Cooperative Wireless Multicast: Performance Analysis and Time Allocation

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***Abstract*—Cooperative wireless multicast is investigated, in which a source sends multicast messages to a number of users. For a multicast message from the source, some users do not successfully receive the message (called unsuccessful users). For a successful user (who successfully receives the message from the source), we define its worst relaying channel gain as the smallest channel gain among its channel gains to all unsuccessful users. It is proposed that the successful user whose worst relaying channel gain is the highest among the worst relaying channel gains of all successful users is selected to serve as a relay. Considering that the channels in the system are independent but non-identically distributed Rayleigh fading, we derive a closed-form outage probability expression for the proposed scheme. It is shown that the proposed scheme can achieve full diversity, and thus, having a larger number of users can improve the outage performance. Further, we study the time allocation strategy that determines the durations used by the source and the selected relay for their transmissions, respectively. An approximate optimal time allocation is derived. In addition, we also investigate the case with relay selection error and the case with mixed Rayleigh/Rician fading. Simulation results verify the performance of the proposed cooperative multicast scheme and time allocation strategy.**

***Index Terms*—Cooperative multicast, outage probability, time allocation.**

I. INTRODUCTION

In wireless multicast, where a source needs to send the same data to a number of users, some users may not successfully receive the data from the source. This is because the channels to the users are independent and some users may experience deep channel fading. To solve this problem, the idea of cooperative multicast has been introduced and investigated recently [1]–[8], in which dedicated relays or successful users (i.e., those who successfully decode data from the source) help forward the received data to those unsuccessful users (i.e., those who are unable to decode data from the source).

Dedicated relays are employed in [2]–[6]. The work in [2] uses a single amplify-and-forward relay, and studies the outage performance over Rayleigh fading channels as well as the power allocation of the source and the relay. When there are multiple relays available, the schemes in [3] and [4] select the relay that maximizes the worst end-to-end signal-to-noise ratio (SNR), referred to as the best-relay-selection. The best-relay-selection based cooperative multicast (BRS-CM) is shown to achieve a full diversity order (i.e., the diversity order equals the number of relays plus one). Also for the multiple-relay case, the work in [5] considers the selection of the N-th best relay. A genie-aided cooperative multicast (GA-CM) scheme is proposed in [6], which allows all successful relays (i.e., the relays who successfully decode data from the source) to simultaneously forward the received data to the users. Each user combines signals from the source and from the successful relays. A diversity order of two is achieved in the system.

Successful users are employed to help forward the received data in [6]–[8]. The work in [7] proposes that some successful users are selected to sequentially forward the received data. The optimal number of users to be selected as relays is derived. In [8], if a successful user forwards its received data, it gets some payment from the unsuccessful users. The problem is formulated as a multiple-seller multiple-buyer Stackelberg game. The distributed cooperative multicast (D-CM) scheme proposed in [6] allows all successful users to simultaneously forward the received data. Similar to the GA-CM scheme, the D-CM scheme also achieves a diversity order of two.

In this paper, different from the D-CM scheme in which all successful users simultaneously serve as relays, we propose to select one successful user to serve as a relay. Specifically, for each successful user, we define its *worst relaying channel gain* as the smallest channel gain among its channel gains to all unsuccessful users. We propose that the successful user whose worst relaying channel gain is the highest among the worst relaying channel gains of all successful users is selected to serve as a relay. The main contribution of this paper is threefold. First, we propose a new cooperative wireless multicast scheme, which is energy-efficient as only one successful user is selected to help. For the selection of a relay among the successful users, a major challenge is how to obtain the required channel state information (CSI). To this end, we propose a channel gain acquisition process. Second, in the literature, the channels among users are usually assumed to be independent and identically distributed (i.i.d.). This may not be accurate, since for a number of users, the channels among them have different path loss attenuation. Thus, we consider that the channels in the system are independent but non-identically distributed (i.n.i.d) Rayleigh fading. Based on this channel model, we derive closed-form expressions for both exact outage probability and its high-SNR asymptote, which further shows that our proposed scheme achieves full diversity.

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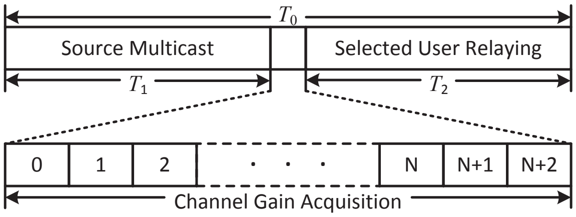

Fig. 1. Three portions for transmission of a multicast message.

We also investigate the case when the channels in the system follow mixed Rayleigh/Rician fading. Third, we propose a time allocation strategy for the source's broadcasting and the selected user's forwarding, which minimizes the outage probability of the system. An optimal time allocation can be approximately determined based on a simple root-finding approach (e.g., a bisection search).

II. System Model and Proposed Cooperative Scheme

Consider that a source s sends multicast messages to N (≥ 2) users, called user $1, 2, ..., N$, respectively. Each channel (from the source to users and between users) experiences path loss attenuation and Rayleigh fading. Channel reciprocity is assumed. Denote f_n and $h_{n,n'}$ as the channel gains of the links from the source s to user n and between users n and n', respectively. Thus, the channel gains of these links follow an exponential distribution with mean $d_n^{-\eta}$ and $d_{n,n'}^{-\eta}$, respectively, with d_n being the distance from source s to user n, $d_{n,n'}$ the distance between users n and n', and η the path loss exponent. Since the distances of the links are different, the channels are i.n.i.d. The transmit power of the source is P_s. If a user is selected to serve as a relay, its transmit power is P_r. The system provides a delay-sensitive multicast service. Each multicast message has L-bit information, and should be delivered to all users within a duration of T_0. Block-fading channel model is assumed, i.e., the instantaneous channel gain of any link does not change within one duration T_0, but it may change from one duration to the next.

For the transmission of a multicast message, T_0 contains three portions of time as shown in Fig. 1: Source Multicast Portion with duration T_1, Channel Gain Acquisition Portion, and Selected User Relaying Portion with duration T_2. The total duration of the three portions is T_0.

Source Multicast Portion: In this portion of time, source s broadcasts an L-bit message to all users. Therefore, the transmission rate is L/T_1. Recall that f_n is the channel gain from the source to user n. The channel capacity is given as $\log_2(1 + P_s f_n/\sigma_0^2)$, where σ_0^2 is the variance of the additive white Gaussian noise (AWGN). If the capacity is not less than the source transmission rate, i.e., $\log_2(1+P_s f_n/\sigma_0^2) \geq L/T_1$, user n can successfully receive the message, and thus, is called a successful user. In other words, user n is a successful user if $f_n \geq (2^{L/T_1} - 1)\sigma_0^2/P_s$. Denote the set of all successful users as $\mathcal{S}$, and the set of all unsuccessful users as $\overline{\mathcal{S}} \triangleq \mathcal{N} \backslash \mathcal{S}$. Here $\mathcal{N} \triangleq \{1, 2, ..., N\}$ is the set of all users.

Channel Gain Acquisition Portion: The purpose of this portion of time is to let each successful user get their channel gain information to all unsuccessful users. There are $N + 3$ minislots, indexed as minislot $0, 1, 2, ..., N + 2$, as shown in Fig. 1.

1) In minislot 0, source s broadcasts a flag message `Channel Gain Acquisition Request` (`CGAR`) to all users. The source also keeps a counter for the number of successful users. The initial value of the counter is 0.
2) Upon receiving the `CGAR`, user $i \in \mathcal{N}$ broadcasts a flag message `Success` or `Failure` at minislot i according to whether it is a successful user or not. If the flag message is `Success`, the source s increases its counter by 1. If the flag message is `Failure`, each successful user, say user n ($n \in \mathcal{S}$), measures (by reception of the flag message of user i) the channel gain between user i and itself, denoted as $h_{n,i}$.
At the end of minislot N, each successful user, say user n ($n \in \mathcal{S}$), knows the channel gain between itself and every unsuccessful user, and thereby computes its worst relaying channel gain as $g_n^{\text{wc}} = \min_{n' \in \overline{\mathcal{S}}} h_{n,n'}$.
3) In minislot $N + 1$, if the counter at the source is N (i.e., all N users are successful) or 0 (i.e., no user is successful), the source broadcasts a flag message `Non User Selection Request` (`NUSR`), which cancels the subsequent minislot $N+2$ and Selected User Relaying Portion, and immediately starts transmission of the next multicast message; otherwise, the source broadcasts a flag message `User Selection Request` (`USR`).
4) In minislot $N+2$ which has duration t_0, upon receiving the flag message `USR`, successful user $n \in \mathcal{S}$ starts a virtual timer initiated by $\tau_n = t_0 \exp(-g_n^{\text{wc}}) < t_0$, similar to [9]. As a result, the virtual timer of the successful user $n^* = \text{argmax}_{n \in \mathcal{S}} g_n^{\text{wc}}$ expires first, which means that user n^* (whose worst relaying channel gain is the highest among the worst relaying channel gains of all successful users) is selected. Then user n^* broadcasts a flag message `Expire` to announce its presence. Upon receiving this message, all other successful users remain silent in the subsequent Selected User Relaying Portion.

In the Channel Gain Acquisition Portion, there are totally six flag messages. So each flag message can be encoded by a 3-bit codeword, and strong channel coding can be used. It is therefore reasonable to assume that the transmissions of the flag messages are error-free.

Selected User Relaying Portion: In this portion of time with duration T_2, the selected user n^* re-encodes its received message and forwards the message using a power level P_r. Unsuccessful users try to receive data from user n^*.[1] For unsuccessful user n, its received SNR is given as $P_r h_{n^*,n}/\sigma_0^2$. Similar to the discussion for Source Multicast Portion, unsuccessful user n can correctly decode the message from user n^* if $h_{n^*,n} \geq \left(2^{L/T_2} - 1\right) \sigma_0^2/P_r$.

III. Outage Performance Analysis

We define $T \triangleq T_1 + T_2$ as the total transmission duration of the source and the selected user. T is a constant since the duration of the Channel Gain Acquisition Portion is fixed. The transmission duration of the source and the selected user is expressed as $T_1 = \alpha T$ and $T_2 = (1 - \alpha)T$, respectively, with $0 < \alpha \leq 1$. Here α is called the time allocation factor.

[1]For operation simplicity, unsuccessful users do not combine the signals received from the source and from user n^*. Thus, the results in this paper can be treated as a lower bound for system performance.

A. Exact Outage Probability Analysis

Since the purpose of multicast is to deliver messages to all users, we declare an outage if there exists a user that cannot successfully decode a multicast message from either the source or the selected successful user. In the sequel, denote the set of successful users as $\mathcal{S} = \{n_1, n_2, ..., n_k\} \subset \mathcal{N}$, in which $n_1, n_2, ..., n_k$ are successful users, and denote the set of unsuccessful users as $\overline{\mathcal{S}} = \mathcal{N} \backslash \{n_1, n_2, ..., n_k\}$. Thus, conditioned on a non-empty set $\mathcal{S}$, the conditional outage probability is the probability that the selected successful user's worst relaying channel gain, given as $\max_{m\in\mathcal{S}} \min_{n\in\overline{\mathcal{S}}} h_{m,n}$, is less than $(2^{L/T_2}-1)\sigma_0^2/P_r$, which can be expressed as

$$\begin{aligned}
&P_{\mathrm{out}|\mathcal{S}=\{n_1,n_2,...,n_k\}} \\
&= \mathrm{Prob}\Big(\max_{m\in\mathcal{S}}\min_{n\in\overline{\mathcal{S}}} h_{m,n} < (2^{\frac{L}{T_2}}-1)\sigma_0^2/P_r \Big| \\
&\qquad \mathcal{S} = \{n_1, n_2, ..., n_k\}\Big) \\
&\overset{(a)}{=} \prod_{i=1}^{k} \mathrm{Prob}\Big(\min_{n\in\overline{\mathcal{S}}} h_{n_i,n} < (2^{\frac{R}{1-\alpha}}-1)\sigma_0^2/P_r\Big) \\
&\overset{(b)}{=} \prod_{i=1}^{k}\Big(1 - \prod_{n\in\overline{\mathcal{S}}}\mathrm{Prob}\big(h_{n_i,n} \geq (2^{\frac{R}{1-\alpha}}-1)\sigma_0^2/P_r\big)\Big) \\
&\overset{(c)}{=} \prod_{i=1}^{k}\Big(1-\exp\Big(-\frac{(2^{\frac{R}{1-\alpha}}-1)\sigma_0^2}{P_r}\sum_{n\in\overline{\mathcal{S}}} d_{n_i,n}^{\eta}\Big)\Big) \qquad (1)
\end{aligned}$$

where $\mathrm{Prob}(\cdot)$ means the probability of an event, $R \triangleq L/T$ is the expected spectral efficiency, $\overline{\mathcal{S}} = \mathcal{N}\backslash\{n_1, n_2, ..., n_k\}$, and $(a), (b), (c)$ come from the fact that the channel gains of links among users are independent and exponentially distributed. Moreover, if $\mathcal{S} = \emptyset$ (null set), there is no successful user, and thus, $P_{\mathrm{out}|\mathcal{S}=\emptyset} = 1$. Recalling that user n is a successful user if $f_n \geq (2^{L/T_1}-1)\sigma_0^2/P_s$, the probability for the condition $\mathcal{S} = \{n_1, n_2, ..., n_k\}$ is

$$\begin{aligned}
&\mathrm{Prob}\left(\mathcal{S} = \{n_1, n_2, ..., n_k\}\right) \\
&= \prod_{i=1}^{k}\mathrm{Prob}\Big(f_{n_i} \geq \frac{(2^{\frac{R}{\alpha}}-1)\sigma_0^2}{P_s}\Big) \\
&\quad \times \prod_{n\in\overline{\mathcal{S}}}\mathrm{Prob}\Big(f_n < \frac{(2^{\frac{R}{\alpha}}-1)\sigma_0^2}{P_s}\Big) \\
&= \exp\Big(-\frac{(2^{\frac{R}{\alpha}}-1)\sigma_0^2}{P_s}\sum_{i=1}^{k} d_{n_i}^{\eta}\Big) \\
&\quad \times \prod_{n\in\overline{\mathcal{S}}}\Big(1-\exp\Big(-\frac{(2^{\frac{R}{\alpha}}-1)\sigma_0^2}{P_s}d_n^{\eta}\Big)\Big) \qquad (2)
\end{aligned}$$

and the probability for $\mathcal{S} = \emptyset$ is given as

$$\begin{aligned}
\mathrm{Prob}\left(\mathcal{S} = \emptyset\right) &= \prod_{n\in\mathcal{N}}\mathrm{Prob}\Big(f_n < \frac{(2^{\frac{R}{\alpha}}-1)\sigma_0^2}{P_s}\Big) \\
&= \prod_{n\in\mathcal{N}}\Big(1-\exp\Big(-\frac{(2^{\frac{R}{\alpha}}-1)\sigma_0^2}{P_s}d_n^{\eta}\Big)\Big). \qquad (3)
\end{aligned}$$

Based on the Total Probability Theorem, the outage probability is given in a closed-form as

$$\begin{aligned}
&P_{\mathrm{out}} \\
&= P_{\mathrm{out}|\mathcal{S}=\emptyset}\mathrm{Prob}\left(\mathcal{S}=\emptyset\right) \\
&\quad + \sum_{k=1}^{N-1}\sum_{1\leq n_1<n_2<...<n_k\leq N} P_{\mathrm{out}|\mathcal{S}=\{n_1,n_2,...,n_k\}} \\
&\qquad \times \mathrm{Prob}\left(\mathcal{S}=\{n_1,n_2,...,n_k\}\right) \\
&= \prod_{n\in\mathcal{N}}\Big(1-\exp\Big(-\frac{(2^{\frac{R}{\alpha}}-1)\sigma_0^2}{P_s}d_n^{\eta}\Big)\Big) \\
&\quad + \sum_{k=1}^{N-1}\sum_{1\leq n_1<n_2<...<n_k\leq N}\Big\{\exp\Big(-\frac{(2^{\frac{R}{\alpha}}-1)\sigma_0^2}{P_s}\sum_{i=1}^{k}d_{n_i}^{\eta}\Big) \\
&\qquad \times \Big[\prod_{n\in\overline{\mathcal{S}}}\Big(1-\exp\Big(-\frac{(2^{\frac{R}{\alpha}}-1)\sigma_0^2}{P_s}d_n^{\eta}\Big)\Big)\Big] \\
&\qquad \times \prod_{i=1}^{k}\Big(1-\exp\Big(-\frac{(2^{\frac{R}{1-\alpha}}-1)\sigma_0^2}{P_r}\sum_{n\in\overline{\mathcal{S}}}d_{n_i,n}^{\eta}\Big)\Big)\Big\} \qquad (4)
\end{aligned}$$

where $\sum_{1\leq n_1<n_2<...<n_k\leq N}$ is the shorthand notation of $\sum_{n_1=1}^{N-k+1}\sum_{n_2=n_1+1}^{N-k+2}\cdots\sum_{n_k=n_{k-1}+1}^{N}$. When $\alpha \to 0$, the outage probability in (4) simplifies to $P_{\mathrm{out}} = 1$. It means that the multicast message cannot be delivered to any user if the time duration for source multicast is too short. If $\alpha \to 1$, the outage probability in (4) simplifies to

$$P_{\mathrm{out}} = 1-\exp\left(-\frac{(2^R-1)\sigma_0^2}{P_s}\sum_{n=1}^{N}d_n^{\eta}\right)$$

which is exactly the outage probability of the direct multicast without relaying.

B. Asymptotic Outage Probability Analysis

We define the *system SNR* as $\bar{\gamma} \triangleq P_s/\sigma_0^2 = \mu P_r/\sigma_0^2$ with $\mu = P_s/P_r > 0$. With the help of the series representation of the exponential function [10, eq. (1.211.1)], the asymptotic expression for the outage probability in (4) is obtained as

$$P_{\mathrm{out}} \overset{\bar{\gamma}\to\infty}{\simeq} P_{\mathrm{out}}^{\mathrm{asym}} = \begin{cases} 1, & \alpha\to 0 \\ \dfrac{2^R-1}{\bar{\gamma}}\displaystyle\sum_{n=1}^{N}d_n^{\eta}, & \alpha\to 1 \\ \dfrac{1}{\bar{\gamma}^N}\displaystyle\sum_{k=0}^{N-1}Q(k,N)F(\alpha;k,N), & \text{otherwise} \end{cases} \qquad (5)$$

where $F(\alpha;k,N) \triangleq [G_1(\alpha)]^{N-k}[G_2(\alpha)]^k$ with $G_1(\alpha) \triangleq 2^{R/\alpha}-1$ and $G_2(\alpha) \triangleq 2^{R/(1-\alpha)}-1$, and

$$Q(k,N) \triangleq \begin{cases} \displaystyle\sum_{1\leq n_1<n_2<...<n_k\leq N}\Big\{\Big(\prod_{i=1}^{k}\Big(\sum_{n\in\overline{\mathcal{S}}}d_{n_i,n}^{\eta}\Big)\Big) \\ \quad \times\Big(\prod_{n\in\overline{\mathcal{S}}}d_n^{\eta}\Big)\mu^k\Big\}, & k=1,2,...,N-1 \\ \displaystyle\prod_{n=1}^{N}d_n^{\eta}, & k=0. \end{cases} \qquad (6)$$

It can be seen from (5) that the diversity order G_d of the proposed scheme is $G_d = 0$ if $\alpha \to 0$ (since no user can successfully receive from the source), $G_d = 1$ if $\alpha \to 1$ (since the scheme essentially reduces to the direct multicast), and $G_d = N$ (which means full diversity) if α is sufficiently away from 0 and 1.

From our numerical results in Section VI, the optimal α that minimizes the outage probability is equal to 1 only in extreme cases. In other cases, the optimal α is sufficiently away from 0 and 1, and thus, the proposed scheme with the optimal α can achieve a full diversity order.

As a comparison, the diversity order of the D-CM and GA-CM schemes in [6] are fixed to 2. So the proposed scheme outperforms these two schemes if the system contains more than two users. Furthermore, the schemes in [6] require all successful users/relays to forward the multicast message, whereas in the proposed scheme, only the selected user forwards the multicast message. Therefore, the power consumption of the proposed scheme is much lower.

Furthermore, for cooperative multicast schemes in [2]–[6] which use dedicated relays, if the number of users increases, the outage probability becomes larger. This is because if there are more users, the probability that all users can successfully receive the multicast message (either from the source or from dedicated relays) is smaller. On the contrary, since the proposed scheme can achieve a full diversity order, the outage probability is expected to decrease significantly if the number of users increases, as verified in Section VI. Thus, the proposed scheme is suitable for large-scale wireless networks.

Here we compare the communication overhead needed in our proposed scheme with that of the existing schemes. In our proposed scheme, we need to know CSI of $|\mathcal{S}| \times |\overline{\mathcal{S}}|$ channels. In the worst case, the CSI of $N^2/4$ channels is needed. As a comparison, in the worst case, the D-CM scheme needs the CSI of $(N^2/4 + N/2)$ channels, the GA-CM scheme needs the CSI of $(M+1)(N-1)$ channels, while the BRS-CM scheme needs the CSI of $(M+1)N$ channels, where M is the number of relays (if applicable) and N is the number of users. So in the worst setup, if $M \approx N$ or $M \gg N$, the proposed scheme needs the CSI of a fewer number of channels than both GA-CM and BRS-CM. If $M \ll N$ (which is more practical than $M \approx N$ and $M \gg N$), the proposed scheme needs the CSI of more channels than GA-CM and BRS-CM. However, when $M \ll N$, the proposed scheme achieves much better outage performance than the GA-CM, D-CM, and BRS-CM schemes. This is because the diversity order of the proposed scheme is N, which is much larger than the diversity order of two achieved by the GA-CM and D-CM schemes and the diversity order of $M+1$ by the BRS-CM scheme.

IV. Time Allocation for outage probability Minimization

Recall that $T_1 = \alpha T$, $T_2 = (1-\alpha)T$, and $T = T_1 + T_2$ is a constant. Now we target at outage probability minimization in the proposed scheme by choosing the time allocation factor α. First we take a look at how the outage probability changes when α varies. Fig. 2 shows the outage probability (4) in a system with four users. It can be seen that we can roughly partition the interval of $\alpha \in (0,1]$ into three regions. In Region I, α is very small, which means that the source needs to transmit at a very high rate. Thus, almost no user successfully receives the source's message, and the outage probability is almost 1. In Region II, α begins to be large enough to guarantee that there are successful users. So the outage probability starts to decrease. As α further increases beyond 0.7, the outage probability begins to increase, because $T_2 = (1-\alpha)T$ becomes insufficient to perform the relay transmission. In Region III, α is close to 1, and the outage probability slightly decreases with α. This is because almost no unsuccessful user correctly decodes the message from the selected successful user due to the very small value of T_2. Therefore, a larger value of α increases the probability that a user correctly decodes the message from the source. When $\alpha = 1$, the system reduces to direct multicast.

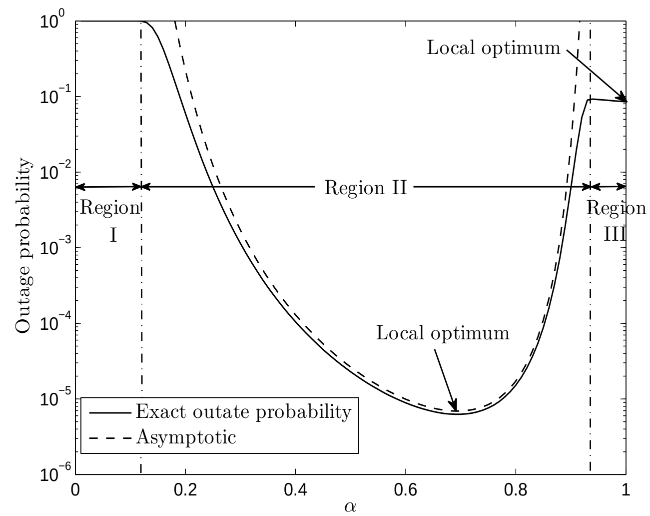

Fig. 2. The outage probability versus the time allocation factor α with $\eta = 2.6$, $\mu = 1$, and $\bar{\gamma} = 30$ dB. Here the source is located at (0,0), and four users are randomly distributed within a circle centered at $(3,0)$ and with radius of 1 (specifically, the four users have randomly generated locations (3.5401, 0.0264), (3.3860, -0.4696), (2.9944, 0.3468), and (3.0764, 0.7981)).

To find out optimal α, apparently Region I can be ignored. In Region III, the locally optimal point is at $\alpha = 1$, with the outage probability given as

$$P_{\text{out}}(\alpha = 1) = 1 - \exp\left(-\frac{(2^R - 1)\sigma_0^2}{P_s}\sum_{n=1}^{N} d_n^{\eta}\right). \quad (7)$$

Thus, we focus on the locally optimal point in Region II, denoted as $\alpha^\dagger$. However, it is difficult to derive $\alpha^\dagger$ due to the complicated expression of P_{out} in (4). Instead, as the asymptotic outage probability in (5) is a tight upper bound of P_{out} in the high SNR regime (also verified in Fig. 2), an approximation for $\alpha^\dagger$ can be obtained by minimizing the asymptotic outage probability as

$$\min_{\alpha} \Phi(\alpha, N) \triangleq \frac{1}{\bar{\gamma}^N}\sum_{k=0}^{N-1} Q(k,N)F(\alpha;k,N) \quad \text{s.t.} \quad 0 < \alpha < 1. \quad (8)$$

The accuracy of this approximation is to be evaluated in Section VI.

We have the following lemmas.

***Lemma** 1:* The following two inequalities hold for $\alpha \in$

$(0,1)$:

$$G_1(\alpha)\frac{\mathrm{d}^2G_1(\alpha)}{\mathrm{d}\alpha^2} > \left(\frac{\mathrm{d}G_1(\alpha)}{\mathrm{d}\alpha}\right)^2,$$
$$G_2(\alpha)\frac{\mathrm{d}^2G_2(\alpha)}{\mathrm{d}\alpha^2} > \left(\frac{\mathrm{d}G_2(\alpha)}{\mathrm{d}\alpha}\right)^2.$$

Proof: The proof is provided in Appendix A. ■

***Lemma** 2:* We have

$$\lim_{\alpha\to 0^+}\frac{\mathrm{d}\Phi(\alpha,N)}{\mathrm{d}\alpha} = -\infty,$$
$$\lim_{\alpha\to 1^-}\frac{\mathrm{d}\Phi(\alpha,N)}{\mathrm{d}\alpha} = +\infty.$$

Proof: The proof is provided in Appendix B. ■

Now we can prove the following theorem.

***Theorem** 1:* The equation $\frac{\mathrm{d}\Phi(\alpha,N)}{\mathrm{d}\alpha} = 0$ has a unique root α on interval $(0,1)$, and the root is exactly the optimal solution for the optimization problem (8).

Proof: The proof is provided in Appendix C. ■

As shown in the proof of Theorem 1, $\frac{\mathrm{d}^2\Phi(\alpha,N)}{\mathrm{d}\alpha^2} > 0$ holds for $\alpha \in (0,1)$, which means that $\frac{\mathrm{d}\Phi(\alpha,N)}{\mathrm{d}\alpha}$ is monotonically increasing with α. Thus, the root for $\frac{\mathrm{d}\Phi(\alpha,N)}{\mathrm{d}\alpha} = 0$ can be numerically found by simple root-finding algorithms such as a bisection search. This root is used to approximate $\alpha^\dagger$, the locally optimal point of outage probability in Region II of α.

Overall, for the proposed scheme, we select α as either $\alpha^\dagger$ or 1, whichever achieves a lower outage probability, expressed as

$$\alpha^* = \begin{cases} \alpha^\dagger & \text{if } P_{\text{out}}(\alpha=\alpha^\dagger) < P_{\text{out}}(\alpha=1) \\ 1 & \text{otherwise} \end{cases}$$

in which $P_{\text{out}}(\alpha=\alpha^\dagger)$ can be evaluated via (4) with $\alpha=\alpha^\dagger$, and the expression of $P_{\text{out}}(\alpha=1)$ is given in (7).

V. Discussion for Relay Selection Error and Mixed Rayleigh/Rician Fading

In this section, we further discuss the performance of the proposed cooperative multicast scheme and time allocation when there exists a relay selection error and when the channels in the system follow mixed Rayleigh/Rician fading.

A. Worst-Case Analysis When There Exists a Relay Selection Error

In the relay selection of the proposed cooperative multicast scheme, the aim is to select the successful user whose worst relaying channel gain is the highest among the worst relaying channel gains of all successful users. However, since the estimated channel gain information in the Channel Gain Acquisition Portion may not be accurate, it is possible that a different successful user is selected. Here we investigate the worst case, i.e., the case when the successful user whose worst relaying channel gain is the *lowest* is selected as the relay. In other words, the selected successful user is $n^* = \arg\min_{n\in\mathcal{S}} g_n^{\text{wc}}$.

The conditional outage probability $P_{\text{out}|\mathcal{S}=\{n_1,n_2,\ldots,n_k\}}$ is now given as

$$\begin{aligned}
&P_{\text{out}|\mathcal{S}=\{n_1,n_2,\ldots,n_k\}}\\
&=\mathrm{Prob}\Big(\min_{m\in\mathcal{S}}\min_{n\in\overline{\mathcal{S}}} h_{m,n} < \frac{(2^{\frac{R}{1-\alpha}}-1)\sigma_0^2}{P_r}\Big|\mathcal{S}=\{n_1,n_2,\ldots,n_k\}\Big)\\
&=1-\mathrm{Prob}\Big(\min_{m\in\mathcal{S}}\min_{n\in\overline{\mathcal{S}}} h_{m,n} > \frac{(2^{\frac{R}{1-\alpha}}-1)\sigma_0^2}{P_r}\Big|\\
&\qquad \mathcal{S}=\{n_1,n_2,\ldots,n_k\}\Big)\\
&=1-\prod_{i=1}^{k}\prod_{n\in\overline{\mathcal{S}}}\mathrm{Prob}\Big(h_{n_i,n} > \frac{(2^{\frac{R}{1-\alpha}}-1)\sigma_0^2}{P_r}\Big)\\
&=1-\exp\Big(-\frac{(2^{\frac{R}{1-\alpha}}-1)\sigma_0^2}{P_r}\sum_{i=1}^{k}\sum_{n\in\overline{\mathcal{S}}} d_{n_i,n}^{\eta}\Big).
\end{aligned} \tag{9}$$

Then, using also (2) and (3) and the Total Probability Theorem, the outage probability of the proposed scheme for the worst case with relay selection error can be found as

$$\begin{aligned}
&P_{\text{out}} = \prod_{n\in\mathcal{N}}\Big(1-\exp\Big(-\frac{(2^{\frac{R}{\alpha}}-1)\sigma_0^2}{P_s}d_n^{\eta}\Big)\Big)\\
&+\sum_{k=1}^{N-1}\sum_{1\le n_1<n_2<\ldots<n_k\le N}\Big\{\exp\Big(-\frac{(2^{\frac{R}{\alpha}}-1)\sigma_0^2}{P_s}\sum_{i=1}^{k}d_{n_i}^{\eta}\Big)\\
&\times\Big[\prod_{n\in\overline{\mathcal{S}}}\Big(1-\exp\Big(-\frac{(2^{\frac{R}{\alpha}}-1)\sigma_0^2}{P_s}d_n^{\eta}\Big)\Big)\Big]\\
&\times\Big(1-\exp\Big(-\frac{(2^{\frac{R}{1-\alpha}}-1)\sigma_0^2}{P_r}\sum_{i=1}^{k}\sum_{n\in\overline{\mathcal{S}}}d_{n_i,n}^{\eta}\Big)\Big)\Big\}.
\end{aligned} \tag{10}$$

Furthermore, in the high SNR regime, we have

$$P_{\text{out}} \overset{\bar{\gamma}\to\infty}{\simeq} P_{\text{out}}^{\text{asym}} = \begin{cases} 1, & \alpha\to 0\\ \dfrac{2^R-1}{\bar{\gamma}}\displaystyle\sum_{n=1}^{N} d_n^{\eta}, & \alpha\to 1\\ \dfrac{1}{\bar{\gamma}^2}Q'F(\alpha;1,2), & \text{otherwise}\end{cases} \tag{11}$$

where $Q' \triangleq \mu\sum_{n'=1}^{N}\Big(d_{n'}^{\eta}\sum_{n\in\mathcal{N}\setminus\{n'\}}d_{n,n'}^{\eta}\Big)$. This result shows that the diversity order G_d of the proposed scheme is $G_d=0$ if $\alpha\to 0$, $G_d=1$ if $\alpha\to 1$, and $G_d=2$ if α is sufficiently away from 0 and 1.

B. Mixed Rayleigh/Rician Fading

Mixed Rayleigh/Rician fading has been considered as a useful channel model in many practical scenarios, such as in micro/macro cellular multi-hop transmissions [11]. Here, we further discuss the outage performance of the proposed cooperative multicast scheme in mixed Rayleigh/Rician fading. We assume that the links between the source and users experience Rayleigh fading and the links between users experience Rician fading.[2] The probability density function (pdf) of channel gain

[2]The results in Section V-B can be straightforwardly extended to the case when the links between the source and users experience Rician fading and the links between users experience Rayleigh fading.

$h_{n,n'}$ is given by [12, eq. (2.16)]

$$p_{h_{n,n'}}(x) = A_{n,n'} e^{-K_{n,n'}} \exp(-A_{n,n'}x) I_0(2\sqrt{A_{n,n'}K_{n,n'}x}) \tag{12}$$

where $A_{n,n'} = d_{n,n'}^{\eta}(1+K_{n,n'})$, $K_{n,n'}$ is the Rician K-factor of the link between users n and n', and $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind [10, p. 911]. Similar to (1), the conditional outage probability $P_{\text{out}|\mathcal{S}=\{n_1,n_2,...,n_k\}}$ in mixed Rayleigh/Rician fading can be computed as

$$\begin{aligned}
&P_{\text{out}|\mathcal{S}=\{n_1,n_2,...,n_k\}} \\
&= \prod_{i=1}^{k}\Big(1-\prod_{n\in\overline{\mathcal{S}}}\text{Prob}\Big(h_{n_i,n} \geq \frac{(2^{\frac{R}{1-\alpha}}-1)\sigma_0^2}{P_r}\Big)\Big) \\
&= \prod_{i=1}^{k}\Big(1-\prod_{n\in\overline{\mathcal{S}}}\int_{\frac{(2^{\frac{R}{1-\alpha}}-1)\sigma_0^2}{P_r}}^{\infty} A_{n_i,n}e^{-K_{n_i,n}}\exp(-A_{n_i,n}x) \\
&\qquad \times I_0(2\sqrt{A_{n_i,n}K_{n_i,n}x})\mathrm{d}x\Big) \\
&\overset{(d)}{=} \prod_{i=1}^{k}\Big(1-\prod_{n\in\overline{\mathcal{S}}}Q_1\Big(\sqrt{2K_{n_i,n}},\sqrt{\frac{2A_{n_i,n}(2^{\frac{R}{1-\alpha}}-1)\sigma_0^2}{P_r}}\Big)\Big)
\end{aligned}$$

where substitution $x = t^2$ and $\int_b^{\infty} t\exp(-p^2t^2/2)I_0(at)\mathrm{d}t = (1/p^2)\exp(a^2/(2p^2))Q_1(a/p, bp)$ [13, eq. (9)] are used in step (d) and $Q_1(\cdot,\cdot)$ denotes the first-order Marcum Q-function [12, eq. (4.34)]. Then, using (2) and (3) and the Total Probability Theorem, the outage probability is obtained as

$$\begin{aligned}
P_{\text{out}} &= \prod_{n\in\mathcal{N}}\Big(1-\exp\Big(-\frac{(2^{\frac{R}{\alpha}}-1)\sigma_0^2}{P_s}d_n^{\eta}\Big)\Big) \\
&+\sum_{k=1}^{N-1}\sum_{1\leq n_1<n_2<...<n_k\leq N}\Big\{\exp\Big(-\frac{(2^{\frac{R}{\alpha}}-1)\sigma_0^2}{P_s}\sum_{i=1}^{k}d_{n_i}^{\eta}\Big) \\
&\times\Big[\prod_{n\in\overline{\mathcal{S}}}\Big(1-\exp\Big(-\frac{(2^{\frac{R}{\alpha}}-1)\sigma_0^2}{P_s}d_n^{\eta}\Big)\Big)\Big] \\
&\times\prod_{i=1}^{k}\Big(1-\prod_{n\in\overline{\mathcal{S}}}Q_1\Big(\sqrt{2K_{n_i,n}},\sqrt{\frac{2A_{n_i,n}(2^{\frac{R}{1-\alpha}}-1)\sigma_0^2}{P_r}}\Big)\Big)\Big\}.
\end{aligned} \tag{13}$$

Using the series representation of the Marcum Q-function [12, eq. (4.41)] and considering that $\bar{\gamma} = \mu P_r/\sigma_0^2$ goes to infinity, the term $Q_1(\sqrt{2K_{n_i,n}}, \sqrt{2A_{n_i,n}(2^{\frac{R}{1-\alpha}}-1)\sigma_0^2/P_r})$ can be asymptotically expressed as

$$\begin{aligned}
&Q_1\Big(\sqrt{2K_{n_i,n}},\sqrt{\frac{2\mu A_{n_i,n}(2^{\frac{R}{1-\alpha}}-1)}{\bar{\gamma}}}\Big) \\
&\overset{\bar{\gamma}\to\infty}{\simeq} 1-\exp(-K_{n_i,n})\sqrt{\frac{\mu A_{n_i,n}(2^{\frac{R}{1-\alpha}}-1)}{K_{n_i,n}\bar{\gamma}}} \\
&\qquad\times I_1\Big(2\sqrt{\frac{\mu A_{n_i,n}K_{n_i,n}(2^{\frac{R}{1-\alpha}}-1)}{\bar{\gamma}}}\Big) \\
&\overset{(e)}{\simeq} 1-\exp(-K_{n_i,n})\frac{\mu A_{n_i,n}(2^{\frac{R}{1-\alpha}}-1)}{\bar{\gamma}}
\end{aligned} \tag{14}$$

where $I_v(\cdot)$ means the vth-order modified Bessel function of the first kind [10, pp. 911], and step (e) uses $I_v(z) \overset{z\to 0}{\simeq} (z/2)^v/v!$ [14, eq. (9.6.7)]. Using (14), the high SNR asymptotic expression for outage probability in (13) can be obtained as

$$P_{\text{out}} \overset{\bar{\gamma}\to\infty}{\simeq} P_{\text{out}}^{\text{asym}} = \begin{cases} 1, & \alpha\to 0 \\ \dfrac{2^R-1}{\bar{\gamma}}\displaystyle\sum_{n=1}^{N}d_n^{\eta}, & \alpha\to 1 \\ \dfrac{1}{\bar{\gamma}^N}\displaystyle\sum_{k=0}^{N-1}\hat{Q}(k,N)F(\alpha;k,N), & \text{otherwise} \end{cases} \tag{15}$$

with $\hat{Q}(k,N)$ being defined as

$$\hat{Q}(k,N) \triangleq \begin{cases} \displaystyle\sum_{1\leq n_1<...<n_k\leq N}\Big\{\Big(\prod_{i=1}^{k}\Big(\sum_{n\in\overline{\mathcal{S}}}A_{n_i,n}\exp(-K_{n_i,n})\Big)\Big) \\ \qquad\times\Big(\displaystyle\prod_{n\in\overline{\mathcal{S}}}d_n^{\eta}\Big)\mu^k\Big\}, \quad k=1,...,N-1 \\ \displaystyle\prod_{n=1}^{N}d_n^{\eta}, \quad k=0. \end{cases} \tag{16}$$

From (15), it can be seen that the proposed cooperative multicast scheme in mixed Rayleigh/Rician fading can still achieve a full diversity order when α is sufficiently away from 0 and 1. Furthermore, when comparing (15) with the asymptotic outage probability in (5), the only difference is that the term $Q(k,N)$ in (5) is changed to $\hat{Q}(k,N)$ in (15). Replacing $Q(k,N)$ in (8) with $\hat{Q}(k,N)$, it can be seen that Lemmas 1 and 2 as well as Theorem 1 are still valid when the mixed Rayleigh/Rician fading is considered. Therefore, the same method as in Section IV can be used to find the optimal time allocation factor α^* in mixed Rayleigh/Rician fading.

VI. Performance Evaluation

In this section, we evaluate the performance of the proposed cooperative multicast scheme. System parameters are $R = 1$bps/Hz, the noise power is $\sigma_0^2 = 1$, the path loss exponent is $\eta = 2.6$, and $\mu = P_s/P_r = 1$.

First, we verify our proposed time allocation method for the i.n.i.d. fading scenario. Source s is located at the coordinate $(0,0)$ and all users are randomly distributed inside a circle centered at (0,10) and with radius of 5. When the system SNR is $\bar{\gamma} = 30$ dB, Fig. 3 shows the exact outage probability (calculated from (4)) when α varies. The derived time allocation factor α^* is also shown in the figure, together with the actual optimal time allocation factor obtained by an exhaustive search. The derived α^* is very close to the actual optimal point, thus verifying the accuracy of our approximation presented in Section IV.

In our approximation, since the asymptotic outage probability is used, it is accurate for high SNR values. Therefore, it is necessary to evaluate the extreme case when the SNR is low and the distance between users is longer than the distance from the source to users. Consider that source s is located at $(0,0)$, and there are two users, located at $(2,0)$ and $(-2,0)$ (i.e., the source is in the middle point of the two users). A low SNR is considered as $\bar{\gamma} = 20$ dB. Since $\bar{\gamma}$ is the ratio of the source transmit power (rather than received power at users) to noise variance, 20 dB is considered a low SNR value. The numerical results are shown in Fig. 4. As a comparison, the case with $\bar{\gamma} = 30$ dB is also shown. When $\bar{\gamma} = 30$ dB, the

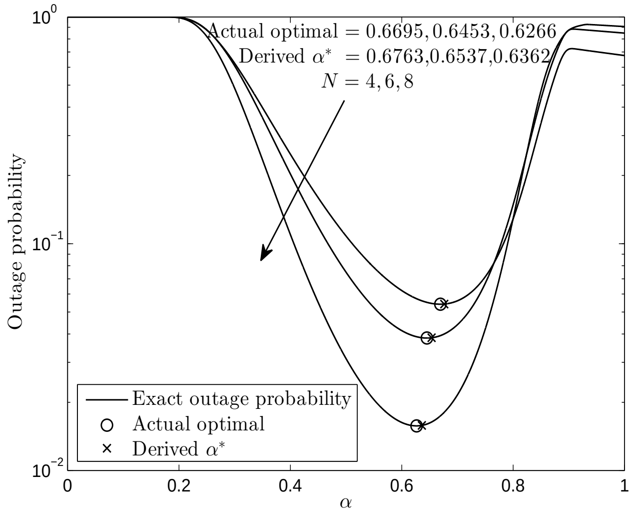

Fig. 3. Comparison of the derived time allocation factor α^* with the actual optimal time allocation factor when the system SNR is $\bar{\gamma} = 30$ dB.

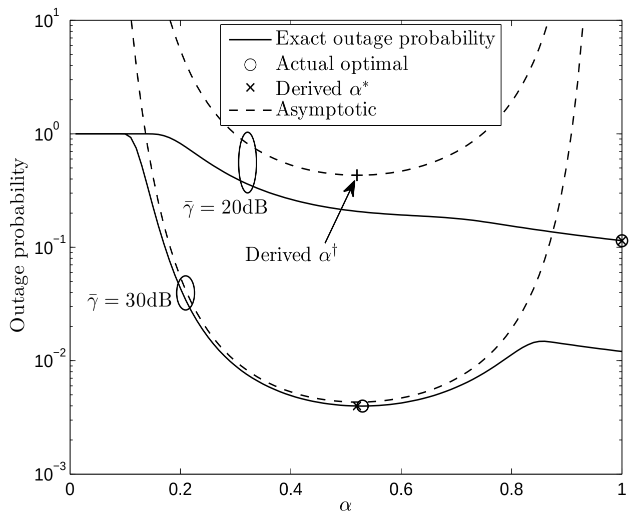

Fig. 4. Comparison of the derived time allocation factor α^* with the actual optimal time allocation factor when there are only two users. Here the source is in the middle point of the two users, and the system SNR is $\bar{\gamma} = 20$ dB or 30 dB.

derived $\alpha^\dagger$ (approximate locally optimal point in Region II of α), which is also the derived α^*, is still close to the actual optimal point. When $\bar{\gamma} = 20$ dB, the derived $\alpha^\dagger$ is not the locally optimal point of outage probability in Region II of α. However, since the locally optimal point in Region III of α (i.e., $\alpha = 1$) achieves a lower outage probability than the derived $\alpha^\dagger$ does, our proposed scheme selects $\alpha^* = 1$, which is the actual overall optimal point. Therefore, although the derived $\alpha^\dagger$ is not accurate in the extreme case, our proposed method can still select the actual overall optimal point. The above observations verify the effectiveness of our proposed time allocation method for the high SNR cases as well as for the extreme cases (i.e., with low SNR, a fewer number of users $N = 2$, and a longer distance between users than the distance from the source to users).

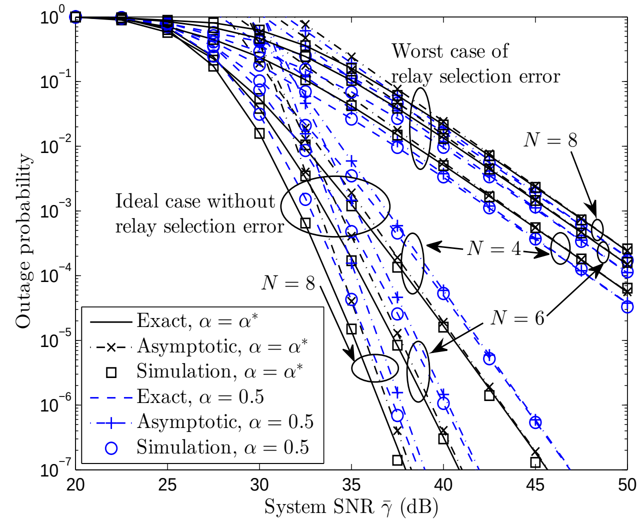

Fig. 5. Outage probability of the proposed scheme with the proposed time allocation and fixed time allocation ($\alpha = 0.5$) when the channels follow Rayleigh fading.

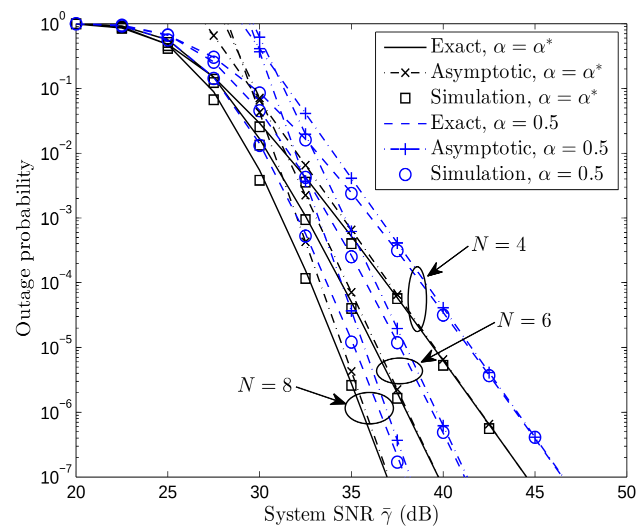

Fig. 6. Outage probability of the proposed scheme with the proposed time allocation and fixed time allocation ($\alpha = 0.5$) in mixed Rayleigh/Rician fading channels, where the Rician K-factor is $K = 5$ dB.

Now we verify our theoretical results by simulation. For the proposed scheme, the numerically calculated exact outage probability (based on (4) and (13)) and the asymptotic outage probability (based on (5) and (15)) are shown in Fig. 5 for the case with Rayleigh fading and in Fig. 6 for the case with mixed Rayleigh/Rician fading. In the two figures, the results for the proposed time allocation and fixed time allocation ($\alpha = 0.5$) are shown, where the number of users is $N = 4, 6, 8$. The source s is located at $(0, 0)$ and all users are randomly distributed inside a circle centered at (0,10) and with radius of 5. It is clear from the figures that our derived outage probability perfectly matches the simulations and the derived asymptotic outage probability is a tight bound for the exact outage probability in the high SNR regime ($\bar{\gamma} \geq 35$ dB), which validates our theoretical analysis. For the proposed scheme, as the number of users increases from 4 to 8, the magnitude of the slope of the outage probability curves also increases from 4 to 8, which verifies that the proposed scheme can achieve a full diversity order. Furthermore, the proposed scheme with the derived α^* has lower outage probabilities than the scheme with fixed time allocation $\alpha = 0.5$, which demonstrates the efficiency of the proposed time allocation method.

For the worst case with relay selection error (i.e., the case when the successful user whose worst relaying channel gain is the lowest is selected as the relay), Fig. 5 also shows the outage performance when the proposed time allocation and fixed time allocation ($\alpha = 0.5$) are used. Here all channels follow Rayleigh fading. The magnitude of the slopes of the outage probability curves are equal to 2, which verifies that a diversity order of two is achieved in the considered worst case. Recall that the proposed time allocation is to minimize the outage probability in the *ideal case* without relay selection error. Therefore, for the considered worst case with relay selection error, the proposed time allocation does not achieve a better performance (in terms of a lower outage probability) than the fixed time allocation, as shown in Fig. 5. Some insight about this is provided below. For the considered worst case, based on (11), we can minimize the asymptotic outage probability $P_{\text{out}}^{\text{asym}} = (1/\bar{\gamma}^2)Q'F(\alpha; 1, 2)$ in Region II of α, i.e., when α is sufficiently away from 0 and 1, by using a similar method to that in Section IV. From (17) in Appendix B, it is known that

$$\frac{\mathrm{d}F(\alpha; 1, 2)}{\mathrm{d}\alpha} = R\ln 2\left(\frac{(2^{\frac{R}{\alpha}}-1)2^{\frac{R}{1-\alpha}}}{(1-\alpha)^2} - \frac{(2^{\frac{R}{1-\alpha}}-1)2^{\frac{R}{\alpha}}}{\alpha^2}\right).$$

Thus, the approximation of the locally optimal point in Region II of α is $\alpha^\dagger = 0.5$ due to

$$\left.\frac{\mathrm{d}F(\alpha; 1, 2)}{\mathrm{d}\alpha}\right|_{\alpha=0.5} = 0.$$

Therefore, it can be concluded that the fixed time allocation ($\alpha = 0.5$) minimizes the outage probability in the considered worst case.

Next we perform comparison with the following existing schemes: the D-CM scheme [6], the GA-CM scheme [6], and the BRS-CM scheme [3]. Since these schemes consider i.i.d. channels, we use i.i.d. Rayleigh fading channels for these schemes and our scheme. For the user-aided cooperative schemes (i.e., the proposed scheme and the D-CM scheme), we set $d_n = 10$ and $d_{n,n'} = 3$ for $n, n' \in \mathcal{N}, n \neq n'$. For the relay-aided cooperative schemes (i.e., the GA-CM scheme and the BRS-CM scheme), we have $d_{SR} = d_{SD} = 10$ and $d_{RD} = 3$, where d_{SR}, d_{SD} and d_{RD} denote the distance of the source-relay links, source-destination links, and relay-destination links, respectively, and the number of relays is 8.

Fig. 7 shows the outage performance of all aforementioned schemes when the number of users increases from $N = 2$ to $N = 30$. It is observed that the outage probabilities of the GA-CM and BRS-CM schemes increase with N. This is because a larger number of users means that the probability that not all users can be eventually successful is larger. For the D-CM scheme, its outage probability decreases with N when $N \leq 10$. When N further increases, the outage probability converges. Different from these schemes, the outage probability of the proposed scheme significantly decreases with N,

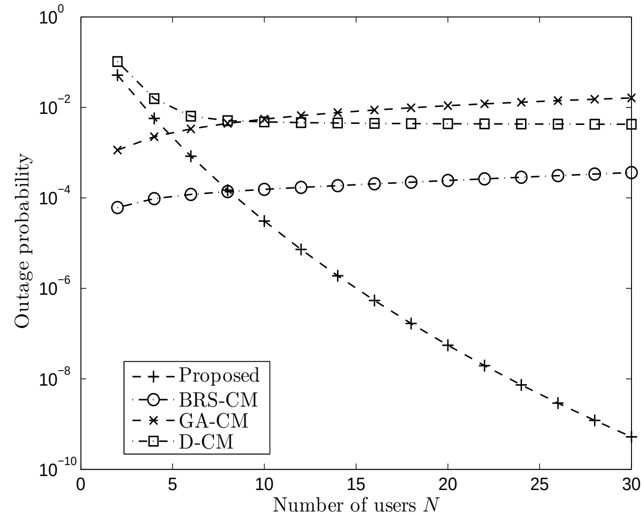

Fig. 7. Outage probability of the proposed scheme, the D-CM scheme, the GA-CM scheme, and the BRS-CM scheme, with $\bar{\gamma} = 35$ dB.

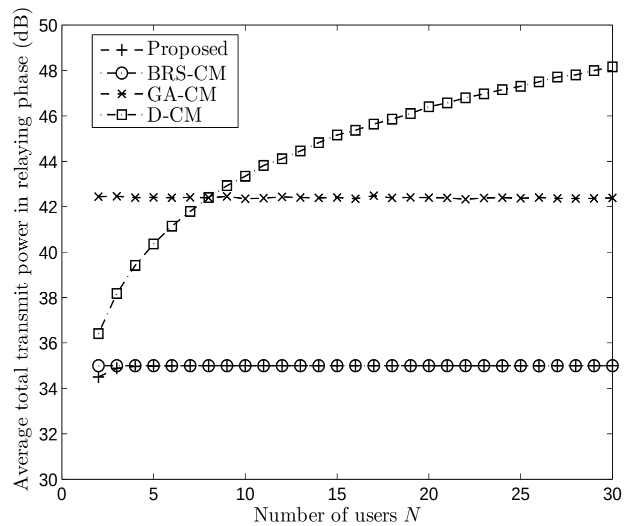

Fig. 8. Power consumption in the relaying phase of the proposed scheme, the D-CM scheme, the GA-CM scheme, and the BRS-CM scheme, with $\bar{\gamma} = 35$ dB.

since the diversity order of the proposed scheme is N.

Fig. 8 shows the power consumption in the relaying phase of the schemes when the number of users increases from $N = 2$ to $N = 30$. It is seen that the proposed scheme and the BRS-CM scheme have almost the same power consumption in the relaying phase, since only one user or one dedicated relay is selected to forward information. On the other hand, the power consumption of the D-CM and GA-CM schemes is much higher than that of the proposed scheme and the BRS-scheme. Furthermore, the power consumption of the D-CM scheme increases with the number of users, because when there are a larger number of users, it is likely that more users are successful and thus, help forward their received message. For the GA-CM scheme, since the number of dedicated relays is fixed to 8 in our simulation, its power consumption remains unchanged with the number of users.

VII. Conclusion

In this paper, we propose a cooperative multicast scheme where a successful user is selected to further relay its received message to unsuccessful users. A procedure is also proposed to obtain the required CSI. The outage performance of the proposed scheme is derived in i.n.i.d channels, where both exact and asymptotic outage probabilities are derived in a closed form. The analytical result indicates that the proposed scheme can achieve a full diversity order. We also investigate the optimal time allocation, and derive an approximately optimal time allocation factor.

Appendix A
Proof of Lemma 1

The first and second order derivatives of $G_1(\alpha)$ are

$$\frac{\mathrm{d}G_1(\alpha)}{\mathrm{d}\alpha} = -\frac{R(\ln 2)2^{\frac{R}{\alpha}}}{\alpha^2},$$
$$\frac{\mathrm{d}^2G_1(\alpha)}{\mathrm{d}\alpha^2} = \frac{R(\ln 2)2^{\frac{R}{\alpha}}}{\alpha^4}(R\ln 2 + 2\alpha).$$

Letting $x = R/\alpha$, the inequality

$$G_1(\alpha)\frac{\mathrm{d}^2G_1(\alpha)}{\mathrm{d}\alpha^2} > \left(\frac{\mathrm{d}G_1(\alpha)}{\mathrm{d}\alpha}\right)^2$$

with $\alpha \in (0,1)$ is equivalent to the inequality $2^x - 1 > (0.5\ln 2)x$ with $x \in (R,+\infty)$. For notational convenience, we define $I_1(x) = 2^x - 1$ and $I_2(x) = (0.5\ln 2)x$, whose first order derivatives are readily obtained as

$$\frac{\mathrm{d}I_1(x)}{\mathrm{d}x} = 2^x \ln 2,$$
$$\frac{\mathrm{d}I_2(x)}{\mathrm{d}x} = 0.5\ln 2.$$

It is known that $I_1(0) = I_2(0) = 0$ and

$$\frac{\mathrm{d}I_1(x)}{\mathrm{d}x} > \frac{\mathrm{d}I_2(x)}{\mathrm{d}x} > 0$$

for $x > -1$. Therefore, the inequality $2^x - 1 > (0.5\ln 2)x$ holds for $x \in (R,+\infty)$, which means that the inequality

$$G_1(\alpha)\frac{\mathrm{d}^2G_1(\alpha)}{\mathrm{d}\alpha^2} > \left(\frac{\mathrm{d}G_1(\alpha)}{\mathrm{d}\alpha}\right)^2$$

holds for $\alpha \in (0,1)$. Similarly, the first and second order derivatives of $G_2(\alpha)$ are

$$\frac{\mathrm{d}G_2(\alpha)}{\mathrm{d}\alpha} = \frac{R(\ln 2)2^{\frac{R}{1-\alpha}}}{(1-\alpha)^2},$$
$$\frac{\mathrm{d}^2G_2(\alpha)}{\mathrm{d}\alpha^2} = \frac{R(\ln 2)2^{\frac{R}{1-\alpha}}}{(1-\alpha)^4}(R\ln 2 + 2(1-\alpha)).$$

By letting $x = R/(1-\alpha)$, the inequality

$$G_2(\alpha)\frac{\mathrm{d}^2G_2(\alpha)}{\mathrm{d}\alpha^2} > \left(\frac{\mathrm{d}G_2(\alpha)}{\mathrm{d}\alpha}\right)^2$$

for $\alpha \in (0,1)$ is also equivalent to the inequality $2^x - 1 > (0.5\ln 2)x$ for $x \in (R,+\infty)$. Then, with the same procedure, it can also be proved that the inequality

$$G_2(\alpha)\frac{\mathrm{d}^2G_2(\alpha)}{\mathrm{d}\alpha^2} > \left(\frac{\mathrm{d}G_2(\alpha)}{\mathrm{d}\alpha}\right)^2$$

holds for $\alpha \in (0,1)$.

Appendix B
Proof of Lemma 2

The first order derivative of $\Phi(\alpha,N)$ is

$$\frac{\mathrm{d}\Phi(\alpha,N)}{\mathrm{d}\alpha} = \frac{1}{\bar{\gamma}^N}\sum_{k=0}^{N-1} Q(k,N)\frac{\mathrm{d}F(\alpha;k,N)}{\mathrm{d}\alpha}$$

with

$$\begin{aligned}
&\frac{\mathrm{d}F(\alpha;k,N)}{\mathrm{d}\alpha}\\
&= [G_1(\alpha)]^{N-k-1}[G_2(\alpha)]^{k-1}\\
&\quad\times\left[kG_1(\alpha)\frac{\mathrm{d}G_2(\alpha)}{\mathrm{d}\alpha} + (N-k)G_2(\alpha)\frac{\mathrm{d}G_1(\alpha)}{\mathrm{d}\alpha}\right]\\
&= (2^{\frac{R}{\alpha}}-1)^{N-k-1}(2^{\frac{R}{1-\alpha}}-1)^{k-1}\Big[k(2^{\frac{R}{\alpha}}-1)\\
&\quad\times\frac{R2^{\frac{R}{1-\alpha}}\ln 2}{(1-\alpha)^2} - (N-k)(2^{\frac{R}{1-\alpha}}-1)\frac{R2^{\frac{R}{\alpha}}\ln 2}{\alpha^2}\Big].
\end{aligned} \tag{17}$$

When $\alpha \to 0^+$, for $k = 0,\ldots,N-1$, following (17) with some algebraic manipulations, we have

$$\begin{aligned}
&\lim_{\alpha\to 0^+}\frac{\mathrm{d}F(\alpha;k,N)}{\mathrm{d}\alpha}\\
&= (\ln 2)R(2^R-1)^{k-1}\lim_{\alpha\to 0^+}\Big(\frac{2^{\frac{R}{\alpha}}}{(1-\alpha)^2/(2^{\frac{R}{\alpha}}-1)^{N-k-1}}\\
&\quad\times[k(1-2^{-\frac{R}{\alpha}})2^{\frac{R}{1-\alpha}} - (N-k)(\alpha^{-1}-1)^2(2^{\frac{R}{1-\alpha}}-1)]\Big)\\
&= -\infty.
\end{aligned}$$

Furthermore, since $Q(k,N) > 0$ as shown in (6), we have

$$\begin{aligned}
\lim_{\alpha\to 0^+}\frac{\mathrm{d}\Phi(\alpha,N)}{\mathrm{d}\alpha} &= \lim_{\alpha\to 0^+}\frac{1}{\bar{\gamma}^N}\sum_{k=0}^{N-1}Q(k,N)\frac{\mathrm{d}F(\alpha;k,N)}{\mathrm{d}\alpha}\\
&= -\infty.
\end{aligned}$$

Similarly, when α is close to 1, following (17) with some algebraic manipulations, we have

$$\begin{aligned}
&\lim_{\alpha\to 1^-}\frac{\mathrm{d}F(\alpha;k,N)}{\mathrm{d}\alpha}\\
&= (\ln 2)R(2^R-1)^{N-k-1}\\
&\quad\times\lim_{\alpha\to 1^-}\frac{2^{\frac{R}{1-\alpha}}[k(\frac{\alpha}{1-\alpha})^2(2^{\frac{R}{\alpha}}-1) - (N-k)(1-2^{-\frac{R}{1-\alpha}})2^{\frac{R}{\alpha}}]}{\alpha^2/(2^{\frac{R}{1-\alpha}}-1)^{k-1}}\\
&= \begin{cases}-(\ln 2)RN2^R(2^R-1)^{N-1}, & k=0,\\ +\infty & k = 1,2,\ldots,N-1.\end{cases}
\end{aligned} \tag{18}$$

Thus, we have

$$\lim_{\alpha\to 1^-}\frac{\mathrm{d}\Phi(\alpha,N)}{\mathrm{d}\alpha} = \lim_{\alpha\to 1^-}\frac{1}{\bar{\gamma}^N}\sum_{k=0}^{N-1}Q(k,N)\frac{\mathrm{d}F(\alpha;k,N)}{\mathrm{d}\alpha} = +\infty.$$

APPENDIX C
PROOF OF THEOREM 1

The second-order derivative of $F(\alpha; k, N)$ can be expressed as

$$\begin{aligned}
&\frac{\mathrm{d}^2}{\mathrm{d}\alpha^2} F(\alpha; k, N) \\
&= (G_1(\alpha))^{N-k-2} (G_2(\alpha))^{k-2} \\
&\quad \times \Bigg[(N-k)(N-k-1)[G_2(\alpha)]^2 \left(\frac{\mathrm{d}G_1(\alpha)}{\mathrm{d}\alpha} \right)^2 \\
&\quad + (N-k) G_1(\alpha) [G_2(\alpha)]^2 \frac{\mathrm{d}^2 G_1(\alpha)}{\mathrm{d}\alpha^2} \\
&\quad + 2k(N-k) G_1(\alpha) G_2(\alpha) \frac{\mathrm{d}G_1(\alpha)}{\mathrm{d}\alpha} \frac{\mathrm{d}G_2(\alpha)}{\mathrm{d}\alpha} \\
&\quad + k[G_1(\alpha)]^2 G_2(\alpha) \frac{\mathrm{d}^2 G_2(\alpha)}{\mathrm{d}\alpha^2} \\
&\quad + k(k-1)[G_1(\alpha)]^2 \left(\frac{\mathrm{d}G_2(\alpha)}{\mathrm{d}\alpha} \right)^2 \Bigg] \\
&> (G_1(\alpha))^{N-k-2} (G_2(\alpha))^{k-2} \Big[k G_1(\alpha) \frac{\mathrm{d}G_2(\alpha)}{\mathrm{d}\alpha} \\
&\quad + (N-k) G_2(\alpha) \frac{\mathrm{d}G_1(\alpha)}{\mathrm{d}\alpha} \Big]^2
\end{aligned} \tag{19}$$

where the inequality follows by applying Lemma 1, i.e., in the big square brackets of (19) we use

$$G_1(\alpha) \frac{\mathrm{d}^2 G_1(\alpha)}{\mathrm{d}\alpha^2} > \left(\frac{\mathrm{d}G_1(\alpha)}{\mathrm{d}\alpha} \right)^2$$

in the second summation term, and use

$$G_2(\alpha) \frac{\mathrm{d}^2 G_2(\alpha)}{\mathrm{d}\alpha^2} > \left(\frac{\mathrm{d}G_2(\alpha)}{\mathrm{d}\alpha} \right)^2$$

in the fourth summation term.

Since $G_1(\alpha) > 0$ and $G_2(\alpha) > 0$, from (19) we have

$$\frac{\mathrm{d}^2 F(\alpha; k, N)}{\mathrm{d}\alpha^2} > 0$$

which means that $F(\alpha; k, N)$ is strictly convex. Moreover, it is known from (6) that $Q(k, N) > 0$, $k = 0, 1, \ldots, N-1$. Thus, it can be seen that

$$\frac{\mathrm{d}^2 \Phi(\alpha, N)}{\mathrm{d}\alpha^2} = \frac{1}{\bar{\gamma}^N} \sum_{k=0}^{N-1} Q(k, N) \frac{\mathrm{d}^2 F(\alpha; k, N)}{\mathrm{d}\alpha^2} > 0$$

holds for $\alpha \in (0, 1)$. So $\Phi(\alpha, N)$ is strictly convex for $\alpha \in (0, 1)$, which means that the equation

$$\frac{\mathrm{d}\Phi(\alpha, N)}{\mathrm{d}\alpha} = 0$$

has at most one root α within the interval $(0, 1)$. Together with the results in Lemma 2, it can be seen that the equation $\frac{\mathrm{d}\Phi(\alpha,N)}{\mathrm{d}\alpha} = 0$ has a unique root α within $(0, 1)$, which is exactly the optimal solution for the optimization problem (8).

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