_t}\min_{m=1,\dots,M_i}\frac{M_i y_m}{\lambda_{ss,im}}\right) \simeq \left(\frac{1}{\rho_t}\min_{m=1,\dots,M_i}\frac{M_i y_m}{\lambda_{ss,im}}\right)^{|\mathcal{A}|-1}$. Applying this approximation into (20) and dividing $\mathcal{R}$ into $\mathcal{R}_1$, $\mathcal{R}_{21}$, and $\mathcal{R}_{22}$, the asymptotic secondary outage probability achieved by the F-SUS scheme in the special case of $M_i = 2$ for $i \in \{1, 2, ..., N\}$ can be derived as

$$P_{\text{F-SUS}}^{\text{s,asy}} = \rho_t^{-N}\Bigg[\prod_{n=1}^{N}\left(\frac{\epsilon_0}{\lambda_{ps,n}}+\frac{\xi_0^n}{\lambda_{sp,n}}\right)$$
$$+\sum_{\substack{\mathcal{A}\subseteq\{s_1,\cdots,s_N\}\\ |\mathcal{A}|\geq 1}}\frac{2^{|\mathcal{A}|-1}}{|\mathcal{A}|}\prod_{s_n\in\bar{\mathcal{A}}}\left(\frac{\epsilon_0}{\lambda_{ps,n}}+\frac{\xi_0^n}{\lambda_{sp,n}}\right)$$
$$\times\sum_{s_i\in\mathcal{A}}\left(\frac{1}{(\pi_1^i\lambda_{ss,i1})^{|\mathcal{A}|}}+\frac{(1+a_i/b_i)^{-|\mathcal{A}|}}{(\pi_2^i\lambda_{ss,i2})^{|\mathcal{A}|}}\right)\Bigg]. \quad (25)$$

***Remark** 2:* It is observed from (24) and (25) that both the R-SUS and F-SUS scheme can achieve the same full diversity order of N for the secondary transmission.

V. PERFORMANCE EVALUATION

In this section, we present numerical results for the proposed framework. Unless otherwise specified, we consider $M_n = 2$ ($n \in \{1, 2, ..., N\}$) with the power allocation coefficients being $\alpha_0^n = 0.7$, $\alpha_1^n = 0.2$, and $\alpha_2^n = 0.1$. The target data rates for primary and secondary signals are set as $R_0 = 0.6$ and $R_1^n = R_2^n = 0.2$. The exponential decay model is adopted for the average channel gains, given by $\lambda_{ps,n} = \lambda_{sp,n} = e^{\delta_p(n-1)}$, and $\lambda_{ss,nm} = e^{\delta_s(m-1)}$, in which $\delta_p, \delta_s \geq 0$ represent the exponentially decaying factors, both taking value of 1.

Fig. 2 shows the analytically derived outage probabilities, asymptotic outage probabilities, and simulated outage probabilities of the primary and secondary networks versus transmit SNR ρ_t for the NOMA assisted cooperation with the R-SUS and F-SUS schemes. It can be seen that the analytical results agree perfectly with the simulated ones, validating the accuracy of the theoretical analysis. As can be observed from the figure, the proposed R-SUS and F-SUS schemes can ensure a full diversity order for both primary and secondary transmissions, i.e., the slopes of the outage probability curves in high ρ_t regime have magnitude equal to N. The R-SUS scheme achieves a lower secondary outage probability than the F-SUS scheme. This is because the R-SUS scheme aims at minimal outage probabilities, while the F-SUS scheme aims at enhanced fairness. For $M_n = 5$ with $(\alpha_0^n, \alpha_1^n, ..., \alpha_5^n) = (0.7, 0.1, 0.08, 0.06, 0.04, 0.02)$ and $(R_0, R_1^n, R_2^n, ..., R_5^n) = (0.6, 0.2, 0.2, 0.2, 0.2, 0.2)$, Fig. 2

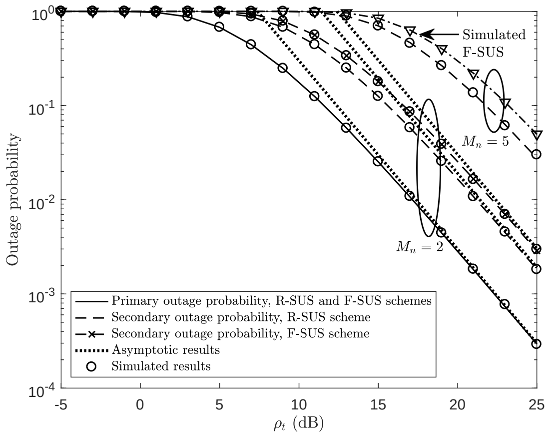

Fig. 2. Outage performance as a function of ρ_t for the considered NOMA assisted cooperative spectrum sharing scenario ($N = 2$).

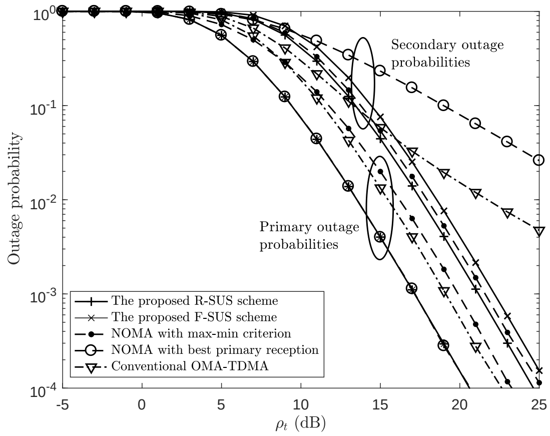

Fig. 3. Outage probabilities of the proposed R-SUS and F-SUS schemes and the three benchmark schemes ($N = 3$).

also shows the analytically derived and simulated secondary outage probability of the R-SUS scheme, and simulated secondary outage probability of the F-SUS scheme (primary outage probability curve of the two schemes with $M_n = 5$ is the same as that of the two schemes with $M_n = 2$ in Fig. 2). We can see that the two schemes achieve a full diversity order for secondary transmissions with $M_n = 5$.

Next we compare the proposed framework with other secondary user scheduling schemes. More particularly, we consider three benchmark schemes in the following. (1) NOMA with max-min criterion: secondary transmitter $s_{n^\ddagger}$ which has the strongest worst-case channel gain is selected, that is, $n^\ddagger = \arg\max_n\{\min(|h_{p,s_n}|^2, |h_{s_n,p}|^2, \min_m |h_{s_n,r_m^n}|^2)\}$. (2) NOMA with best primary reception: similar to the reactive criterion [10], the secondary transmitters which correctly decode x_0 constitute a decoding set $\mathcal{D}_c$, and then $s_{\hat{n}}$ in $\mathcal{D}_c$ which maximizes the primary reception quality is selected, that is, $\hat{n} = \arg\max_{s_n \in \mathcal{D}_c}\{|h_{s_n,p}|^2\}$. (3) Conventional orthogonal multiple access-time division multiple access (OMA-TDMA): the secondary transmitter, say s_n, that maximizes the primary reception reliability is selected, and the transmissions for signals x_0, x_1^n, x_2^n are completed within four orthogonal phases as follows. The first two phases are used by the primary transmitter to send its signal and by s_n to forward the primary signal, and the remaining two phases are used by s_n to transmit to its two receivers based on a TDMA manner. To make a fair comparison, we assume that all the schemes have the same target data rate requirement. Fig. 3 shows outage probabilities of the proposed secondary user scheduling schemes and the three benchmark schemes. We have three observations:

- The NOMA scheme with max-min criterion also guarantees a full diversity order for the primary and secondary transmissions. But it suffers an SNR gain loss, i.e., the outage probability curves are on the right-hand side of outage probability curves of our proposed R-SUS scheme.
- The NOMA scheme with best primary reception yields the same primary outage behavior as that of our proposed schemes, but its secondary outage performance is much worse. This is due to the fact that the NOMA scheme with best primary reception considers only $|h_{s_n,p}|^2$ to determine the “best” secondary transmitter, which leads to a diversity order loss at the secondary network.
- Our proposed NOMA schemes have lower outage probabilities for both primary and secondary networks than the OMA-TDMA scheme. The performance gain of our schemes is mainly attributed to an efficient use of the spectrum, i.e., the primary and secondary receivers are simultaneously served in one resource block, while separated resource blocks are needed by OMA-TDMA.

VI. CONCLUSION

We have developed a NOMA assisted cooperative spectrum sharing framework, and proposed the R-SUS and F-SUS schemes to exploit multiuser diversity. Analytical expressions of outage probabilities have been derived in closed form. The results manifest that the R-SUS scheme achieves the outage-optimal performance for the primary and secondary transmissions, and the F-SUS scheme guarantees user fairness. Both schemes achieve a full diversity order.

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